Structural Analysis of Historical Constructions, New Delhi 2006
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Survey, Analysis and Structural Modelling of Ancient Masonry Building: the Case of the “Insula del Centenario [IX, 8]” at Pompeii

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ABSTRACT: This work deals with the structural analysis of the masonries of the “Insula del Centenario” in Pompeii, with the purpose of recovering and consolidating the existing buildings, as well as proposing new hypothesis for the covers with the outlook of a probable reopening to visitors. For the structural analysis, the model of a equivalent continuum of the masonries in linear elasticity has been studied with various techniques available in literature and applied to this particular case. Then a 3D model using finite element has been realized for the structural analysis, done for several static and dynamics load conditions. The analysis has provided interesting results for future recovering solutions, and in particular for the realization of new covers or the substitution of the covers now available. With the same purpose a prototype solution using light materials has been realized to cover part of the main atrium.

1 ANCIENT MASONRY AND STRUCTURAL ANALYSIS

Dealing with masonry structures, especially if ancient, the structural analysis becomes extremely complex due to the masonry texture, for the irregularity in dimensions and the disposition of the stones, for the lack of homogeneity in the distribution of the mortar joint, for the uncertain mechanics parameters of the materials and for the high dispersion of their value and, finally, for the uncertainty on the behaviour of the whole structure.

In fact, the masonry buildings are difficult to model with the common rules used in structural engineering and moreover, for the ancient and monumental historical buildings, the uncertainties on the response of the entire organism are worsen by the fact that often the buildings are in a different static configuration from the designed one, turning out in a ‘unicum’ scenario unfeasible to treat with standard techniques.

The first step when there is the need of the recovering or the consolidation of historical buildings is the study of the ancient construction techniques and the survey of the existing building, with particular attention for the variations of the structural configuration, for the damage and cracks and for the degree of actual reliability.

The role of structural engineering arises in the framework of a specific study for the consolidation which needs knowledge related to the structural analysis as well as to the history and the materials. For these reasons it is important to improve the technique of diagnostic of the conservative being and the mechanical characterization of the material and structural analysis.

2 SURVEY OF THE ‘INSULA DEL CENTENARIO’

The ‘Insula del Centenario’ is not totally excavated and so a complete plan does not exist; for the structural analysis the data of the survey of the building done by the archaeological team has been linked to the topographic survey of the plane of the pavement. From the analysis of the masonry in situ, the following main classification can be done:

- masonry called ‘opera a telaio’ (opus africanum): based upon a main structure ‘a telaio’ made by big regular block (limestone of Sarno, stone or foam of lava) posed vertically and horizontally with the interposed panels filled with ‘opera incerta’ (see below) o with little block of limestone of Sarno, of variable dimensions originally posed without mortar and only in subsequent period with mortar;
- masonry called ‘opera incerta’ (opus incertum) which is made by three layers: the external layers and the interior core. This is the most popular technique. Particular attention was posed on the location of the stones (limestone of Sarno, bricks or volcanic stones) avoiding the vertical mortar joints with the purpose to avoid cracks (Giuliani 1990);
- masonry called ‘opera vittata’ (opus vittatum mixtum), similar to the ‘opera incerta’, together also called ‘a sacco’, used above all for the toothing, and for subsequent modification or consolidation. The masonry is constituted by little regular stone block (usually of tuff) located in horizontal layer similar in the shape to the bandages (the ‘vittae’) and often alternated with layer of masonry brick with location very different from masonry to masonry;
- masonry called ‘opera laterizia’ (opus latericium): constituted of only brick, seldom seen in the Insula, with the almost unique purpose of consolidation and recovery of the damaged masonries.

3 MECHANICAL MODELS OF THE MASONRIES OF THE ‘INSULA DEL CENTENARIO’

The results obtained with the guideline proposed by Italian law (D.M. 20/11/87) are reported.

Two different approaches to the same mechanical model are later proposed: the first one related to the ‘opera a telaio’ the second one related to the ‘opera incerta’ and ‘opera vittata’.

3.1 Mechanical model in accordance with the guideline of the Italian law

The analysis of the masonry panel in Pompeii has highlighted a prevalence of the limestone of Sarno (about 70%). The elastic modulus (E), the Poisson coefficient (ν) and the compressive strength (f_b) of the limestone of Sarno have been recovered in literature (Pegon *et al.* 2001, Badalà and Cuomo 1996, Colombo 1998, Desio 1973, Leger 1875):

$E = 12000$ MPa; $\nu = 0.2$; $f_b = 12$ MPa.

The compressive strength of the pozzolanic mortar is assumed 7.8 MPa (Leger 1875) and the presence of plaster is considered, excluding any structural role, as a superficial load with a specific weight equal to 15 KN/m^3 (Giuliani 1990, Leger 1875). From these data the mechanical characterization of the masonry made by limestone of Sarno and genuine pozzolanic mortar can be obtained, after the homogenization suggested by the Italian law (D.M. 20/11/87):

$f_k = 4.46$ MPa $E = 4460$ MPa $G = 1784$ MPa $\nu = 0.25$

where f_k = compressive characteristic strength and G = shear modulus.

3.2 The model of the masonry called ‘a telaio’

The model is based upon an algorithm of compatible constitutive identification (Salerno and De Felice 1998, Trovalusci and Masiani 1999, Formica and Casciaro 2000).

The masonry panel is made by two classes of elements: blocks and mortar joints with the following hypotheses:

· the blocks are considered infinitely stiff,
· the deformation is located in the mortar joints.

The 20% of the global masonry is constituted by mortar as it appears by archaeological survey. The assembly of the blocks and joints (with a periodic location called ‘tassellatura’) is described as a planar lagrangian system in which some points (whose components describe the degree of freedom of the system) are related through a discrete number of springs where the elastic energy is located. The kinematical variables are related to the blocks, while the stress variables are related to mortar joints. The mortar joint is described by three springs reacting respectively to vertical, horizontal and rotational loading. The system blocks-joints (represented by a selected elementary volume of reference: VER), is called fine because in it there are defined: the kinematics, the deformations, the balance equations, and the constitutive law. The continuum equivalent to the system block-joints is called coarse.

The procedure of identification is an algorithm which allows to derive information from the fine model to the coarse model thinking to a correspondence between the two models. In particular the correspondence is represented by the principle of virtual works.

Applying to a periodic masonry (obtained through symmetric transformations of VER) and assuming that all the joints have the same geometric characteristics, the same material parameters and the same stiffness, the definition of the elastic matrix is achieved.

The parameters used for the modelling are: the mechanical characterization of the mortar and the geometric characterization of the whole masonry. The different building techniques are taken into account without loosing the global mechanics lack of homogeneity of the masonry, due above all to the geometric lack of homogeneity in the layers of block.

The mechanical characterization of blocks and joints used in the model is reported in the Table 1 (where σ_c = compressive strength and γ = specific weight):

Table 1 : Mechanical characterization of blocks and joints

Materials	E MPa	ν	σ_c MPa	γ KN/m^3
Hydraulic mortar	4000	0.25	6	14
Limestone of Sarno	12000	0.20	12	15

The identification technique previously described has been applied to the interposed panels among the blocks of limestone of Sarno (see Fig. 1, where all units are in centimetres).

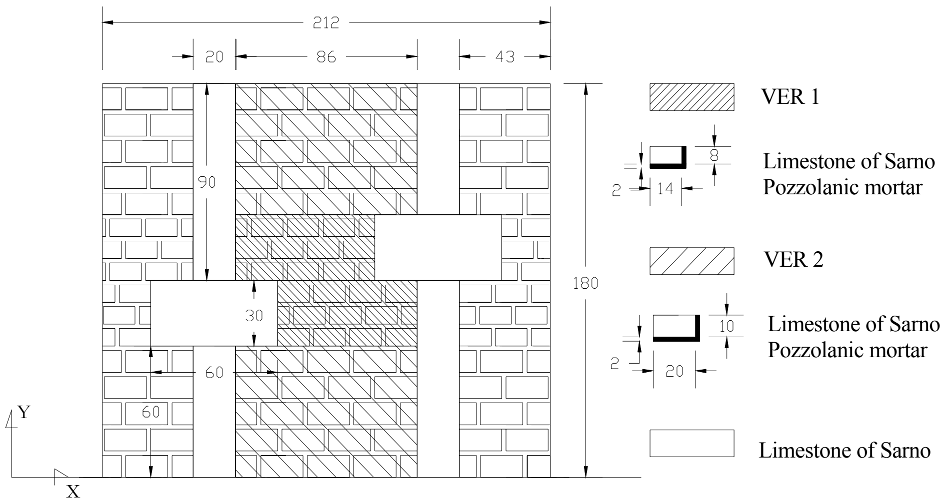

Figure 1 **:** Volume of reference for the masonry called ‘opera a telaio’.

The hypotheses that the mean width of the joint is the same for the two different joint classes (head and bed) and that the two classes share the same material constants, are considered. Thinking to a masonry *opus* made perfectly, the shifting between blocks is 50%.

Two different models (‘moduli’) have been selected (Fig. 1) and lighted two homogeneous zones with similar geometric and mechanics characteristic. In this way the periodic disposition of the two elementary references volumes : VER1 and VER2.

The data obtained through the identification of the two VER are reported in the Table 2:

Table 2 : Continuum mechanical characterization

Continuum	E_x MPa	E_y MPa	G_{xy} MPa	ν_{xy}
VER 1	10900	14655	14925	0.25
VER 2	14750	18437	22000	0.25

The new model obtained is again a continuum model with lack of homogeneity, constituted by two continuum systems (VER1 and VER2) integrated among the limestone blocks.

A finite element micromodelling has been realized with a planar mesh of the entire panel.

The numerical specimen has been subjected to three tests: a compression test in direction parallel to the head joint; a compression test in direction parallel to the bed joint; and a shear test. From the tests some interesting data about the compression, the lateral expansion and the sleeping are derived to evaluate elastic parameters (see Table 3).

Table 3 : Elastic parameters of masonry ‘a telaio’

Masonry	E_x MPa	E_y MPa	G_{xy} MPa	ν_{xy}
‘Opera a telaio’	5874	24567	4974	0.19

3.3 An alternative mechanical characterization of masonry called ‘opera incerta’ and ‘opera vittata’

A technique of homogenization has been applied to the ‘opera incerta’ and ‘opera vittata’ derived from the formulation of P. B. Lourenço (Lourenço 1996a,b). The structure is thought as a set of layers in the direction of the thickness, with external and internal layer (masonry called ‘a sacco’). The masonry is considered constituted by external layers and a core, it is necessary to act with a homogenization in several steps which take into account all the discontinuities among stones and joints as well as among layers.

The mechanical characterization of the core, considered isotropic and homogeneous, is assumed like in (Lamprecht 1995) with the following values:

density $\gamma_n = 2.58$ Kg/cm^3 ; breakdown compression stress $\sigma_m = 35$ MPa

$E = 18000$ MPa $G = (0.4\ E) = 720$ MPa $\nu = 0.25$

3.4 The model of the masonry ‘opera incerta’

The ‘opera incerta’ is mostly composed by external layers made by limestone of Sarno and pozzolanic mortar, and a ‘caementa’-made core. It has been already talked about limestone of Sarno and pozzolanic mortar. About the external layer’s specific weight, it results $\gamma_p = 12.4$ KN/m^3, considering about 20% of mortar.

Considering the three layers of the whole masonry in ‘opera incerta’ (external layer-interior core-external layer) of the same thickness, it results that the specific weight of the entire wall is $\gamma_o = 16.9$ KN/m^3.

Using for the core the previous data and for the external layers those ones defined by the guidelines in the Italian law, for the material of the ‘opera incerta’ in its unity (stratified material made by three layers and composed by two different materials with a recurrence to refer to the whole structure) from the numerical elaboration of the homogenisation, it has been obtained (see Table 4, where x and y are the axes in the plane of the layer and z is the axis normal to this):

Table 4 : Elastic parameters of masonry ‘opera incerta’

Masonry	E_x MPa	E_y MPa	E_z MPa	G_{xy} MPa	G_{xz} MPa	G_{yz} MPa	ν_{xy}	ν_{xz}	ν_{yz}
‘Opera incerta’	8973	8973	6307	3589	3589	2381	0.25	0.17	0.17

3.5 The model of the masonry ‘opera vittata’

The external layer (Fig. 2) of the masonry in ‘opera vittata’ is realized by a series of regular courses (made alternating one or more courses of limestones of Sarno and several brick-made courses) and the ‘caementa’-made core.

From literature, the mechanical brick features are: $E = 7000$ Mpa; $\nu = 0.1$; $\sigma_c = 8$ MPa.

Because of the particular shape of blocks and bricks of the external layer, it possible to assume for the thickness of the external layer the value of about 15 centimetres.

Considered the percentage and the specific weights of the constituent materials, for the whole external layer it is obtained the specific weight: $\gamma_p = 13.8$ kN/m^3.

After defining the thickness and the specific weight of the external layer and remembering the weight of the core, it is obtained for the thickness of 35, 45 and 55 centimetres the respectively specific weights of: 15.51, 17.80, and 19.25 KN/m^3.

For the mechanical identification, instead of the application of the guidelines suggested by Italian law, an original application of the Lourenço's procedure has been used with the homogenization of the external layer in more steps.

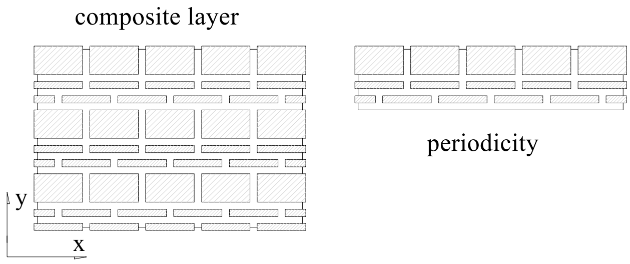

Figure 2 : External layer model, made by 'opera vittata'

For the external layer, considering the typical geometric regularity of the masonry, a homogenization in three steps has been done: two in X direction, with the parallel homogenization of the courses of bricks and blocks of limestone, and the third step in Y direction. In the first, double homogenization, in X direction, it has been regarded separately the stone-made courses and the brick one. About the stone blocks and the bricks the periodicity is referred to a course of stone-mortar and brick-mortar, and remembering the already seen mechanical features for the limestone of Sarno, pozzolanic mortar and brick, from the homogenization procedure, it is obtained (see Table 5):

Table 5 : Elastic parameters for the courses of blocks and bricks

Courses	E_x MPa	E_y MPa	E_z MPa	G_{xy} MPa	G_{xz} MPa	G_{yz} MPa	ν_{xy}	ν_{xz}	ν_{yz}
Blocks	9781	10961	10961	3918	3918	4558	0.21	0.21	0.20
Bricks	6499	6621	6621	2819	2819	2976	0.12	0.12	0.11

In the second homogenization, in Y direction, there's a new composite material whose layers are: stone courses (homogenized in the 1st step on X; it's an orthotropic material), brick courses (homogenized material in 2nd step on X; it's an orthotropic material) and mortar (an isotropic material). From the homogenization procedure for the full external layer it's obtained (see Table 6):

Table 6 : Elastic parameters for the external layer of 'opera vittata'

Masonry	E_x MPa	E_y MPa	E_z MPa	G_{xy} MPa	G_{xz} MPa	G_{yz} MPa	ν_{xy}	ν_{xz}	ν_{yz}
External layer	7137	7179	7689	2496	2901	2628	0.25	0.20	0.28

After homogenizing the external layer, using the same data for the core, for the entire masonry in 'opera vittata', made by three layers and two different materials, with the periodicity referred to the entire structure, from the last homogenization, it is obtained, with dependence on thickness, the values showed in Table 7 for 35 and 55 centimetres.

Table 7 : Elastic parameters for the entire masonry 'opera vittata'

Masonry depth cm	E_x MPa	E_y MPa	E_z MPa	G_{xy} MPa	G_{xz} MPa	G_{yz} MPa	ν_{xy}	ν_{xz}	ν_{yz}
35	11917	11940	10608	4566	3935	3647	0.25	0.21	0.23
55	14089	14104	12500	5507	4695	4427	0.25	0.22	0.23

4 STRUCTURAL ANALYSIS OF THE 'INSULA DEL CENTENARIO'

After the definition of the mechanical characteristic of the different type of masonry, a three dimensional model of the masonries of the 'Insula del Centenario' has been built up.

For the geometric description of the model, the starting point has been the examination of the data collected by topographers and archaeologists: the survey of the plan and the walls, with the masonry stratigraphic unit (USM) with the indication of the materials, building techniques, the consolidation and the plaster.

The middle surface of each masonry panel and cover, has been modelled in a three dimensional reference, using a 2D finite element. The mesh has been fitted to describe the actual shape of the walls and covers taking into account the plaster as dead loads. From the comparison between the pictures and the data available wall by wall, the characterization of the finite elements has been done taking into account the particular type of masonry and the acting loads.

The boundary conditions related to the soil (with the exact altimetry), among the walls and covers have been carefully evaluated. Links more or less strong has been taken into account in relation to the actual conditions. Further investigation has been posed on the structures without the covers which will be substituted, to evaluate the configuration during the demolition phase of the old covers and also in the final configuration with new wood covers (the weight of these covers does not influence too much the stresses in the masonries due to the lack in the box effect of these structures). A last geometric model has been realized taking into account new covers able to do an efficient link with the perimeter walls (reinforced concrete floor with perimeter concrete beam) analyzing the situation of material continuity.

After all the previous definitions, several numerical analyses have been performed.

The structure shows a behaviour really sensitive to the seismic load, in particular in x-direction (parallel to the front on Di Nola street). In fact there are large and particularly high masonry panel, up to 800 cm height, posed perpendicular to X-direction and not wind-braced. The analysis of the results, compared with the guideline of the Italian law, which suggests an admissible compression stress of 0,580 MPa, does not highlight particular problems in the actual configuration, but, under seismic load in X direction (see Fig. 3), high increment in the stresses, with a stress equal to the double of the admissible stress previously reported, has been recovered. Other important increments in the other stress components have been found.

The model allows the study of the effects of the interaction among various loading conditions. These simulations provided compact information about stresses, represented in general by colour maps. The simulation has been carried out using a very popular software like SAP 2000 and the final results of the structural analysis have been collected in database now on the web.

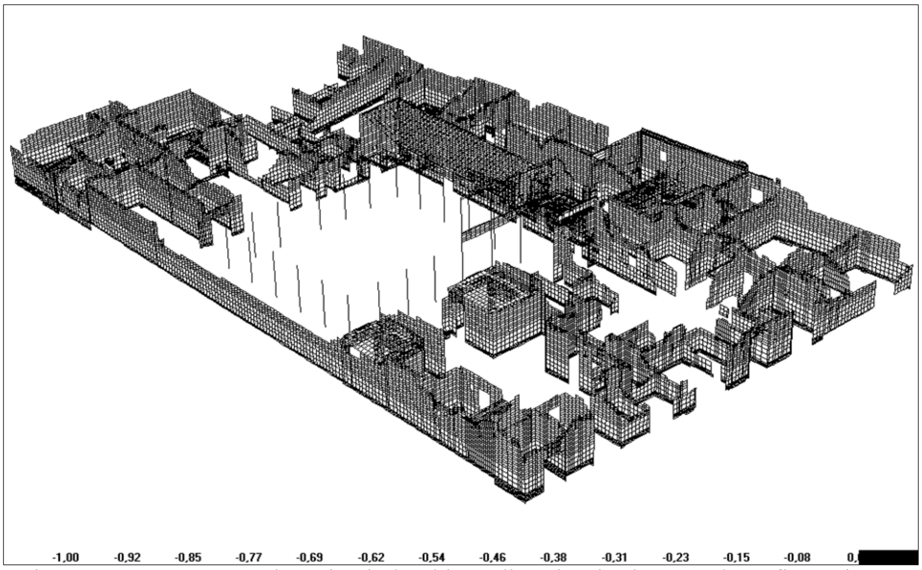

Figure 3 : Response to the seismic load in x-direction in the actual configuration.

5 HYPOTHESIS OF NEW COVERS AND PROTOTYPE

In the framework of new technologies which can be applied in this context, related to the structural analysis, an experimental application of polycarbonat plate (solid and alveolar; free supplied from Bayer Sheet Europe) has been done to cover part of the Insula, to closing window opening and to protect in fresco painting and wall inscription (Custodi 2001, Custodi and Santarelli 2001, Custodi 2002).

Fig. 4 shows the prototypal cover of a portion of the main atrium using a light structure in alveolar polycarbonate plate and aluminium, (the used vertical supports are necessary only to the prototype).

Such prototypal application has been used as reference for a finite element model of a new cover structure in the area of the main atrium. In Fig. 5 the stresses under wind effect of such model has been reported.

Figure 4 : Prototypal cover of a portion of the main atrium.

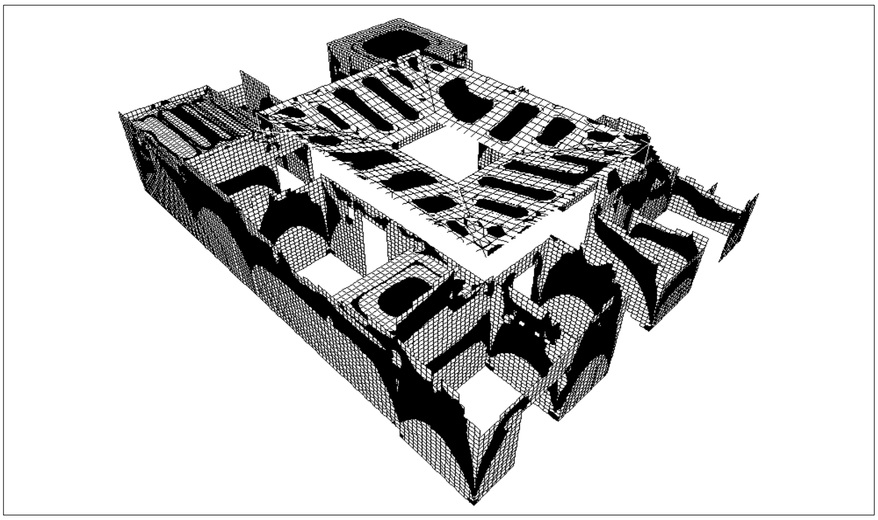

Figure 5 : Model of the main atrium with a new cover : map of the stresses due to wind.

6 CONCLUSIONS

The activity done involves some aspects proper of the structural engineering: from the actual prototypal model to the mathematical model, to the analysis of data used, to the analysis of the results and their degree of reliability. A deep interaction with other teams of research is necessary, in particular for the archaeometric survey, the final effect is an enrichment of the knowledge in the conservation of the archaeological estate.

ACKNOWLEDGEMENTS

Under 'Insula del Centenario' Project (Scagliarini and Santoro 2001), an agreement stipulated in the framework of 'Pompeii Law' between the Archaeological Superintendence of Pompeii and the University of Bologna, the DISTART (Department of Structural Engineering, of Transportation, of Water, of Survey and of Territory) has contributed sharing the knowledge in structural and topographical field.

The Authors, in particular, belong to the DISTART, ("Scienza delle Costruzioni" Section) and they deal with the structural analysis of the masonry of the Insula. In detail this work shows the final results of the activity developed by the research unit whose program is : "Pompei, Insula IX, 8: sperimentazione e modellazione di materiali e strutture" inside the national research program (COFIN 2003) : "Pompei, Insula IX, 8: ricerche archeologiche e archeometriche per la conoscenza, la conservazione e la valorizzazione dell'edilizia pompeiana".

Numerical facilities have been provided by LAMC, Laboratory of Computational Mechanics of the DISTART Department.

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