Structural Analysis of Historical Constructions, New Delhi 2006
P.B. Lourenço, P. Roca, C. Modena, S. Agrawal (Eds.)

Possible Geometric Genesis of a Medieval Cathedral (Alba, Piedmont, Italy)

F. Antonino
Free-lance architect, Savigliano (Cuneo), Italy

G. Pistone
Politecnico di Torino – 1st School of Architecture, Department of Structural Engineering, Turin, Italy

D. Zorgniotti
Free-lance architect, Cherasco (Cuneo), Italy

ABSTRACT: Since 1998 extensive studies have been conducted on the cathedral of the city of Alba, both in order to assess the severity of a number of deterioration phenomena, and hence be able to assess the current safety margins of the building, and in order to plan possible strengthening works.
These studies were geared to the identification of materials and models of behaviour.
A F.E.M. model was also developed as a valuable aid to gain an understanding of the structural behaviour of the cathedral: the results of the numerical model turned out to be in good agreement with the experimental findings.
Of special interest was the analogy observed between the geometric genesis that, in all likelihood, inspired the original builders of the cathedral and the schemes that underlie the numerical model produced in the course of present-day studies.
This article summarises main results of the studies conducted on the building so far.

1 FOREWORD

In recent years several studies have been conducted on the behaviour of the cathedral of the city of Alba – the Church of S. Lorenzo – in order to evaluate the stability of the structure and to assess a number of static problems in preparation for the strengthening works designed to restore the building to acceptable safety conditions.

The goal of the preliminary studies was to gain a better understanding of all the aspects characterising the building, to serve as a valuable support for a correct formulation of the structural diagnosis. Accordingly, the first stage consisted of making surveys of the building and the foundations, identifying the cracking patterns, drilling cores from the masonry walls for the analysis of its constituent materials, performing flat-jack tests at different points in the piers and the facade, and monitoring a number of major lesions by means of strain gauges.

The surveys were carried out in parallel with an examination of the historical records, and the information collected provided an overall picture of the building stages of the complex through the centuries.

The investigation also supplied significant data for the production of a series of finite element numerical models and the interpretation of the structural behaviour of the cathedral.

Surveying a monument or a building means much more than merely recording the numerical data collected in the field – albeit extremely accurate ones, as made possible by present day technologies and tools – but having no critical significance. It must also be an attempt to decipher the laws underlying the measurements: when we look at the architectural works of the past we must try to identify the criteria that determined their proportions.

All the foregoing considerations and the questions arising from them have persuaded us to adopt a different approach to the study of the Cathedral of S. Lorenzo, conducted in parallel with the method adopted in modern engineering science.

2 THE MATHEMATICS AND GEOMETRY OF MEDIEVAL CATHEDRALS

2.1 The School of Chartres and geometry

The school was founded in 1020 by the bishop of Chartres, Fulbert, a disciple of Gerbert (pope in 998 with the name of Sylvester II).

The School of Chartres, where future priests were educated, had an immense influence on the construction of the cathedrals of western Europe. At the school, they also studied and recorded the principles of Christian thought, which, in the construction of the cathedrals, found expression through symbols, shapes, statues, numbers and special colours designed to get across the Christian message to the – largely illiterate – populations of the Middle Ages.[1]

The science that was called "geometry" during the Middle Ages played a decisive role in determining the dimensions of the floor plan and the cross sections of a cathedral, as well as in the construction of the elements it was composed of.

Thierry of Chartres, the most influential representative of the School, tried to explain the mystery of the Trinity through a geometric demonstration. The identity of the three Persons, in his view, was represented by the equilateral triangle.

It is necessary to take into account the influence exercised on the followers of the School of Chartres by Saint Augustine: the philosopher used architecture and music to demonstrate that the number, as borne out by the simplest proportions that are based on “perfect” ratios, is the root cause of all aesthetic perfection.

The application of "perfect proportions" was achieved through rigorous geometric reasoning to define the requirements to be fulfilled in terms of both stability and beauty: gothic architecture seems to indicate that all static problems were in actual fact solved by resorting to purely geometric methods.[2]

3 THE CATHEDRAL OF S. LORENZO: TRANSFORMATIONS AND GEOMETRY

3.1 Historical background

We have no certain knowledge about the origins of the cathedral, about its initial shape and size or the time it was founded, though it certainly dates from before the tenth century. It is believed that it was erected on the ruins of an ancient pagan temple, dedicated to Apollo, as was often the case, but there are no documents to confirm this hypothesis.

The reconstruction of the cathedral, between 1486 and 1517, was prompted by an initiative by bishop Andrea Novelli.

In 1624 is was announced by "*expertis fabriis murariis*" that the church was in danger of crumbling "*in tegulis et voltis*" and, in fact, a few years later the vaults of the nave and some beams of a chapel collapsed.[3]

In the 1650’s, through the intervention of Monsignor Paolo Brizio, the vaults were reconstructed, the foundations of various chapels were built and the entire structure was restored.

On 24 July 1863, the definitive Restoration Commitee was convened upon the initiative of Andrea Formica, and on 24 February 1864, Count Edoardo Arborio Mella was entrusted with *disegno* and site management tasks.

In a letter of 20 May 1865, Mella signed his *cenni di schiarimento* (explanatory notes) to the restoration drawings submitted.

In August 1872 the rose window has already been opened in the facade and the two side windows had been modified.

It is believed that the works performed in the years 1866-72 included: replacement of the six lateral chapels with new polygonal ones; construction of the eight lateral buttresses topped by small spires; restoration of the vertical alignment of the side walls through the construction of a covering layer of new bricks; demolition of the walls bearing down on the arches of the side vaults, replaced with a demi-arch resting on the main vault (serving as a buttress); demolition of

[1] Gout (2001)
[2] Von Simson (1988)
[3] Morgantini (1988)

the church vaults (save for those of the two main chapels), which were subsequently reconstructed in the shape of pointed vaults; demolition of the apse, the entire vault of the presbytery and the crypt. New foundations were dug for the apse, which was reconstructed in polygonal form. The inner walls were also modified to improve their verticality. Finally, the three vaults of the pronaos were torn down and rebuilt.

3.2 Current structural set-up

In its present-day configuration, the structural set-up of the cathedral consists of a nave and two aisles, with chapels opening on either side, two of which, the ones flanking the new altar, are much bigger than the others. The presbytery terminates in a polygonal apse, with five ogive shaped windows.

The dimensions of the building are imposing, with a plan developing over a length of ca 47 m, in the main section, and a further 23.50 m in the presbytery; the building is 36 m wide and its height, at the ridge of the intrados of the nave vaults is 23 m from floor level.

The front part of the cathedral has an entrance portico. Thus, the facade is perforated at the base by the three arcades of the porch. A central rose window and the two ogive shaped windows on its sides break the massive surface of the masonry facade.

The interior of the main section of the church is characterised by the slender four-lobed columns supporting the system of ogee vaults marked by diagonal and contour ribs.

3.2.1 The equilateral triangle inside the cathedral: chance or geometric precision?

The present-day structure of the building clearly reveals the influence of the restructuring works performed between 1866 and 1872 by Edoardo Arborio Mella, an architect from Vercelli who was active in Piedmont primarily as a restorer of medieval buildings. From the consultation of his drawings and the relative documents[4], supported by extensive readings of the earlier writings by architect Filippo Morgantini, we gained some insight into the approach adopted by Mella.

The plan[5] found illustrates the major interventions performed by Mella and makes it possible to identify some pre-existing structures dating back to 1486 (Fig. 1), the conditions of the cathedral after 1486 and in 1868 (Fig. 2).

In trying to gain an understanding of Mella's approach, who surely was familiar with the Gothic architecture of the region, we tried to identify the guide lines and the geometric criteria underlying his project. Here we shall briefly summarise what we have ascertained so far.

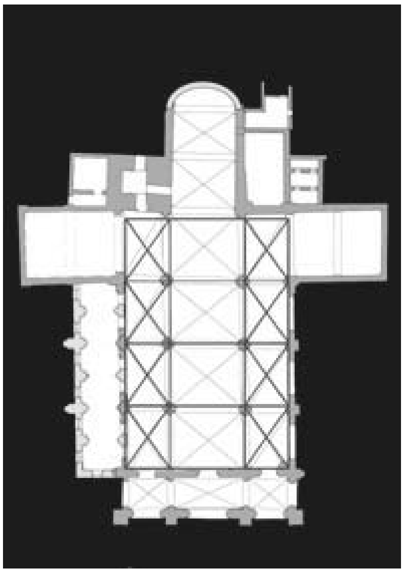

Figure 1: Situation in 1486

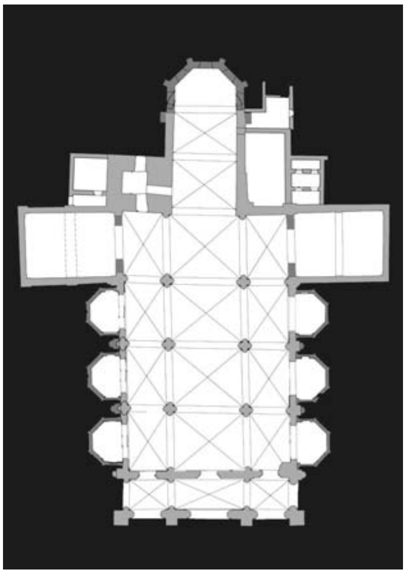

Figure 2 : Restoration by Mella (1868)

[4] Archivio Arborio Mella Vercelli (AAMV) and Archivio Istituto Belle Arti Vercelli (AIBAV)

[5] AIBAV (N. 700)

Starting from the plan and taking into consideration the portion that can be ascribed to Novelli (Fig. 1), we can isolate the nave and the two aisles, developing in length over four bays. By measuring for each bay the width of the two aisles and that of the nave relative to the centre line of each element, we get:

Table 1 : Dimensions of the cathedral

Left aisle	Nave	Right aisle	Average width of aisles	Nave/aisle ratio
683	1129	710	697	1.62
679	1130	701	690	1.64
678	1126	693	686	1.64

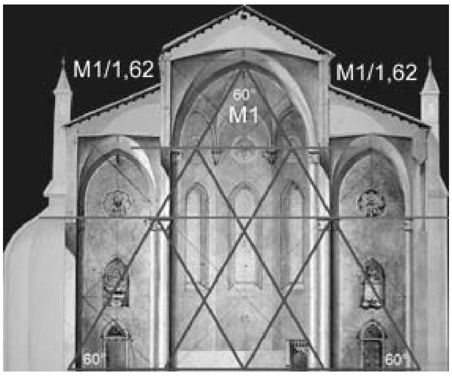

Figure 3 : Cross-sectional view

As the readers will have noticed, this ratio comes very close to the *divina proporzione*, (1,6180339887...). At this point, it may be objected that this proportion is not reproduced exactly to the second decimal number, but we believe that a difference this small would not have been much of a concern for the medieval builders.

Having defined the overall dimensions of the nave and the aisles, we took into consideration the cross-section of the church (Fig. 3), always with reference to the planimetric portion produced by Novelli.

On this cross-section, we plotted, starting from the midline of the exterior walls, a line slanted at an angle of 60° so as to define an equilateral triangle that marks the heights at the keystones of the main arches of the nave. By plotting lines running parallel to the sides of this equilateral triangle and making them pass through the intersection between the horizontal and the fundamental midspan axes identified before, we find the imposts of the arches of the aisles and the impost of the arches of the nave.

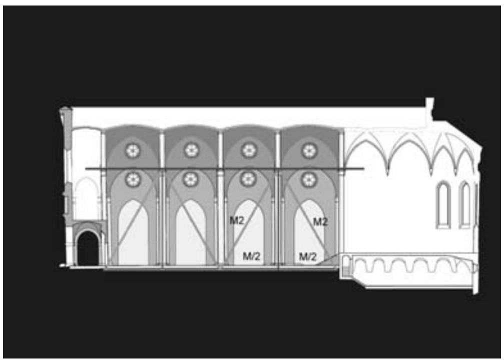

Figure 4 : Longitudinal section

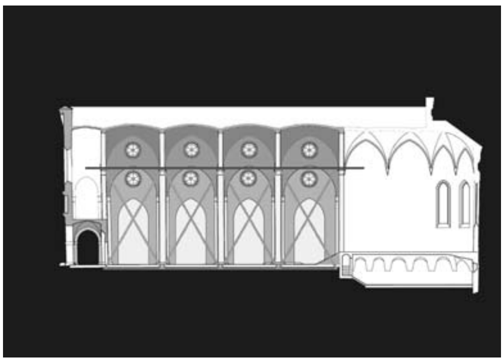

Figure 5 : Longitudinal section

In the cross section too, the equilateral triangle can be used to define the positions of the columns delimiting the nave: starting from the apse, let us plot a straight line set at a 60° angle until it meets ideally the straight line marking the imposts of the main arches defined before. By doing so, we obtain another module marking the distances between one column and the next (Fig. 4). On these two modules, i.e., the one marking the distances between columns in the longitudinal direction and the other, showing the distance between the columns and the outer walls in the transverse direction, we have plotted a parallel line in either direction and the shape obtained is another equilateral triangle (Fig. 5).

Now the question is, are these modules, the geometry and the proportions obtained due to chance or to a deliberate search for geometric correspondences?

It is probably the same question posed by French theorist Jean Villette, when he considered the positions of the four main columns at the crossing of the nave and transepts in the Cathedral of Chartres. The subtitle to his work was *Hasard ou Stricte Gèometrie?*[6]

4 THE CATHEDRAL OF S. LORENZO: INVESTIGATIONS AND STRUCTURAL ANALYSES

4.1 Structural survey and cracking configuration

The geometric data was supplemented with information of a structural nature, indispensable for the creation of a correct numerical model.

This applies in particular to the elements of decisive importance from the standpoint of stability: the intersections between the nave partitions and the façade and the transverse links between the nave vault arches and the longitudinal partitions between the naves.

The determination of the cracking configuration encompassed the system of cracks proper and the set of permanent deformations that have appeared in the building over time.

The system of cracks includes a number of major lesions cutting the building in the transverse direction in the proximity of the facade. In particular, cracks are observed in the outer walls of the two aisles, in the second span starting from the facade: these cracks are seen to become progressively wider, going from the bottom to the top of the walls; under the roof, cracks up to several centimetres wide run at a slant in the vertical walls.

The vaults of the first bay, close to the facade, are cut by lesions set at different slants, which, as a whole, reproduce the detachment of the front part from the body at rear. Approximately in the centre of the church, in the fourth span from the facade, there is crack that clearly separates the transverse arch from the vault.

The permanent deformations produced in the building over time have given rise to a translation of the upper part (vaults, nave partitions, roofing) relative to the base plan; the direction of this translation is toward the square with an off-plumb of up to 41,9 cm. Albeit to a lesser extent (ca 10 cm) the same fate has befallen the piers inside the building.

Smaller transverse deformations have also been observed in the interior piers; in this case, the displacements suggest a predominant southward component.

The foregoing information indubitably represents the vicissitudes of the building through the centuries. It should be noted, however, that the values determined with these measurements are probably due to the numerous repair works performed in the past to attenuate the misalignments present at the time and therefore the present-day situation does not reflect the displacements that occurred in the distant past.

4.2 Surveys of the foundations – Core drilling, quality and state of preservation of the masonry

Measurements were made both on the ground where the building rises and on the foundations of the isolated buttresses supporting the facade. The overall picture that emerges has revealed, under layers of fill and soil reworked to a depth of 3,90 m, the presence of clayey-silty soil strata down to 10 m below ground, with sand underneath. In other words, the foundations of the facade are constructed in soils having rather poor mechanical properties.

[6] Gout (2001)

The conditions and quality of the constituent materials of the masonry were documented by drilling cores and then inspecting the faces of the holes with TV probes. The masonry of the facade is characterised by an inner core enclosed by walls, about 25 cm thick, made up of bricks, brick fragments and stones of different sizes embedded in lime mortar, without any big voids but having limited strength.

The presence of many cracked bricks, especially in the outer courses, was probably caused by crushing, indicating a state of compression and bending in the masonry. In better conditions are the inner piers, consisting of brickwork free of voids, set in mortar and displaying good adhesion and compactness as well as excellent mechanical properties.

4.3 Flat-jack tests

The tests revealed high compressive stresses, of between 0,9 and 3,0 MPa, especially in the piers. Equally high were the values obtained from the tests performed on the facade buttresses at ground level, with a peak of 3,1 MPa in one of the central piers.

4.4 Monitoring a number of appreciable lesions

Some cracks have been monitored since 1999 by means of strain gauges, by recording the measurements at regular intervals. The cracks were present both in the nave vaults and in the partitions above them.

4.5 Numerical analysis of the structure

The complexity of the structure made it necessary to perform a series of numerical analyses taking into account different assumptions regarding the restraint conditions that might have occurred through the centuries, also in relation to the various historical transformations. To this end, models had to be produced by taking into account the spatial behaviour of the building. A short description of the final model deemed most significant is given below.

The construction of the various models was facilitates by the considerations on the geometry of the cathedral formulated above, which made it possible to introduce considerable simplifications, without, however, deviating from the reality of the construction.

Finite elements models were developed, reproducing the materials in the elastic-linear field.

The modelling process was performed on the basis of the surveys conducted according to modalities selected precisely to this end.

Mechanical properties: were determined from the results obtained on the cores and from the observation of the walls of the holes drilled into the masonry; the parameters assumed were determined by analogy with tests conducted on elements having similar construction features and components and dating back to the same period. They are: E = 2.500 MPa; μ = 0,15; γ = 180 KN/m^3

Finite elements used: the finite elements types used were: shell elements for the vaults and solid elements for everything else.

Restraints: the structure is restrained at the base. Even the bell tower constitutes a stiff restraint. For the models reproducing only the main section of the church, the connection with the apse also consists of a fixed joint.

Actions considered: only dead weight was taken into account, on account of its predominance compared with live loads the building may be subject to.
The seismic behaviour of the structure was not analysed.

Size of the model: the models of the main section of the church use 13.246 elements, linked by 18.833 nodes, resulting in a system with 112.998 degrees of freedom.

4.6 Results of the analyses

First of all it should be noted that the model expresses the most important aspects of the structural behaviour of the building and is useful to obtain a full picture of strain and stress conditions.

The salient aspects of the behaviour of the cathedral are summarised below.

The deformed configuration is characterised by a slight projection of the nave towards the square, mated to a widening of the cross-section at the centre of the building, which, however, is a much less noticeable (Fig. 6).

Such deformations, affecting the central core of the church, are countered by a number of stiffer zones.

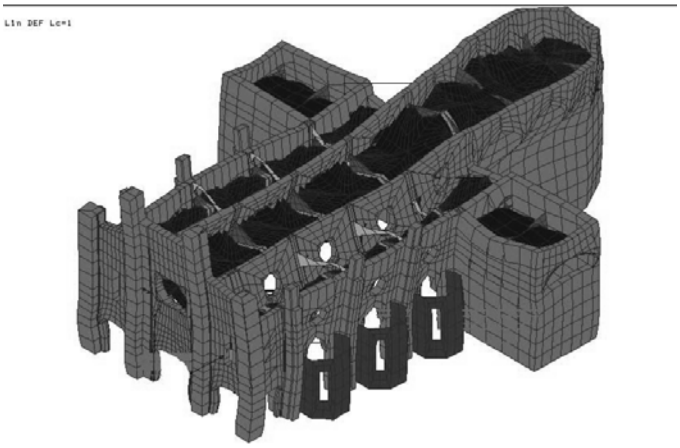

Figure 6 : Overall view of the model

Laterally, albeit weakened by the presence of the chapels, the exterior walls serve as blocking elements, in a much as they constitute a substantially rigid partition in their own plane; accordingly, they are able to retain in place the facade that is locked into them.

In the back of the church, the presbytery and the apse, on account of their massive and close configuration, prevent all movements.

In the transverse direction, the chapels of the transepts, of a much bigger size than the others, prevent any significant displacement.

The foregoing displacements find full confirmation in the verticality measurements obtained on the walls and the inner piers. It should be noted that the displacements determined with the numerical model cannot be correlated with the values measured, which are much greater: this is obviously due to the material employed in the model, which is perfectly elastic and tension resistant. Nevertheless, the theoretical representation obtained is of fundamental importance in the interpretation of the experimental data.

As a result of this behaviour, major tensile stresses are produced in the nave partitions and also at the connection of the facade to the body behind, as can be seen in the figure (Fig. 7).

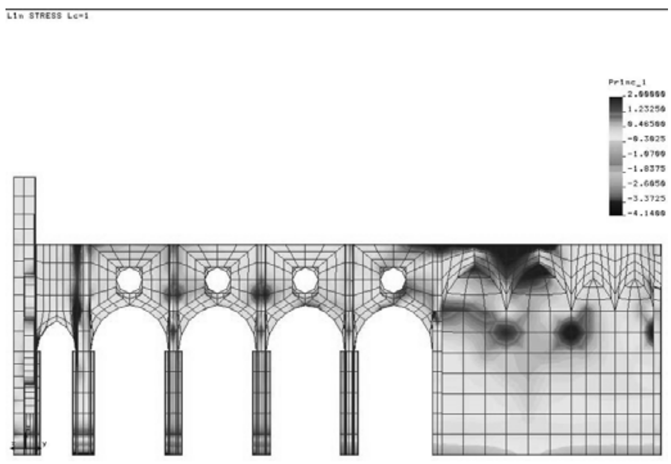

Figure 7 : Longitudinal view of the model

A confirmation of the results supplied by numerical modelling comes from the cracks that cut the church transversally next to the facade: they are present both in the partitions and in the vaults.

4.7 Analysis of collapse mechanisms

The presence of two large masonry elements resting on the nave columns (the partitions separating the nave from the aisles) results in a behaviour of greater complexity than might be suggested by the series of arches on which the partitions are rested. In actual fact, the partitions can be likened to two diaphragms of considerable size (45 m long and 11 m high) albeit perforated and weakened by the presence of the round windows.

The behaviour of these diaphragms, resting on the nave columns along their entire development, reflects the formation of an upper zone in compression and a zone underneath, partly diminished by the presence of the arches, where the stresses are mostly tensile (Fig. 8).

The distribution of stresses in the partitions, which can be represented more accurately by isostatic lines, sheds light on the behaviour of the partition elements separating the nave from the aisles and the effects this has on the facade: it is possible to identify the formation of a single big arch, which starts from the transept with an impost and reaches the facade with the other; in reality, the arch is supported along its development by the arches of the nave and the pillars of the latter. However, the thrust exercised on the facade is considerable, much higher than the thrust generated by the series of arches separating the nave from the aisles.

The foregoing considerations were the starting point for an analysis of the mechanisms that may arise from this situation and, finally, what the ultimate limit state might be like.

The structure as a whole tries to find a condition of greater stability, a condition that can be reached through a progressive reduction of the thrust against the facade. The latter is directly proportional to the upper load and the span of the idealised arch, and is inversely proportional to the rise of the arch itself.

The thrust at the facade and the ensuing horizontal displacement of the facade have resulted in the formation of a series of transverse cracks in the body of the church, acting as veritable hinges at the connection between the pillar and the partition.

The two ideal arches that can be identified in the uncracked model are progressively fragmented into a series of smaller arches, whose span corresponds to the length of the portions separated by the lesions.

As the span decreases the resultant is verticalised, resulting in an attenuation of the horizontal thrust component. In the final configuration, corresponding to the minimum thrust at the facade, we may discern 5 macro-elements delimited by the pillars, as shown in Fig. 9.

From the studies conducted, it is believed that collapse may even occur before the kinematism depicted in the figure is reached, if the displacements brought about by the formation of the macro-elements cause the base of the facade to be displaced from the vertical: in these circumstances, the facade would topple towards the front square due to the presence of tensile stresses in the facade columns. It should be noted that absolute displacements at the top of each pillar gradually reduce as we move towards the transept, whilst the absolute displacement at the facade is maximum, being the sum of the partial displacements of all the various macro-elements: this was confirmed by strain gauge readings.

A different problem to be addressed is the possible effects of dynamic loads as may be due to seismic actions: the horizontal forces and hence the additional displacements that would be generated in these conditions would be able to bring about the collapse of the building, Broadly speaking it is possible to consider two collapse modes compatible with the assumptions made: one envisages the fall of the facade, according to the modalities analysed above, the other envisages the partial collapse of one of the nave vaults, probably the first or the last, or possibly even both.

The vaults in fact should be viewed as many macro-elements totally disconnected from one another and from the nave partitions, owing both to construction and to deformation factors.

Hence, in the event of an earthquake, the vaults would tend to behave in counterphase relative to the partitions and to one another, thereby promoting collapse.

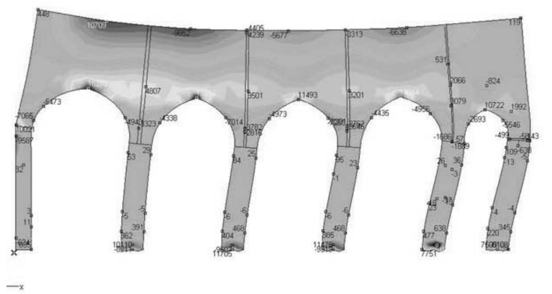

Figure 8 : Initial ideal arch – Stresses expressed in daN/m

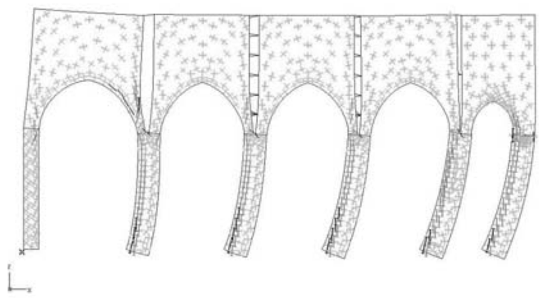

Figure 9 : Collapse kinematism with 5 macro-elements

5 CONCLUSIONS

Based on the information obtained from the strain gauge readings and the numerical model, it can be concluded that hinges have been progressively forming at the bays, both in the nave partitions and the vaults, and especially at the connections between the vaults and the main transverse arches.

The first hinge probably formed at the apse and was soon followed by the hinges at the façade, as also evidenced by the numerical model.

This process continued with the formation of additional hinges accompanied by continuous forward displacements towards the façade.

The total collapse of the nave might occur if, for the lack of further resources, the first hinge converted into a carriage, or the local collapse of the first vault adjacent to the façade could be caused by excessive relative displacements.

ACKNOWLEDGMENTS

For the materials supplied and their helpful collaboration, the authors wish to thank in particular:
Mr. Paolo Franchetti (AIBAV Archivio Istituto Belle Arti - Vercelli)
Mrs. Rosso (AAMV Archivio Arborio Mella - Vercelli)

REFERENCES

Carbone, I.V., and Fiore, A. and Pistone, G. 2001, *Le costruzioni in muratura*. Milan: Hoepli

Di Pasquale, S. 1996. *L'arte del costruire - Tra conoscenza e scienza*. Venice: Marsilio Editori

Jean Gimpel, J. 1982. *Costruttori di cattedrali.* Translation by Giulia Veronesi e Monica Girardi. Milan: Jaca Book

Gout, M. 2001. *Symboliek in kathedralenbou*. Den Haag: Mirananda uigevers B.V.

Heyman, J. 1982. *The masonry arch*. Chichester: Ellis Horwood

Heyman, J. 1996. *Elements of the theory of structures*. Cambrigde: University Press

Morgantini, F. 1988. *Edoardo Arborio Mella restauratore (1808-1884)*. Milan: Franco Angeli

Von Simson, O. 1988. *La cattedrale gotica, il concetto medievale di ordine*. Translation by Maria Augusta Coppola. Bologna: società editrice il Mulino

Viollet-le-Duc, E. 1981. *L'Architettura ragionata - estratti dal Dizionario*. Translation by Adriana Maria Colombini Mantovani - Introduction by M. Antonietta Crippa, Milano: Jaca Book