Some considerations on out-of-plane collapse modes of masonry walls

G. Brandonisio, E. Mele & A. De Luca

Department of Structural Engineer (DIST), University of Naples “Federico II”, Naples, Italy

ABSTRACT: In this paper the problem of out-of-plane collapse of masonry walls under vertical and horizontal loads is addressed through the review of ancient building rules and the study of the European (Eurocode 6 and 8) and new Italian seismic codes on unreinforced masonry buildings.

The limitations of the analysed seismic codes and the empirical geometrical rules of ancient constructors are compared with the results of some structural analyses carried out on rectangular masonry panels with different slenderness h/s, aspect ratio h/b (height/width) and boundary conditions. In particular, the analyses have been carried out both with the limit analyses and non linear FEM analyses with Abaqus computer code.

The results of these parametric analyses show that the respect of the seismic codes limitations on the geometrical slenderness h/s allows to implicitly have a minimum resistance to out-of-plane collapses of the masonry walls.

A proposal is made to overcome the unjustified provisions of some codes which do not allow to have distance among orthogonal walls larger then 7 m.

1 INTRODUCTION

Traditionally, the study of the behaviour of masonry panels focuses the attention on the in-plane failures and resistant mechanisms. Nevertheless, the out-of-plane collapses of the walls are often activated both under vertical loads and horizontal forces. These failure modes depend on many aspects, as quality of bricks and mortar, bond pattern, geometrical characteristics and boundary conditions of the wall.

In this paper the influence of the last two above aspects (geometrical characteristics and restraint conditions) are investigated. Particularly, the target of the paper is to underline the relationships among: the h/s ratio (height to thickness ratio) of the wall; the aspect ratio h/b (height to width ratio); the edge restraint conditions; and the out-of-plane strength of masonry wall subjected to vertical loads and horizontal forces.

In this aim, in the first part of the paper the problem of the buckling of a masonry wall under axial load is addressed in order to find a relationship between the wall slenderness λ (as defined in the stability theory of columns or plates) and the load-carrying capacity.

Then, the behaviour of masonry wall under horizontal actions is analyzed through both the evaluation of the collapse multipliers and the study of geometrical limitations, recommended both by the ancient rules of building and modern European seismic codes (EC8’03 and NTC’07). The results are presented in a homogeneous approach in order to explicit the relationship between the recommended h/s ratio and the aspect ratio h/b. Then, for several edge restraints conditions of walls, designed according to required h/s ratio, the relationships between the h/b ratio and the collapse multipliers are illustrated and compared with the results of a parametric nonlinear F.E.M. analysis.

The results herein obtained provide useful indications on the seismic capacity of the masonry walls.

2 WALL UNDER VERTICAL LOADS

As in to the case of a steel column under axial force, the resistance of a masonry wall under pure axial load depends on the slenderness λ, defined by the following relationship:

$$\lambda = \frac{h_o}{i} = \frac{\beta \cdot h}{i} = \beta \cdot \sqrt{\frac{12}{s}} \cdot \frac{h}{s}$$ (1)

where: h_o is the buckling length of the wall in the buckling plane considered; i is the radius of gyration of the cross section of the wall; \(\beta\) is a coefficient that depends on the type of restraint; h and s are the height and the thickness of the wall, respectively.

From the Eq. (1) it is possible to note that the slenderness \(\lambda\) is proportional to h/s ratio, which can be defined as geometrical slenderness of the masonry panel.

The Figure 1 shows the relationship between the axial resistance \(N_f\) of the wall and the slenderness \(\lambda\).
1. The effect of slenderness on resistance of a masonry wall under axial load.

1.0

Figure 1. Effect of slenderness on resistance of a masonry wall under axial load.

The effect of slenderness on resistance of a masonry wall under axial load.

Nf/Npl

λ

N

Elastic buckling

Crushing

Transition region

Figure 2. Edge restraints of the wall: (a) wall restrained at the bottom (wall 1L); (b) wall restrained at the top and bottom (wall 2L); (c) wall restrained at the top, at the bottom and on one vertical edge, with the other vertical edge free (wall 3L); (d) wall with 4 edges restrained (wall 4L).

1.0

Figure 3. Comparison among the limitations of EC6’03 and NTC’07 on h/s ratio.

– wall 3L (Fig. 2c): the value of ρ (appointed as ρ3) is depending on the degree of restraint ρ2 at the top and bottom and on aspect ratio h/b:

ρ1 = f(ρ2; h/b)  (3)

– wall 4L (Fig. 2d): the value of ρ (appointed as ρ4) is again depending on the degree of restraint ρ2 at the top and bottom and on aspect ratio h/b:

ρ4 = f(ρ2; h/b)  (4)

In the EC6 the slenderness λc should not be greater than 27 when the wall is subjected mainly to vertical loading. This limitation on λc, together with the suggested limitations on ρ, may be used to establish the maximum h/s ratio allowed by the EC6.

In Figure 3 the relationships between h/s and h/b are shown in graphical form. In particular, the curves (1)–(2) are associated to the limits of the wall 2L, while the curves (3)–(4) and curves (5)–(6) are associated to the limitations (3) and (4), for wall 3L and 4L, respectively.

The comparison among the curves (1)–(6) of Figure 3 underlines the effect of the lateral edge restraints on the required h/s ratio. For example, in the case of wall with aspect ratio h/b = 1, the maximum values of geometrical slenderness h/s is equal to 36 (ρ2 = 0.75) or 27 (ρ2 = 1.0), for the wall 2L; it is equal to 38 (ρ2 = 0.75) or 30 (ρ2 = 1.0), for the wall 3L; it is equal to 56 (ρ2 = 0.75) or 54 (ρ2 = 1.0), for the wall 4L.

Instead, the new Italian code NTC’07 does not consider the case of wall 3L. In fact, in NTC’07 are

\[
\lambda_c = \frac{h_{ef}}{s_{ef}} = \frac{\rho \cdot h}{s_{ef}}  \tag{2}
\]

where: h_{ef} is the effective height of the wall; s_{ef} is the effective thickness of the wall, that is usually taken equal to the actual thickness s (for cavity walls the EC6’03 suggests the following formula: s_{ef} = (t_1 + t_2) \frac{1}{3} where t_1 and t_2 are the actual thickness of the leaves; the Italian code NTC’07, instead, in every case assumes s_{ef} = s); ρ is a reduction factor depending on the edge restraint or stiffening of the wall.

In the examined codes (EC6’03 and NTC’07) the suggested values of the coefficient of restraint ρ are related to the aspect ratio h/b and to the degree of the edge restraints. Particularly, with reference to the walls of Figure 2, the EC6’03 makes the difference among the following three cases:

– wall 2L (Fig. 2b): the value of ρ (appointed as ρ2) is depending on the degree of restraint (ρ2 = 0.75 in presence of r. c. floors at the top and bottom of the wall; ρ2 = 1.0 for walls restrained at the top and bottom by timber floors);

\[ h_{ef} = \frac{h}{s} \]

In the following part of this paper, we will assume s_{ef} = s.
recommended reduction factors $\rho$ only in the case of wall 2L ($\rho_2 = 1.0$) and wall 4L, for which $\rho$ depends only by the aspect ratio: $\rho_4 = f(h/b)$.

The maximum slenderness $\lambda_c$ permitted in the NTC’07 is 20; this limit, together with the recommended values of $\rho$, allows to plot in Figure 3 the curves (7) and (8) associated to the walls 2L and 4L, respectively.

The comparison among the curves (1)÷(6) associated to the limits of EC6’03 and to the curves (7) and (8) derived by the NTC’07, shows that the Italian code is more conservative than the EC6 both for wall 2L and wall 4L (Fig. 3). For the case of wall 4L, in Figure 4 it is proposed the comparison between the coefficients of restraint $\beta$ and $\rho$, that define the relationships between the slendernesses $\lambda$ and $\lambda_c$ and the h/s ratio (Eqs. (1) and (2), respectively). Therefore, the diagram allows to compare the differences between the theoretical slenderness $\lambda$ (Eq. 1) and the conventional slenderness $\lambda_c$ (Eq. 2).

To define the reduction factors $\beta$ of the wall 4L, it is utilized the elastic theory of the buckling of the rectangular plates under compressive loads. If the wall’s edges may be considered pinned, the critical stress $\sigma_{cr}$ of the wall 4L becomes:

$$\sigma_{cr} = \frac{N_{cr}}{s} = \frac{\pi^2 \cdot E}{12 \cdot (1 - v^2)} \left( m + \frac{h/b}{m} \right) \cdot \frac{1}{(h/s)^2}$$  \hspace{1cm} (5)

where: $m$ is the number of half waves in witch the plate buckles in the vertical direction ($m = 1, 2, \ldots, n$);

$$D = \frac{E \cdot s^3}{12 \cdot (1 - v^2)}$$  \hspace{1cm} E is the modulus of elasticity; $v$ is the Poisson’s ratio (in Figure 4 it is utilized $E = 1100$ MPa and $v = 0.1$).

The condition:

$$\sigma_{cr} = \frac{\pi^2 \cdot E}{\lambda_c^2}$$  \hspace{1cm} (6)

allows to define the slenderness $\lambda$ in the case of wall 4L with all pinned edges:

$$\lambda = \left( m + \frac{h/b}{m} \right) \cdot \frac{h}{s} = \frac{\beta \cdot h}{i} = \beta \cdot \sqrt{12 \cdot \frac{h}{s}}$$  \hspace{1cm} (7)

where the reduction factor $\beta$ assumes the expression:

$$\beta = \frac{1 - v^2}{\sqrt{m + \frac{(h/b)^2}{m}}}$$  \hspace{1cm} (8)

The trends of the curve (1)÷(3) of Figure 4 show that the factors $\beta$ and $\rho$ are equal only in the case of EC6’03 in the hypothesis $\rho_2 = 1$; the associated curve (2), indeed, is situated on the bisecting line. On the contrary, in the cases of EC6 with $\rho_2 = 0.75$, and of NTC’07, only for small values of reduction factors (lower than 0.5) there is coincidence between $\beta$ and $\rho$; for higher values (greater than 0.5), in EC6 ($\rho_2 = 0.75$) the factor $\beta$ is larger than $\rho$, while in the NTC’07 $\beta$ is lower than $\rho$.

By observing the curves plotted in Figure 4, it is possible to found an approximate relationship between $\lambda$ and $\lambda_c$. In fact the curves (1), (2) and (3) suggest the following identities:

$$\text{EC6’03} \rightarrow \lambda \equiv 4 \cdot \lambda_c; \quad \text{NTC’07} \rightarrow \lambda \equiv 3.4 \cdot \lambda_c$$  \hspace{1cm} (9)

The relationships between capacity reduction factor $N_f/N_{pl}$ and conventional slenderness $\lambda_c$, proposed in EC6’03 and NTC’07, are plotted in Figure 5a and b, respectively. In the diagrams are reported several curves corresponding to different values of eccentricity $e$ of axial load $N$.

The application of Eq. (9) allows to individuate the influence of slenderness $\lambda$ on the reduction factor $N_f/N_{pl}$. The Figure 5, indeed, suggests that $N_f/N_{pl}$ is almost equal to 1.0 (almost no reduction) when $\lambda$ is lower than about 20 (stocky wall). When $\lambda$ is greater than about 50÷60, instead, the reduction of resistance is significant (slender wall). Finally, when $\lambda$ is comprised between 20 and 50÷60, the wall shows an intermediate behaviour, with interaction between crushing and elastic buckling phenomena.

3 WALL UNDER HORIZONTAL LOADS

Also the out-of plane resistance of a wall subjected to horizontal loads is influenced by the h/s ratio. Indeed, since the ancient rules of the art of building masonry structures, the h/s ratio has been properly limited in
order to reduce the vulnerability of the walls to the out-of-plane collapses.

In this paragraph, the out-of-plane collapse mechanisms of a wall with different restraint conditions at the edges are initially examined and the collapse multipliers are evaluated; then, the ancient rules of the art and the European (EC8'03) and Italian (NTC'07) seismic provisions for the unreinforced masonry buildings, which mainly consist of geometrical limitations, are analyzed; finally the resistance of masonry walls to horizontal loads, designed according to the above geometrical limitations (ancient rules of the art and seismic codes) is evaluated.

3.1 Collapse multipliers of horizontal loads

In the case of uniform distribution of horizontal loads, the application of the principle of virtual works to the wall 1L of Figure 2a, allows to explicit the relationship between the geometrical h/s ratio and the out-of-plane collapse multiplier \( \alpha_1 \) (defined as maximum horizontal load to self weight load ratio):

wall 1L: \( \alpha_1 = \frac{1}{h/s} \)  \hspace{2cm} (10)

When the wall is sufficiently retrained at the top and bottom by the floor diaphragms (wall 2L) the multiplier \( \alpha \) becomes (Hendry et al. 1997):

wall 2L: \( \alpha_2 = \frac{8}{h/s} \)  \hspace{2cm} (11)

Instead, if the wall is stiffened on one or on two vertical edges (wall 3L and 4L, respectively), in the hypothesis that the yield lines are slanted at 45\(^{\circ}\), the collapse multipliers \( \alpha \) are, respectively:

wall 3L: \( \alpha_3 = \frac{8}{h/s} \cdot \frac{1}{1 - \frac{2}{3} \cdot \frac{h}{b}} \)  \hspace{2cm} (12)

wall 4L: \( \alpha_4 = \frac{8}{h/s} \cdot \frac{1}{1 - \frac{1}{3} \cdot \frac{h}{b}} \)  \hspace{2cm} (13)

The Eqs. (12) and (13) show that the multipliers \( \alpha_3 \) and \( \alpha_4 \) depend on the geometrical slenderness \( h/s \) and on the aspect ratio \( h/b \) of the wall.

In the case of \( h/s = 10 \), the curves (1)-(4) associated to Eqs. (10)-(13) are plotted in Figure 6. It can be observed the influence of both edge restraints conditions and aspect ratio \( h/b \) on the out-of-plane resistance of the wall. In fact, the diagram of Figure 6 shows that: (i) the curves (1) and (2) (corresponding to wall 1L and 2L, respectively) do not depend on the aspect ratio \( h/b \); (ii) in presence of floor diaphragms at the top of the wall (wall 2L), the multiplier \( \alpha \) increases of 8 times with respect to the case of wall 1L; (iii) in presence of wall 3L and 4L with 1 or 2 stiffened vertical edges, the collapse multiplier \( \alpha \) shows a further increase when the \( h/b \) ratio increases. For example, in Figure 6 a comparison among the multipliers \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) is provided when \( h/b = 1 \).

3.2 Ancient rules of art

Regarding the stability of unreinforced masonry walls under horizontal loads, J. B. Rondelet (1802), in his
The historical treatise asserts that a wall will have strong stability if h/s = 8, medium stability if h/s = 10, low stability if h/s = 12.

From the well-known study on the collapse mechanisms for unreinforced masonry panels characterized by different restraint conditions, Rondelet realizes that, for the same degree of stability, the thickness s can be reduced when the distance b between the transversal wall decreases. Particularly, in the case of a wall of one floor and unique room building, when the beams of the roof are well fixed to the wall and there are no roof thrust, the minimum value of the thickness s which guarantees an adequate degree of stability for the wall, can be calculated using the following empirical rule by Rondelet:

\[
s = \frac{b \cdot h}{12 \cdot \sqrt{b^2 + h^2}}
\]  

In term of h/s ratio, the Eq. (14) can be written as follow:

\[
\frac{h}{s} = 12 \sqrt{1 + \left(\frac{h}{b}\right)^2}
\]

The geometrical Rondelet’s rule (15) is plotted in Figure 7 (curve (d)) together with the limits that Rondelet gives for wall 1L: h/s = 8, strong stability (curve (a)); h/s = 10, medium stability (curve (b)); h/s = 12, low stability (curve (c)).

Particularly, the curve (d) shows that the required h/s ratio to guarantee the stability of the wall increases with the aspect ratio h/b of the masonry panel; for example, if h = 3.5 m, for h/b = 1 it is necessary to have a minimum value of h/s = 17, i.e. a wall thickness s = 21 cm; instead, for h/b = 1.5 it is necessary to have a minimum value of h/s = 21, i.e. a wall thickness s = 17 cm.

The above Rondelet’s rules, in terms of collapse multiplier α (Eqs. (10) and (13)), lead to the curves (a)÷(d) reported in Figure 8. The comparison among these curves shows the great increase of resistance to out-of-plane loads of wall 4L (curve (d)) with respect to wall 1L (curves (a)÷(c)). Further, the trend of curve (d) shows that the multiplier α increases with h/b ratio; indeed, the curve (d) is asymptotic to the vertical line h/b = 1.5. In such meaning, Rondelet writes that the collapse of wall is impossible when the distance b between the transversal stiffening walls is low.

### 3.3 Seismic codes for unreinforced masonry buildings

In order to reduce the attitude to out-of-plane collapse of unreinforced masonry panels, also the modern European seismic codes (EC8’03 and NTC’07 (seismic part)) limit the h/s ratio through the conventional slenderness \( \lambda_c \) (Eq. (2)). Particularly, in presence of masonry wall with natural stone units, the Eurocode 8 prescribes the following limit:

\[
\lambda_c = \frac{\rho \cdot h}{s} \leq 9
\]

while, the new Italian code NTC’07 recommends the following limits:

- seismic zone 1 \((a_p=0.35g)\) & 2 \((a_p=0.25g)\): \( \lambda_c = \frac{\rho \cdot h}{s} \leq 10 \)
- seismic zone 3 \((a_p=0.15g)\) & 4 \((a_p=0.05g)\): \( \lambda_c = \frac{\rho \cdot h}{s} \leq 12 \)

The above limitations on conventional slenderness \( \lambda_c \), and consequently on h/s ratio, are more restrictive with respect to the case of buildings in non-seismic zone. In Figure 9 the comparison among the analyzed seismic limitations are provided in term of h/s ratio. In the cases of wall 1L and 4L, it can be observed a substantial coincidence between the limitations of EC8’03 and NTC’07, especially when h/b < 1 (for wall 4L); instead, for h/b > 1 the limitations prescribed by EC8’03 are more conservative than the Italian provisions ones. Finally, also for this diagram it is possible to note the effect of edge restraints on the h/s limits.
In terms of collapse multiplier $\alpha$ (Eqs. (11)-(12)), the condition (16) recommended in EC8’03 allows to obtain the curves plotted in the diagram $h/b-\alpha$ of Figure 10. Particularly in figure are reported: the curves (e) and (f) for the wall 2L; the curves (g) and (h) for the wall 3L; the curves (i) and (l) for the wall 4L. Each couple of curves is associated to the condition $\rho^2 = 0.75$ or $\rho^2 = 1$ that the EC6’03 considers in the definition of $h_{ef}$.

The limitations (17) and (18) of Italian code NTC’07 (seismic part) are plotted in the $h/s-\alpha$ diagrams of Figure 11 for wall 2L (curves (m) and (n)) and for wall 4L (curves (p) and (q)). The diagram shows that in high seismicity zones (curves (m) and (n)) the Italian code recommends a minimum value of collapse multiplier $\alpha$ higher than the case of low seismicity (curves (n) and (q)). Moreover, the curves (p) and (q) associated to wall 4L show a constant trend in the range $h/b = 0.5 \div 1$, i.e. in the most common range of aspect ratio $h/b$ for the masonry walls. The presence of the horizontal plateau in the curves (p) and (q) depends on the fact that the Italian code uses the same expressions of the collapse mechanisms as the ones examined in §3.1 to define the reduction factor $\rho$.

In Figure 12 the comparison among the limitations of Rondelet, EC8’03 and NTC’07 (in seismic zones) on collapse multiplier $\alpha$. walls 2L and 4L it is possible to note a certain coincidence among the plotted curves; particularly, it can be observed that the Rondelet’s curve (d) is overlaid to the curve (q) (associated to NTC’07) and to the curve (g) of EC8’03 (wall 3L, $\rho^2 = 0.75$) when $h/b > 1$ and $h/b < 0.8$, respectively.

4 F.E.M. PARAMETRIC ANALYSIS

In this paragraph, non linear analyses on rectangular walls are carried out through the F.E.M. computer code Abaqus 6.7-1 [Simulia, 2007]. On the basis of these analysis results, some considerations about the collapse multiplier $\alpha$ of walls 1L, 2L, 3L and 4L subjected to horizontal loads, are derived.

4.1 Modelling of the masonry walls

In Table 1 are reported the characteristics of the 42 examined models. The specimens have been labelled according to the following criteria: h-b-s Ln, where the h, b and s are the geometrical dimensions of the wall (in cm) and Ln (n = 1-4) defines the edge restraint of the wall as schematized in Figure 2. Regarding the
geometry, all the walls have height $h = 350$ cm; the width $b$ varies between 233 and 1400 cm in order to have walls with aspect ratio between 1.5 and 0.25. The thickness $s$ is set equal to 30, 35, 45 cm, in order to have walls with $h/s$ ratio equal to 12, 10 and 8, respectively.

Four-nodes shell elements (S4R5 elements) are used to model the masonry tuff walls; reduced integration is used for the shell elements; the number of integration points through the thickness of shell element is equal to five.

All the examined walls have been subjected to non linear analyses using a smeared cracking approach as implemented in the computer code Abaqus.

### Table 1. Examined models.

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<tr>
<th>Specimen</th>
<th>$h/b$</th>
<th>$h/s$</th>
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<td>8</td>
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<tr>
<td>350-350-45 L2</td>
<td>1</td>
<td>8</td>
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In order to correctly calibrate the model parameters, reference has been made to the curve fitting procedure was made by Giordano A. (2002), which utilizes the results of the experimental tests on masonry tuff walls.

The applied loads are the self weight and the horizontal load, which increases with monotonic low up to the end of the analysis.

### 4.2 Results and comparisons

In Figure 13a, a summary of the non linear analyses for the walls 350-350-35 is reported; particularly, the pushover curves are depicted. In this diagram, the collapse multipliers calculated through the limits analyses and already provided in Figure 6 are also reported. The comparison among the nonlinear static and limit analyses shows that the F.E.M. results provide collapse multipliers higher than limit analyses, because the tensile strength of masonry is considered in the Abaqus models.

Indeed, in Figure 13b and c the visualization of the F.E.M. deformed shapes, with stress tensor vectors, and the hypothesized collapse mechanisms are reported. The comparison shows a good agreement between the Abaqus and limit analyses.

In Figure 14 the comparison among the collapse multipliers computed by means of F.E.M. analysis and of limit analyses (Eqs. (10)÷(13)) is provided. Particularly, the diagrams refers to walls characterized by $h/s$ ratio equal to 10. In almost all analyzed specimens (Table 1) the diagrams confirm the previous
Figure 14. Comparison among the collapse multipliers of walls with \( h/s = 10 \) evaluated through F.E.M. and limit analyses.

Figure 15. Comparison among the collapse multipliers evaluated through nonlinear static and limit analyses on walls designed according to the geometrical requirements of EC8’03 (a) and NTC’07 (in seismic zones) (b).

observation regarding the multipliers \( \alpha \); in fact, the values of \( \alpha \) computed through pushover analyses are generally higher than the values calculated with the limit analyses. Finally, also the curves associated to the results of nonlinear static analyses carried out on walls 3L and 4L show an increasing trend with the aspect ratio \( h/b \); this confirms the beneficial effect of the vertical edge restraints on the resistance of the walls to out-of-plane collapses.

The Figure 15 shows the comparison among the F.E.M. results and the curves relative to collapse multipliers \( \alpha \) of the walls 3L and 4L associated to EC8’03 (Fig. 15a) and NTC’07 (Fig. 15b). The thicknesses \( s \) of the analyzed specimens have been obtained by Figure 9 curves (4) (wall 3L) and (6) (wall 4L) for EC8’03, and by Figure 9 curves (8) and (9) for NTC’07. In the case of EC8’03, the results of F.E.M. analyses show an increasing trend of the curves (Fig. 15b), while in the case NTC’07 it possible to note a sub-horizontal trend of the F.E.M. curves.

Moreover, when \( h/b \) ratio is larger than one, the NTC’07 curves show a slight decrease of collapse multipliers; this seems to underline that the limitations of NTC’07 give very large values of maximum \( h/s \) ratio when the wall is characterized by high values of aspect ratio \( h/b \).

Finally, the comparison among the F.E.M. results reported in the curves (a) of Figure 15 shows that the EC8’03 is on the safe side with respect to NTC’07; in fact, it can be observed that the curve “Abaqus 4L” of Figure 15a is characterized by higher values of collapse multipliers \( \alpha \) than the curves “Abaqus 4L” plotted in Figure 15b.

5 CONCLUSIONS

The study of the out-of-plane collapses of masonry walls subjected to vertical loads has suggested the relationship between the slendernesses \( \lambda \) and \( \lambda_c \) or, equivalently, between the coefficients of restraint \( \beta \) and \( \rho \). In term of slenderness \( \lambda \), it has been observed that a wall characterized by \( \lambda < 20 \) can be defined as stocky wall, having a low reduction of axial strength \( N_f \) with respect to the squash resistance \( N_{pl} \) of the wall; when \( \lambda > 50 \div 60 \), instead, the wall can be considered as slender wall. Finally, if the slenderness \( \lambda \) is comprised between 20 and 50\( \div 60 \), the wall is characterized by an intermediate behaviour, with interaction between the crushing and the elastic buckling of the wall.

In the second part of this paper, the issue of out-of-plane collapses of masonry walls subjected to horizontal loads has been studied through (i) the analyses of the collapse mechanisms; (ii) the review of ancient rules of the art; (iii) the study of the European (EC8’03) and new Italian seismic codes (NTC’07) for unreinforced masonry buildings.

In the case of walls 3L and 4L, the comparison among the geometrical limitations underlines that the recommended h/s ratio becomes greater than the values h/s = 8, 10 or 12, that Rondelet suggests for the wall 1L. For example, in the range \( h/b = 0.5 \div 1.5 \), the required geometrical slenderness h/s for the wall 4L is equal to or smaller than: 13\( \div 21 \) according to Rondelet’s rule (Eq. 15); 11\( \div 27 \), according to EC8’03; 10\( \div 32 \), according to NTC’07.

These observations are also confirmed in the practice for a wider range of aspect ratio h/b, as shown in Figure 16 for the 8 classes of macro-elements of 10 masonry churches studied in Brandonisio et al. (2008).
The study of the collapse mechanisms together with the analyses of the geometrical requirements derived from the ancient rules and the modern seismic provisions, show the influence of the $h/s$, $h/b$ and edge restraints conditions on the out-of-plane collapse multiplier $\alpha$.

The $h/b - \alpha$ curves suggest a remark regarding the “Rules for simple masonry buildings” reported in the EC8’03 and NTC’07. In fact, both EC8’03 and NTC’07 require a maximum spacing of 7 m between two bracing walls; this geometrical requirement, for the common values of storey height $h = 3.5 \div 5$ m, conducts to walls characterized by aspect ratios $h/b$ greater than 0.5. In absence of holes, this leads to masonry panels which resist to out-of-plane lateral loads equal to: 87% (when $\rho_2 = 0.75$) or 106% (when $\rho_2 = 1$) of their self weight, in the case of walls designed according to EC8’03; 120% (in seismic areas 1 and 2) or 100% (in seismic areas 3 and 4), in the case of Italian seismic code.

In the last part of the paper, the results of a parametric nonlinear F.E.M. analysis carried out on rectangular masonry walls has been presented. In the F.E.M. analyses both $h/s$ and $h/b$ ratios and the edge restraints have been varied. The results confirm the observation done through the application of the limit analyses, i.e. the respect of the seismic codes limitations on the geometrical slenderness $h/s$ allows to implicitly have a minimum resistance to the out-of-plane collapses of the masonry walls subjected to horizontal loads.

In the light of the previous consideration, it could be suggested to overcome the indications concerning the maximum spacing between the cross walls equal to 7 m often recurrent both in ancient treatises and in the modern seismic codes. In particular, it seem that a more appropriate limitation on orthogonal elements should be given in non dimensional forms, i.e. in terms of aspect ratio of the masonry panel: $h/b \geq 0.5$.

REFERENCES


Simulia 2007. ABAQUS Theory Manual. USA.