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Guastavino dome analysis: A comparative approach for Jefferson's Rotunda at the University of Virginia

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ABSTRACT: Contributing to over 1000 buildings in North America, Rafael Guastavino, Sr. and his son greatly influenced American turn-of-the-century architecture with their trademark vault structures formed by specially designed clay tiles and cement mortar. The heterogeneous material composition and traditional construction methods gave rise to forms much less common to today's building and analysis methods, challenging the resources and training for engineers working with these historic structures. Guastavino outlined his methodology for designing arches, domes and vaults in an essay in 1893. The equations were developed from idealizations in basic mechanics while in combination with numerous tests performed on his vaulting system. This study compares Guastavino's equations for dome analysis to theoretical approaches using Thomas Jefferson's Rotunda dome at the University of Virginia as a case study. As part of a recent structural evaluation of the existing tile dome, Robert Silman Associates employed membrane theory for thin-shell analysis, graphical analysis, and finite element modeling as a comparison to Guastavino's empirical methodology.

1 INTRODUCTION

Rafael Guastavino, Sr. (1842 to 1908) was born in Spain; trained as a builder and architect, he moved to the United States in 1881 and brought with him the art of Catalán tile vaulting (Austin, 1999). Tile vaults have origins in many countries, however they are generally attributed to the Spanish in the Catalonia area. The vaults are characterized by a gently curving profile, large spans, relatively thin cross-sections and a short construction phase due to the fast-setting mortar. Usually the tiles are laid flat, with the length of the tiles in the plane of the vault. Vaults with this brick orientation are defined by Guastavino as a timbrel vault (Guastavino, 1895).

Guastavino employs the Catalán vaulting system, utilizing liberal amounts of cementitious mortar as a binding element connecting the layers of clay tile and as a structural component to resist tension loads during construction. The result of this construction was more monolithic and elastic structural behavior than pure tile systems. Guastavino referred to this building system as 'cohesive construction' (Guastavino 1895). In addition to the pleasing appearance, tiled Catalán vaults have an inherent fire-resistant quality. Arriving in the United States shortly after fires in both Chicago and Boston, Guastavino's fire-resistant vaults quickly became very popular. Realizing the need to prove the advantages of timbrel vaults to American architects and engineers, Guastavino created the Guastavino Fireproof Construction Company and began experimenting with tile and mortar designs.

Fire is also what brought Guastavino to Charlottesville, Virginia. The Rotunda building on the University of Virginia's campus caught fire in 1895 destroying all the interior framing and the laminated wood dome (Figure 1). Designed by Thomas Jefferson and completed in 1826, the Rotunda was the crown of Jefferson's "Academical Village" (Wilson 1995). Based on a one-third-scale model of the Pantheon in Rome, the Rotunda was constructed out of materials found locally to the central Virginia (Figure 2). Jefferson's original design included a laminated wood dome. The dome construction was designed based on domes by Philibert Delorme that Jefferson had seen in France. Delorme domes are constructed from thin laminates of wood, lapped and joined together to create large ribs of a dome structure. The lightweight design of the laminated wood dome and the relatively inexpensive and quick construction were among the great advantages of the system, however its susceptibility to fire was a great shortcoming. The cylindrical exterior walls that survived the fire are composed of multi-wythe masonry brick and the original interior and roof framing was all made from wood. After the fire, the architectural firm of McKim Mead and

Figure 1. Jefferson's Rotunda after the 1895 fire (from Arise and Build).

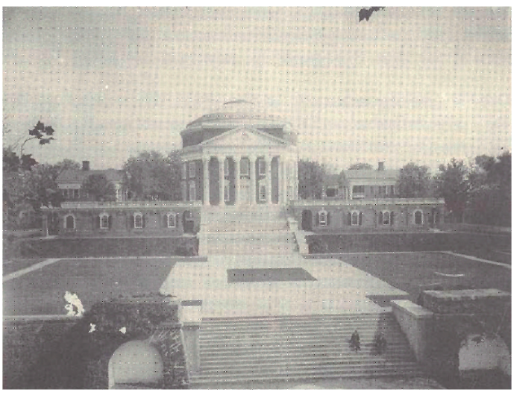

Figure 2. University of Virginia's Rotunda: Reconstructed dome in 1898 (from Arise and Build, 30).

White from New York City was selected to rebuild the Rotunda. The architects then asked Guastavino to design a non-combustible dome over the remaining brick walls.

2 STRUCTURE DESCRIPTION

Guastavino chose a double dome system with an air gap of about 18" between the domes. Both inner and outer domes were constructed using long slender tiles. These tiles were specially designed by Guastavino. The inner dome of the Rotunda (and presumably the exterior dome as well) is constructed with three layers of tiles near the support (1" thick and overlapping at the joints). Between the tiles are two ½" wide layers of cementitious mortar (total 4" thickness).

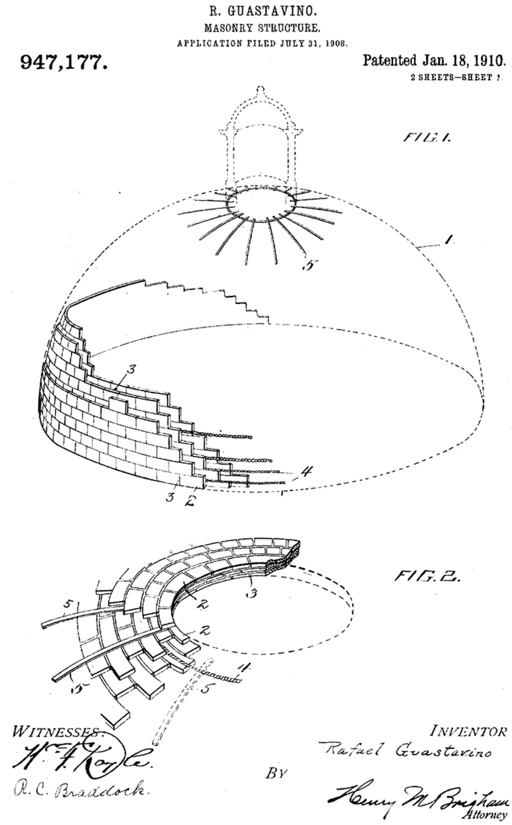

Figure 3. Sketch from Raphael Guastavino's patent issued on January 18, 1910 for a dome with an oculus (from google.com patent search, online).

Similar to the dome depicted in his patent number 947,177, which Guastavino obtained January 18, 1910 (Figure 3), the earlier Rotunda dome is a three-tile system with an oculus. Additionally within this patent, Guastavino illustrates internal iron reinforcement. The presence of such elements within the tile system itself however, has not been observed in the dome of the Rotunda. Guastavino also patented his clay tiles and used a durable and fast-drying cementitious mortar to solidly bind the tiles at the joints. Guastavino experimented with the tile designs and cement quantities to arrive at the optimal cement ratio and tile configuration.

Completed in 1898, the finished Rotunda was greatly different from the original, however the new construction was fire-resistant with much of the framing composed of Guastavino's tile and selective iron and steel tensile components.

Although the new dome had the great advantage of being a non-combustible, fireproof material, it had

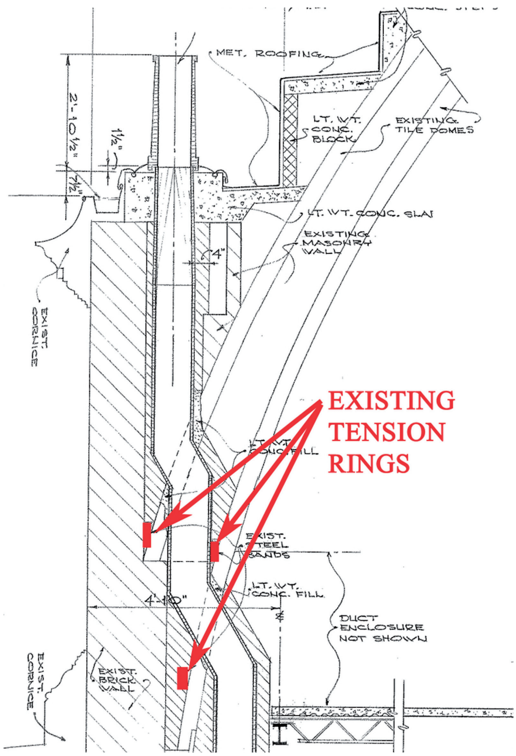

Figure 4. Embedded tension rings at the Rotunda (Ballou, D-4).

the disadvantage of being significantly heavier. Both a vertical and horizontal force had to be resisted at the base. Because the overall load was greater, the horizontal thrust is correspondingly greater. To resist this horizontal thrust, a series of steel plate tension rings were installed (Figure 4). It was common to introduce a series of embedded tension rings within the shell of tile structures. These were often in the form of metallic rods with twisted or deformed cross section to enable an integrated bonding within the tile and mortar assembly.

Unlike today, many of the proprietary structural systems built in the United States at the turn of the century were designed based on experimental load tests. Guastavino was no different. He experi mented with various tiles, cementitous mortars and configurations, until he discovered the optimal design for his purpose. In 1893, at the Society of Arts in Boston, Massachusetts, Guastavino presented his "Essay on the Theory and History of Cohesive Construction, Applied Especially to the Timbrel Vault." In this essay, Guastavino outlines guidelines for analyzing arches, domes and vaults, however acknowledging that "we do not pretend to have an absolute mathematical formula, but a practical one, enough to insure sufficient security for safe construction" (Guastavino 1985). The equations were developed in the 17th century based on the principles of equilibrium (Becchi 2003). Guastavino applies these equations with the results of his numerous load tests to obtain workable formulas to numerically design his vaulted systems.

3 ANALYSIS

Through experimentation, Guastavino obtained average values of ultimate stresses in the tile and cement mortar system. Guastavino's "transverse resistance", or what we would refer to as a modulus of rupture in out-of-plane bending, was found to be 90 psi (621 kPa). His average ultimate tensile strength was 287 psi (1979 kPa) and a crushing strength of 2060 psi (14203 kPa) (Guastavino 1895).

Around the same time period, Columbia University in New York was also performing tests on the structural strengths of terracotta tiles. For a semi-porous clay tile placed perpendicular to the force, Columbia obtained an average crushing strength of 2168 psi (14948 kPa) (Kidder 1921), interestingly within 5% of Guastavino's value.

3.1 *Guastavino analysis*

According to Guastavino's essay the best system for creating a dome consisted of three layers of tiles and two thick layers of cementitious mortar squeezed between the tiles. For this design, Guastavino derived an equation (Equation 1) for tiled domes under uniform loading conditions (Guastavino 1895 – Figure 5). The equation uses a safety factor of 10 to convert the crushing strength of the tiles to an allowable working stress. An allowable compressive stress of 206 psi (1420 kPa) is within a common range for masonry construction of the time, however a safety factor of 10 is noted by Guastavino to be somewhat conservative (Guastavino, p. 149).

The formula is perhaps best understood when comparing its evolution from Guastavino's equation for the arch or barrel vault. At its root is the simple thrust equation for a three-hinged arch under a horizontally projected, uniformly distributed load. The translation to the dome equation simply divides the arch thrust equation by 2. Significant among the initial assumptions is that the arch or vault selected as examples have a 10% rise to span ratio, a relatively flat profile. This is significant in that the loading is presumed to be uniform over a horizontal projection, per "superficial foot," in Guastavino's terminology. Thus, the expectation is that the equation given may have limited applicability for domes of significantly higher rise to span ratios, even though it is not explicitly limited by Guastavino as such. Guastavino also provides an adjustment to his equation to account for changing

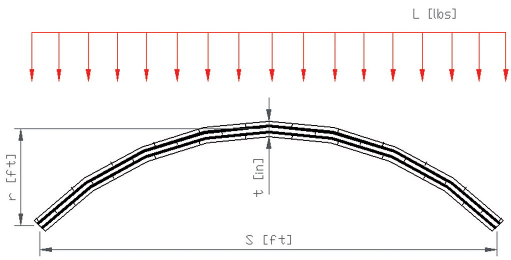

Figure 5. Guastavino's method of analyzing his dome construction involves variables, r, S, T and L depicted above. The load L functions as a horizontally projected, uniformly distributed load in the equation, although it must correspond to the varying surface load of the vault.

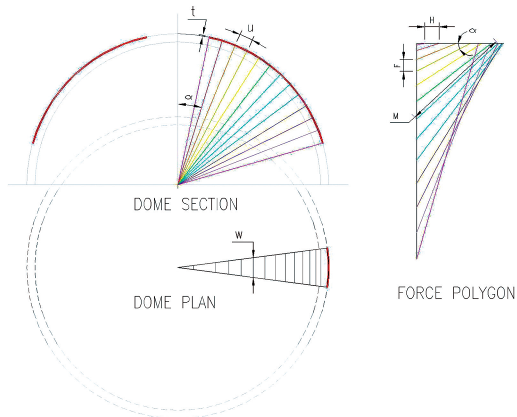

Figure 6. RSA's graphical analysis on the Rotunda dome.

force over the dome profile, however the approach is not fully explained and requires further research to derive.

Guastavino's Dome Analysis:

$$t = \frac{L \cdot S}{8 \cdot r \cdot 12 \cdot C \cdot 2} \qquad (1)$$

t = thickness of the dome at crown (in)
C = coefficient of compression derived from Guastavino's experiments
= 2060 psi (Guastavino, 58)
S = span of the dome (ft)
r = rise of the dome (ft)
L = total projected load at center section (lbs).

At the exterior Rotunda dome, with span of 74.2 feet (22.6 m), rise of 27.9 feet (8.5 m), and assumed crown thickness of 4 inches, the equation results in the ultimate load along its center section, L, equaling a maximum of 595,000 lbs (269,900 kg). To look at allowable capacities, we divide by Guastavino's recommended safety factor of 10, giving an allowable load of 59,500 lbs (26,900 kg). Translating this horizontally projected total load over the surface arc length along the center section (99.67 feet (30.4 m)), a total surface load of 597 lbs/ft sq. (28.6 kPa) is calculated. With the dead load of the dome approximately 50 lbs/ft sq. (30.4 kPa) an allowable surface live load of 547 lbs/ft sq. (26.2 kPa) remains; this value is significantly greater than the standard live load requirement for a steeply sloping roof at the time of construction (perhaps 20 lbs/ft sq. (1.0 kPa)). Thus, according to Guastavino's equation, his tiled dome structure would satisfy the design requirement. Though not explicitly stated, it may be that residual capacity accounts for more nuanced loadings and loading configurations such as lateral and asymmetrical gravity applications. Guastavino would have undoubtedly been aware of such conditions and likely evaluated these to some extent with his empirical load tests.

At first glance the approach and results appear reasonable, however, Guastavino's theory of cohesive construction and application of equilibrium contains many simplifying assumptions and limitations. His essay reiterates the historical view of timbrel vaults as monolithic construction. To achieve such monolithic behavior, the heterogeneous tile assembly relies on the shear and tensile capacity of the cementitious mortar layers (Guastavino 1895). Despite the claims at substantial bending moment capacity, his vault structures consistently are supplemented with thrust-resisting components, such as the steel tension ring bands at the Rotunda.

3.2 *Graphical analysis*

Another design method available at the start of the 20th century was graphical analysis. Graphical analysis is also based on a theory of equilibrium and involves a scaled drawing accurately representing the dome in section and plan. The arch produced by the dome section is then divided into segments and the forces acting on each segment are illustrated by scaled lines connected in a force polygon. The stresses in the dome are derived from scaling lines in the force polygon and converting them into forces. (Becchi 2003). Assisting his father in the building of cohesive construction, Rafael Guastavino, Jr., experimented with this method of design.

Graphical analysis relies heavily on accurate drawings of the curvature, the angled guidelines that divide the structure into segments, and the scaling of force units to units of length. As such, the chance of human error is significant and the process is time consuming in comparison to other methods, despite being visually compelling. However, the fluid use of AutoCAD or other computer graphics does allow for increased precision in generating such graphical solutions.

Table 1. Meridional forces in the rotunda dome.

Meridonial stress				
Angle from y-axis [degrees]	Membrane theory [psi] (kPa)	Finite element modeling [psi] (kPa)	Graphical analysis [psi] (kPa)	Guastavino [psi] (kPa)
0.0	−32.3 (−222.6)		−51.8 (−356.9)	−27.60 (−190.3)
12.0	−32.6 (−225.1)	−16.4 (−113.3)	−38.1 (−262.8)	
18.2	−33.1 (−228.4)	−18.5 (−127.3)	−37.4 (−258.0)	
24.5	−33.8 (−233.1)	−23.7 (−163.4)	−37.5 (−258.3)	
30.6	−34.7 (−239.3)	−27.8 (−191.9)	−37.8 (−260.5)	
36.8	−35.9 (−247.3)	−30.8 (−212.2)	−38.6 (−265.9)	
43.0	−37.3 (−257.2)	−33.5 (−230.6)	−39.7 (−273.8)	
49.2	−39.1 (−269.3)	−35.8 (−247.0)	−41.0 (−282.8)	
55.4	−41.2 (−284.0)	−36.3 (−250.0)	−42.6 (−294.0)	
61.1	−43.5 (−300.2)	−41.3 (−284.6)	−44.6 (−307.3)	
67.8	−46.9 (−323.2)	−63.9 (−440.8)	−46.9 (−323.4)	
74.0	−50.6 (−349.1)	−52.7 (−363.2)	−49.7 (−342.5)	−58.9 (−406.1)

* Compressive stresses are negative.

Table 2. Hoop forces in the rotunda dome.

Hoop stresses				
angle from y-axis [degrees]	Membrane theory [psi] (kPa)	Finite element modeling [psi] (kPa)	Graphical analysis [psi] (kPa)	Guastavino [psi] (kPa)
0.0	−31.8 (−219.6)		−7.7 (−52.9)	
12.0	−29.5 (−203.4)	−64.5 (−444.7)	−7.2 (−49.9)	
18.2	−26.7 (−184.2)	−39.0 (−268.8)	−6.5 (−44.7)	
24.5	−23.0 (−158.8)	−31.2 (−215.0)	−5.6 (−38.4)	
30.6	−18.5 (−127.4)	−25.3 (−174.1)	−4.6 (−31.7)	
36.8	−13.0 (−89.6)	−18.8 (−129.8)	−3.2 (−22.4)	
43.0	−6.6 (−45.8)	−12.3 (−84.8)	−1.9 (−13.0)	
49.2	0.6 (4.0)	−5.7 (−39.3)	−0.3 (−2.2)	
55.4	8.3 (57.4)	2.8 (19.4)	1.4 (9.5)	
61.1	17.3 (119.1)	15.6 (107.9)	3.2 (22.3)	
67.8	27.5 (189.8)	16.8 (115.5)	5.3 (36.3)	
74.0	32.8 (226.3)	−10.8 (−74.6)	7.5 (51.5)	

* Compressive stresses are negative.

The Rotunda dome was analyzed with the traditional graphical analysis techniques to acquire stresses in the dome caused by the material dead load and superimposed live loads distributed uniformly over the surface of the dome (Figure 6). The results are in Tables 1 and 2 below.

3.3 *Membrane theory analysis*

Another analysis technique based on the equilibrium theory is the membrane theory. The approach was popularized in the 1950s as a modern analysis technique for designing thin-shelled domes. The procedure involved two perpendicular forces, meridional forces, and hoop forces. Meridional forces occurred from the bottom edge of the dome, across the top and to the other bottom edge, or in what would be the north-south (longitudinal) direction on a globe. Hoop forces occurred on horizontal planes through the dome, or in what would be the east-west (latitudinal) direction on a globe (Figure 7). The meridional and hoop forces in the domes were calculated using Equations 2 & 3.

Meridional Forces: $$N'_{\phi} = -aq\frac{1}{1+\cos\phi} \qquad (2)$$

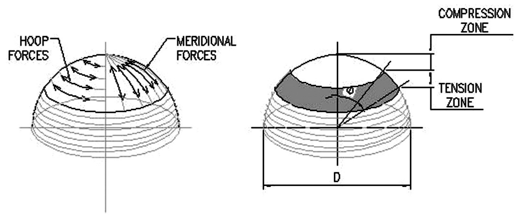

Figure 7. The sketches above were used to analyze the dome using meridian and hoop stresses.

Hoop Forces: $$N'_\theta = aq\left(\frac{1}{1+\cos\phi} + \cos\phi\right) \quad (3)$$

Where a = radius of dome (ft)
q = load (psf)
Φ = angle from vertical to point on dome

The membrane theory is limited by the assumption that all internal resisting forces are axial tension or compression in the plane of the vault, neglecting out of plane bending.

3.4 *Analysis from a finite element model*

The Rotunda was also analyzed with a simplified finite element model which generated results based on linear elastic theory. A principle assumption in elastic theory is that the dome is a homogeneous, monolithic structure, with shell element having capacity for out-of-plane bending resistance. The model is based on a thin-shelled dome with a pinned base. The additional masonry around the base of the dome was not considered in this preliminary comparative analysis.

4 ANALYSIS RESULTS

The results of all four methods of analysis performed on the dome are below in Tables 1 and 2.

The calculated stress levels for the four methods are relatively consistent for the meridional stress. Guastavino's value at the theoretical crown, based upon his thrust calculation, compares well with the membrane theory results and offers a quick estimation of the thrust stresses. The general compressive stress values are well within typical allowable stresses for brick masonry, which is on the order of 200 to 250 psi (1379 to 1724 kPa) (Kidder 1921). This is in-keeping with Guastavino's allowable compressive stress of 206 psi (1420 kPa).

Interestingly, Guastavino does not address the determination of hoop force or stress in his equations, although he seems to clearly understand the importance of this resistance. Although he does not directly address the requirement for external thrust resistance, by way of buttressing or tension rings, his designs consistently include such measures.

Using the membrane theory and modern analysis Equation 4, the tensile force in the steel tension rings was calculated.

Tension Ring Force: $$T = rN'_\Phi \cos\Phi \sin\Phi \quad (4)$$

Where r = radius of dome in plan (ft)

The tension ring force in the outer domes was calculated to be 25 kips (11340 kg). The inner dome would have less of a tension ring force since there was very little live loading applied, but may be con servatively taken as 20 kips (9072 kg), giving a combined ring force of 45 kips (20410 kg). The steel pieces found embedded in the walls were 4"×½" (10.2 cm × 1.3 cm) and 3"×3/4" (7.6 cm × 1.9 cm).

Drawings from a 20th century renovation indicate that there are three metal bands. Assuming an 18,000-psi (124,106-kPa) allowable tensile strength for the early steel bands, an approximate total resisting force was calculated to be 113 kips (51,260 kg). This value is more than sufficient to carry the combined tension ring force for the inner and outer domes. The placement of the rings also demonstrated Guastavino's knowledge of the structure and awareness of a need for external thrust resistance.

5 CONCLUSION

In conclusion, the analysis equation Guastavino provided in his "Essay of Cohesive Construction" obtained results similar to those done by other analysis methods. Guastavino's ultimate stresses were comparable to other testing done at the same time period. His analysis method was rooted in basic mechanical theory and gives the appearance of a reliable scientific approach, however, Guastavino's simple equation is found to be insufficient for anything more than an approximate analysis offering insight into the builder's thinking and assumptions. Any formal structural analysis must be supported with some alternative technique to achieve a reliable prediction of internal stresses or loads imposed by the dome on the surrounding structure. Perhaps the most important evidence in support of Guastavino's overall approach however, is the undeniable success of his constructions. Given the apparent analytical limitations, it seems clear that his program of load testing and empirical study, following upon the building tradition of Catalán vaulting, were essential to his profound achievements.

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