

## From Simple Roof Structure Calculus Based on 2D Modelling to 3D Models–Case study: Reformed Church in Cluj-N., Romania

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**Abstract:** Computer aided modelling, software options, as well as hardware performances have developed rapidly in the last several decades. Though the time required for building up a holistic 3D mechanical computer model for a baroque roof structure decreased significantly, it may be worth to stop at simple calculus, 2D or limited 3D modelling level, in various cases. Conclusions of the present lecture aim to identify the accuracy and reliability of the 4 levels of modelling. The recommendations formulated intend to identify the appropriate level of research according to the span, historic roof-type and state of decay of a given roof that is to be conserved. Research schemes are also suggested, aiming to offer an efficient tool for experts and engineers according to the complexity of the assessment.

**Keywords:** Historic / Baroque roof structure, mechanical modelling, conservation

### Introduction

University curricula (in structural engineering) do not include tuition in *historic load-bearing structures*. In Romania, even modern roof or timber structures were for a long time totally absent from construction and architecture university curricula. Modern (engineered) timber structures slowly made their way back into the curricula, but traditional carpentry-structures are still just occasionally taught, at institutions where professors possess expertise in conserving historic buildings / structures (for example: the Technical University of Cluj-N., Faculty of Architecture and Urbanism).

The historic and aesthetic value of traditional (baroque roof) structures is already a certainty for specialists in built heritage conservation, just like the importance of the message carried by their structural construction rules, historic material and execution technology.

There exist several competing interpretations regarding the necessity and methods of modelling, or regarding the mechanic behaviour of the baroque roof structures and their compounding elements, put forward by theoreticians of the topic. (VÁNDOR et al.)

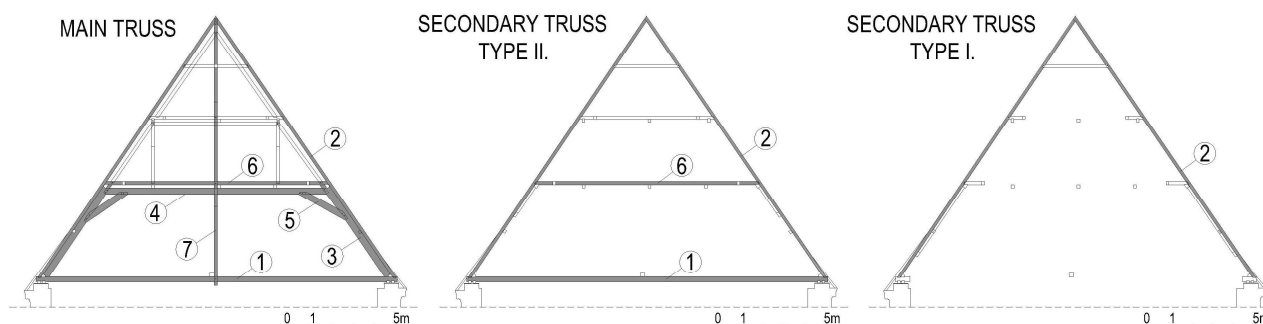


Figure 1: Code according to the typology of Transylvanian baroque roofs: A.2.2(c)–a/c(3)–I-A(1) – (see SAHC 2008, pg.663-671), analysed elements: (1) – tie-beam; (2) – common rafter; (3) – main rafter; (4) – straining beam; (5) counterbrace; (6) collar beam; (7) compound suspension bar

### Levels and Special Questions of Modelling of Historic (baroque) Roof Structures

The analysed models are built up for an existing (outstanding) 15.70m span baroque roof structure (Reformed church, Kogălniceanu street, Cluj-N., Romania). The mechanical behaviour and the safety

of subensembles (main and secondary trusses, longitudinal bracing systems), as well as of the main compounding elements of the baroque roof (common rafters – 2; tie-beams – 1; main rafters – 3; straining beam – 4; counter braces – 5; collar beams – 6; compound suspension bar – 7, etc. – Fig. 1) are analysed according to the following sets of models (Fig. 2):

**(2.1.) Simple element calculus** for common rafters as continuous beams; the baroque straining (and longitudinal bracing) system as pinned simple frames – (models: 2.1.1–6.);

**(2.2.) Plane – 2D models** for secondary trusses, longitudinal bracing system and main trusses

A useful tool in researching the overall behaviour of spatial roof structures that can be divided into two plane systems (transversal and longitudinal ones), for gable roofs, or pitched roofs where pitches are not significant, is the: **(2.3.) Spatial – 3D limited model** – including two bays created between three successive main trusses including secondary trusses and longitudinal bracing systems. The compounding elements of the middle main truss will behave as any of the series of trusses. In special cases, the **(2.4.) Spatial – 3D holistic model** can be used, for spatial structures that cannot be decompounded into plane frames, or in cases when the pitched part of a roof has a significant impact on the overall behaviour of the structure.

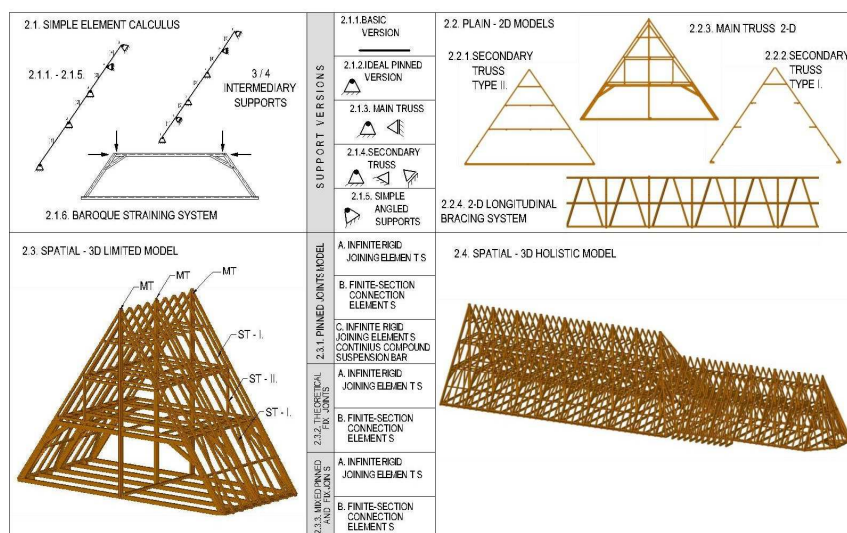


Figure 2: Levels and versions of modelling

In parallel to a number of hypotheses that were used for all models, from each levels of analysis derives a set of special questions.

**Simple element calculus** is taught even in our days as “the way” to check or design a rafter / purlin within an ordinary roof structure (2.1.1. loads projected on a horizontal beam). It also represented the early-engineering way of interpreting the mechanical behaviour of rafters (19<sup>th</sup> century). The mechanical model of continuous beam is perfect for costal roofs (on purlins) (Szabó 2009) and pure eclectic continental roofs (Szabó 2005), where common rafters are “carried elements”. Within *rafter-tie-beam* structures (closed triangle frames – as baroque roof trusses) rafters’ mechanical behaviour can be (partially) described as a continuous beam. Purlins and collars are modelled as supports in 4 different ways (Figure 3). The present set of calculations were carried out by using infinite rigid supports; compression stiffness of collars and bending stiffness of purlins can also be simulated by using spring-supports. Differences of the elements' behaviour by projecting the loads on the horizontal beam to the angled continuous beam with pins and simple supports were also analysed.

The baroque longitudinal bracing system (Fig. 4) – situated in the plane of the rafters – contains a longitudinal element (20) at the half of its height. In ordinary baroque roof structures these have no supporting role for the common rafters (models with 3 intermediary supports); but in the case of large spans they can have such a role (models with 4 intermediary supports).

The baroque straining system in this approach is loaded only by vertical and horizontal concentrated loads, summing up 4 support reactions of joint 3 (Fig. 3).

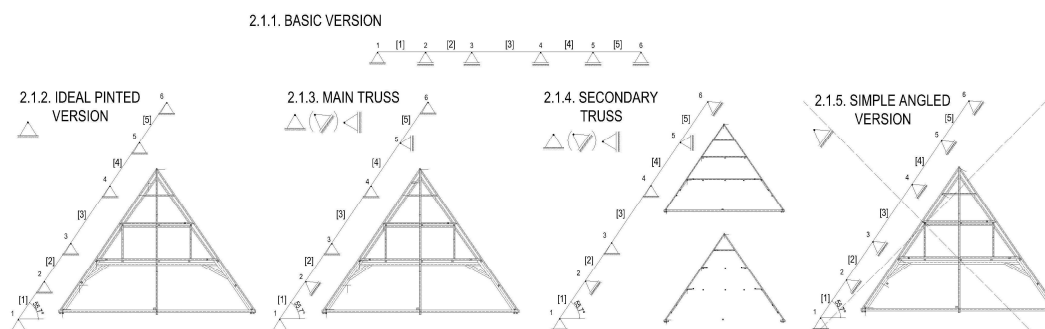


Figure 3: Continuous beam models – 4 intermediate supports

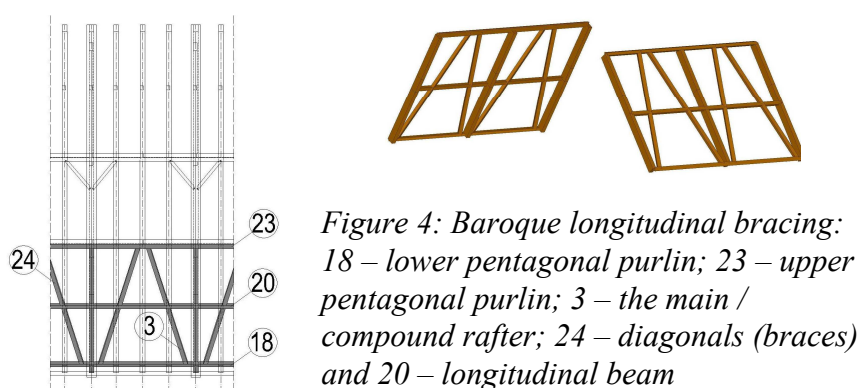


Figure 4: Baroque longitudinal bracing:  
18 – lower pentagonal purlin; 23 – upper  
pentagonal purlin; 3 – the main /  
compound rafter; 24 – diagonals (braces)  
and 20 – longitudinal beam



**Plane – 2D models** of the secondary trusses, longitudinal bracing system and main trusses represented a useful tool in analysing roof structures that can be decomposed into two plane systems in the last decade of the 20<sup>th</sup> century (software such as P-Frame, used for 2D – linear – frame analysis, made possible the quick modelling of main and secondary trusses of historic roof structures). The most common analysis was using linear bar frames with simple pin joints. A series of questions arose concerning load transfer from secondary truss to main ones.

Purlins, header beams in secondary trusses can be considered as pin supports, but their rigidity is actually assured by the bending stiffness in the vertical and horizontal plane of these beams. Therefore a more accurate modelling involves a primary model of longitudinal elements or sub-ensembles in order to identify the spring-force constant ( $k$  – kN/m). Once the constants are calculated, the secondary trusses supported by spring or infinite rigid supports are analysed. Main trusses can be loaded both by distributed loads applied on common rafters, and concentrated loads resulting from summing up the spring / support reactions of the secondary trusses (2 x type I+1 x type II).

**Internal joints** (one of the major questions for all the following models): are mainly x-z plane pins, but complex carpentry joints (common+main rafters-straining beams and counterbraces) are capable for a limited bending moment transmission – 2 pegs – Fig. 5). Therefore it is worth analysing a mixed model including infinite / or finite (mathematically calculated) fixed joints, as well as pins.

A deficiency of the plane frame model is that compound elements (7 – compound suspension bar) are to be modelled as a simple rectangular sections, from which the question of interrupting the suspension bar and / or the collar beams also derives. Both elements are continuous, pinned joints perturb the overall behaviour, fixed joints introduce extra stiffness as well as non existing bending moments into the structure – Fig. 5.

Analysis used in conservation practice preferred the pin joints-version, as the resulting structure was less stiff, so the calculus errors contributed to underestimating the global safety of the structure.

**Spatial – 3D limited model** – has become easy to construct using various software since the end of the last century (the models presented were built in Axis VM 9 – Finite Element Analysis & Design Program, but parallel modelling was also carried out in Robot Structural Analysis, and SAP 2000 or

S-Frame or any other advanced software could also be explored in order to compare reliability, easiness, etc.)

This level eliminated the questions of modelling load transmission, but at the same time a number of further questions arose (related to limitations of the software used). The question of internal pin / infinite or partially fixed joints became even more complicated as the pins plane (not spherical joints) had to be defined carefully according to the plane of the given element.

Modelling the connection between axes that geometrically do not intersect can be carried out by infinite rigid connection elements ("A" models) or short timber elements ("B" set). (Figure 5)

The same question arises when a compound element (7 – suspension bar) meets horizontal transversal elements (6 – collars, 4 – straining beams, 1 – tie-beams): connections have to be modelled in order to assure continuity both for the suspension bar and the horizontal elements, but without allowing bending moment transmission, alike the real joints. Interrupting material continuity (allowing just axial / shear force transmission) would reduce overall stiffness of the structure.

Built heritage conservation practice also prefers models with pin joints; modelling joining not intersecting axes differs from office to office or from school to school.



Figure 5: Complex joints: (1) tie-beam; (2) common rafter; (3) main rafter; (18) pentagonal eaves purlin; (24) diagonals; wall-plates; (7) compound suspension bar; (4) / (6) straining and collar beams.

**Spatial – 3D holistic model** sets the same questions as the former level, but the amount of work that needs to be invested is more significant. Uncertainties deriving from bar-end joints, connection elements can become more determinant. Therefore this level is used only in cases when the geometry of the structures requires the holistic approach, or in the case of extremely important structures.

### Data Used in Modelling

For all models loads were calculated using Romanian (European) codes: (1) dead loads: software generated self weight + tiling; (2) – pressure and suction (according to wind load regulation in Romania, in 3D modelling there are 5 values for wind load for transversal main direction, and 4 values for longitudinal wind direction); (3) snow-loads, when the angle of the roof ( $55.6^\circ$  – originally a  $3/2$  ratio,  $56.3^\circ$ ) is close to  $60^\circ$  for which snow loads become zero, and are not determinant. (4) technological and concentrated loads. In order to check the safety of the structure / elements – load combinations created according to the Eurocode (NP 082-04) were used.

An important data-set would need further (laboratory) studies, such as timber strength analysis. The material built into historic roof structures was trimmed centuries ago, mainly in winter-period, trees were grown in different climate conditions, their strength / quality should be superior to the common (regulated) calculation values (Eurocode 5). The presented studies were carried out using strength, elastic modulus, density etc. values given by contemporary (Romanian) codes.

### Mechanical Behaviour of the Main Elements Resulting Form the Models

Mention should be made of the fact that the modelled roof (due to its large span) presents an anomaly compared to common baroque roofs: common rafters(2) – under common load combination – deflect so much that the longitudinal middle beam within the longitudinal bracing system (20) becomes intermediate support for the them (1.55-2.1cm deflection, more than measured distance 0.8-1.5cm).

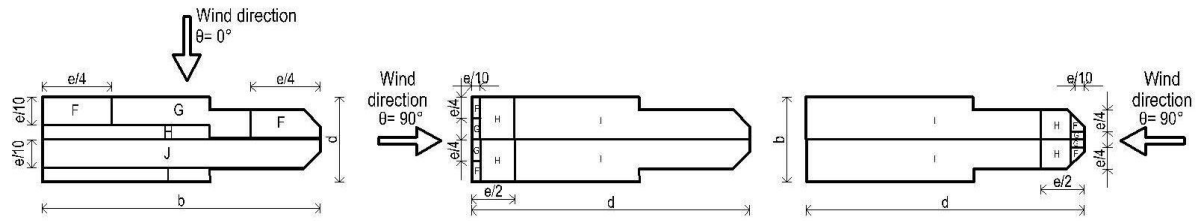


Figure 6: Zones with different wind loads (pressure / suction values) according to the wind direction

The studies of the main elements, their determinant stress / strain under specific loads, as well as the conclusions of the studied models can be synthesised as follows.

(1) *Common rafter* – in main and secondary trusses, type I and II – in all models *compression* and *bending* are the determinant strains, except for the horizontal continuous beam model, though the effect of wind loads are not taken into consideration in simple element calculus when supports are infinite rigid ones (2.1.1. – underestimates stresses, overestimates deflection). Common rafters in secondary trusses type I (the most reduced stiffness for both gravitational and horizontal loads) are most stressed by dead loads, have the largest movement and deflection. Wind loads stress mostly the common rafters within the main trusses (stiffest for horizontal action).

The existence of intermediate support (element – 20) contributes efficiently to the reduction of deflection. Wind loads and dead loads stress the secondary truss's common rafters similarly, and stress the main truss's common rafters more, as dead loads (183%).

(2) *Tie-beams* – in main and secondary trusses type II – are tension elements in all models (bending moment as well as shear present at joint with (7) and (3)). Dead loads determine larger strains than horizontal loads. Tie-beams in main trusses are 50% more stressed than the ones in secondary trusses.

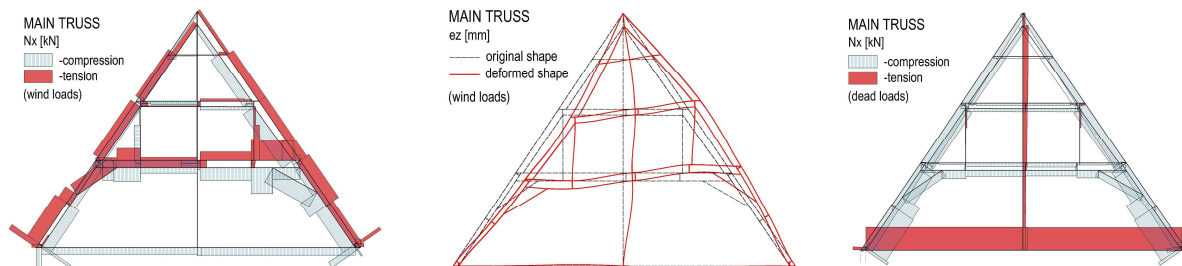


Figure 7: examples of diagrams:  $N_x$  – wind and dead loads, wind displacements

(3) *Main (compound) rafters* – within main trusses; simple compression / tension elements when common rafters (2) are not supported through longitudinal beams (20). Dead loads determine compression in both elements, wind loads determine alternately compression and tension depending on the wind direction. They are overloaded when the continuity of compound suspension bars (7) is interrupted.

(4) *Straining bar* – main trusses; principally compressed elements at dead loads, that can be tensioned under wind loads; compression with bending (at joints with (5) – counterbraces and (7) – suspension bars) result in the largest stresses.

(5) *Counterbraces* – main trusses; compressed or tensioned elements, with a determinant role at wind loads (5 times larger stresses from horizontal loads rather than from gravitational ones).

(6) *Collar beams* – both in main and secondary trusses; mainly compressed elements, with a determinant role at dead loads, being directly loaded by common rafters, where their continuity must be kept in modelling.

(7) *Compound suspension bars* – their most commonly known role is to reduce deflection of the long tie-beam, but they have a determinant role through their continuity at horizontal loads. Suppressing this continuity results in overloading the elements of the baroque straining system, especially (3) straining beam and (4) counterbrace, but also common rafters (1).



The choice of infinite rigid versus finite section joining elements determines stress concentration in rafters (A – version 2.3.1-3). Infinite / partial fixed joints introduce extra capacity and increase overall stiffness.

### Conclusions – Recommendation for the Use of Various Models

Efficient modelling is always determined by the technical level of the time. When only hand static calculations were available there was no other choice. It is important for engineers or undergraduate students to be able to understand basic statics, but analytic approach can also be misleading. Common rafters in baroque roof structures are not supported separate beam systems, they are determinant elements of the closed triangle frames.

**Simple element calculus** – though in Romanian engineering schools common rafters are taught to be calculated as continuous beams (determinant strain: bending), this is clearly a misinterpretation in historic (baroque) roof structures, where common rafters are frame elements that are compressed and bent; an angled beam model would be closer to reality, but even so the role of the triangle-frame element (common rafter) in taking over wind load wouldn't be taken into consideration.

**Plane – 2D models** Building separate 2D models, and calculating spring / rigid support reactions to introduce as loads to longitudinal and main transversal trusses requires more time than building up a limited 3D model, therefore this model level is recommended only for quick calculus in order to describe the general behaviour of a main / secondary truss. It is not recommended (anymore) for checking the elements.

**Spatial – 3D limited model** – is practically the most recommended modelling level at present. The greater safety factor is ensured by using pin joints, infinite rigid joining elements, and by not being allowed to suppress the continuity of compound suspension bars or collars. Partial fixed joints are allowed to be introduced only in cases when on site observations show the correct, safe mechanical behaviour of the roof structure, and pinned model would show the contrary.

**Spatial – 3D holistic model** – involves an increase of invested time, and is recommended only for detailed studies, and for structures where pitched areas present deformations or not controlled displacements.

The present lecture dealt only with the possibilities of modelling ideal (not degraded or deformed) structures; further studies are carried out on modelling various degradations of the historic roof structures.

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