Advanced Materials Research Vols. 133-134 (2010) pp 313-318
© (2010) Trans Tech Publications, Switzerland
doi:10.4028/www.scientific.net/AMR.133-134.313

Masonry Walls as Orthotropic No Tension Structures

BASILI Michela [1,a] and SINOPOLI Anna [2,b]

[1,2] Dept. of Structural and Geotech. Engineering, Sapienza, Rome, Italy

[a]michela.basili@uniroma1.it, [b]anna.sinopoli@uniroma1.it

Abstract In this paper, following the research line proposed by Milankovicth (1910) and Signorini (1925), a semi-analytical approach for the analysis of masonry walls - treated as horizontally multi-layered *strata* of no-tension material - is proposed in order to evaluate the stress distribution generated by applied loads. The main applications concern walls subjected only to vertical loads. The goal is to identify in the masonry wall the actual bearing sub-structures by defining at each vertical and horizontal level the transversal sections generated by the unilateral behaviour. The method proposed will allow identifying the partition of the wall into macro-elements, which depend on the centre of applied loads, and wall and bricks geometrical features. The technique adopted to approach the problem can be referred to as the search for the *best shape*. Two cases are discussed: corbelled masonry walls surrounding windows and arch behaviour of masonry over the window.

Keywords: Masonry, no-tension material, arch and pseudo-arch behavior

Introduction

Mechanical modelling of masonry structures represents a challenging subject even if very complex. The structure is realized of composite material with periodic structure so that difficulties arise to mathematically model such behaviour as a continuum. Several models are proposed in literature. In the last years, continuum micromechanical models became increasingly popular among the masonry community: they basically make use of homogenisation techniques, e.g. (Lourenço et al. 2007, Milani et al. 2006). The advantage of such models is that they are applicable in finite element analysis, therefore developing numerical procedures. The main idea is that masonry can be considered as a periodic structure obtained by regular repetition of elementary inhomogeneous cells of small size with respect to those of the wall structure (Zucchini and Lourenço 2002). In the continuum approach several constitutive relationships have been proposed, basically based on phenomenological and micromechanical models; among the phenomenological ones, the most used is based on the assumption of no-tension material: the masonry is modelled as a homogeneous elastic continuum with infinite to compression and no tensile strength. The first pioneering work is due to Signorini (1925), who studied masonry as a material that cannot support tension, as a prototype of unilateral constitutive relationship; further on, others followed this school of thought e.g. (Giaquinta and Giusti 1985, Romano and Sacco 1987).

There are two major issues to be examined when treating the problem of masonry mechanical modelling: the first is to find the shape of the structure when we know the applied forces; the other is to find the law of forces acting on the structure when we know its shape. Authors maintain that the direct issue, namely the best shape, is more interesting as more complex than the other. The proposed approach is in this line. The key idea is that of renouncing *a priori* to take into account deformation aspects, and to guarantee equilibrium through the identification of a thrust line fully contained into the unknown bearing structure: this is the problem of the best shape to give a masonry structure. The joints, vertical and horizontal, because of the scarce tensile strength are the intrinsic fragility points of the masonry; their behaviour influence significantly the definition of the bearing structure in the presence of external loads. Moreover, it is in the authors opinion that the complexity of masonry mechanical modelling depends not only on the geometry of the microstructure and the mechanical properties of the constituting elements, but also on the circumstance that the formation of either a unique resistant structure or multiple sub-structures divided by fracture lines is variable with applied loads and boundary conditions. At the beginning of 20^{th} century, Milankovicth (1910) made research on the best shape to give to buttresses treated as infinitely resistant to compression and no-tension structures. In his two-dimensional continuum Milankovicth assumed no-tension behaviour at any horizontal plane, and determined, depending on the geometrical lateral profile, the other one corresponding to the minimum thickness. Based on a similar approach one of the authors (Sinopoli, 2010) has already investigated the best shape to give to corbelled pseudo-arches and domes. In this paper the method is extended to masonry walls characterized by the presence of windows.

Masonry as Orthotropic No -Tension Continuum

The approach proposed can be considered a semi-analytical engineering approach, aiming at modeling the masonry as a no-tension orthotropic material. The two-dimensional adopted continuum assumes infinite resistance to compression and, at first step, no-tension behavior concentrated along any horizontal plane, so that masonry is treated as a horizontally layered no-tension continuum. The first step does not explicitly account for the regular offset of vertical joints belonging to two consecutive layered courses. The assumption underlying the proposed model is therefore that the structure is built in regular horizontal courses, and that none have breaks in continuity. The vertical joints are not ignored, but taken implicitly into account, since they become active in case the boundaries of the resistant section cross their position. In this Section two different shapes will be identified for the bearing structure of a masonry wall with windows that meet specific *a priori* requirements and can be considered to be *optimal theoretical shapes*. The starting point of each analysis is the equation of equilibrium, in terms of resultant reaction acting at any horizontal level: the search criterion used to find the optimal solution is therefore based on the minimum thickness of the resistant structure. Since the position of the centre of pressure on the joint depends on the distribution of normal pressure on the joint itself, assumed no-tension behavior, care has to be taken to avoid tensile stress being generated at each joint with consequent reduction of the resistant section: the centre of pressure must then fall within the cross section core, the limits of which represent the boundary defining the minimum thickness, that is the *best shape*.

Corbelled Masonry Wall Surrounding a Window In a reference system *(x, y)* consider a unity depth masonry wall and its elementary element (Fig. 1) containing a window, and let the geometrical lateral side be fixed as a vertical straight line; due to symmetry we shall consider only half part of the wall element. Assume that the span covered by the window is $L=1$ while at same height the extension of the masonry from the window to the lateral profile is $d/2$; assume moreover that an assigned weight V_0 acts at $y=0$. At generic height y the resultant of the forces, due only to the action of the weight, is vertical while the position of the corresponding centre of pressure E depends on the thickness of the resistant section. To identify the *best shape* of the masonry wall element surrounding the window, let us identify the internal profile curve (*intrados line*) in order that the centre of pressure falls on the middle third nearest the intrados at each horizontal joint. At the generic joint MN at height y, let x indicate the coordinate of point E, the unknown position of the middle third to be identified; similarly, let $x+dx$ be the coordinate of point E', the middle third position at $y+dy$. The forces acting on the element of infinitesimal height dy are: weight $V(y)$ hypothetically applied at E, infinitesimal weight dw applied at the centre of mass G and the reaction at $y+dy$, applied at point E'. Equilibrium with respect to E' then requires that:

$$-Vdx + (x_G - x_{E'})dw = 0 \tag{1}$$

where dw is the infinitesimal weight:

$$dw = \tfrac{1}{2}\, p\left[(L+d) - 3x\right]dy \tag{2}$$

and p the specific weight of the material. From Eqs. 1 and 2, assuming that $(x_G - x_{E'})$ is equal to the same quantity assessed at height y, we obtain:

$$V(x,y) = \frac{3}{8} p \left(\frac{L+d}{3} - x \right)^2 \frac{dy}{dx} \tag{3}$$

Moreover, since $V(x, y)$ is equal to:

$$V(x, y) = V_0 + \tfrac{1}{2} p \int_0^y \left[(L+d) - 3x \right] dy \tag{4}$$

by equating the derivatives of Eqs. 3 and 4 with respect to x, we obtain the differential equation defining the level y as a function of the thrust line position x:

$$\frac{d^2 y}{dx^2} - \frac{18}{(L+d) - 3x} \frac{dy}{dx} = 0 \qquad \text{with: } y(0) = 0\,; \qquad \left.\frac{dy}{dx}\right|_{x=0} = \frac{24 V_0}{p (L+d)^2} \tag{5}$$

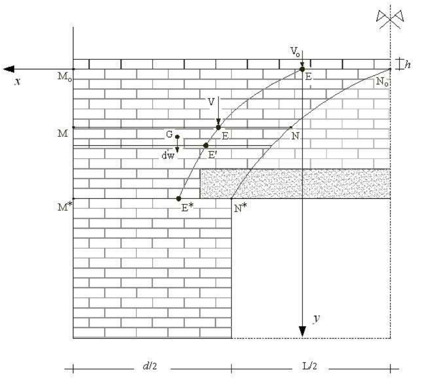


Figure 1: Masonry element without thrust

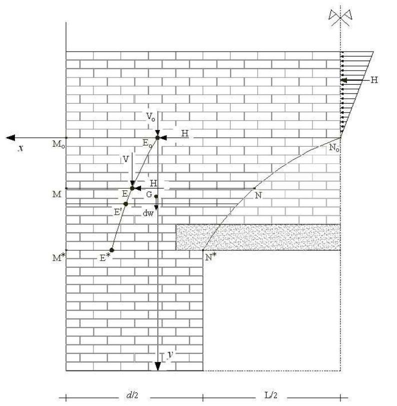


Figure 2: Masonry element with thrust

In Eq. 5 the boundary conditions define the starting of the thrust line and slope of $y(x)$; they correspond to the main purpose of determining the height y_{max} necessary to guarantee that the resistant section covers all the span at $y=0$ and coincides with the extended support at side of the window at bottom. Eq. 5 is a second order non-linear differential equation with variable coefficients, the solution of which has been determined numerically; it should be noted that it does not depend on the specific weight, resistance being guaranteed by the geometry.

The numerical analyses undertaken have considered different values of both d and weight V_0. In particular, several values of weight are considered by changing the height h^* on which V_0 depends. In all the examples reported we refer to a window span of $L=1m$. Figs. 3a-c show some of the results obtained for a constant value of h^* ($h^*=0.05L$) and different values of d: $d=2L$, L and $0.5L$, respectively. We want to show how the masonry portion adjacent the window influences the *best shape* of the pseudo-arch generated. As can be seen from each figure, having fixed the vertical geometrical lateral side and the unitary extension of the window span at bottom, the solution – dashed line – identifies the thrust line of the resistant structure surrounding the window, which is always an arch curved line. At generic height y, the extension from the vertical lateral side to the ideal internal profile (*intrados line*) represents the resistant section; while, as y varies, the vertical lateral profile represents the envelope of the positions of the neutral axes. Let us define the region extending from the ideal intrados to the vertical lateral profile as the *compression strength domain*. As y decreases, the thrust line tends asymptotically towards an oblique straight line, whose distance from the window middle span is always different from zero. By increasing d it is observed that y_{max} becomes shorter. For d great enough ($d=2L$), the pseudo-arch covers the whole span in a very short height

(y_{max}=$0.27L$); while by decreasing d a part of the masonry increasingly heavy will loads the architrave. By comparing the real structure with its *strength domain*, we can then identify the theoretical path of possible fracture lines along the ideal lateral profiles.

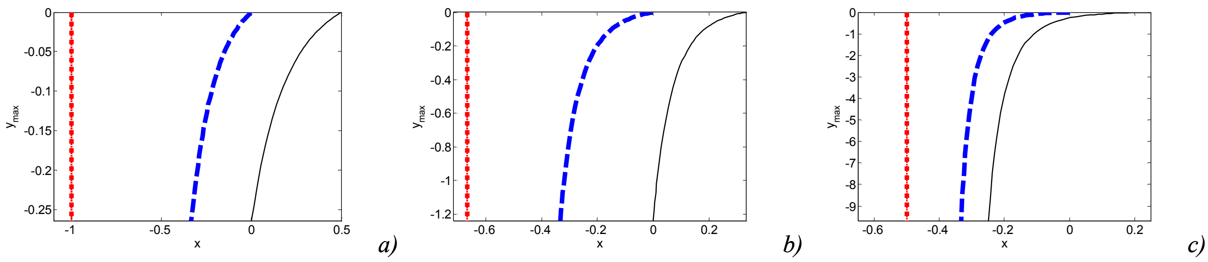


Figure 3: (a) d=2L; (b) d=L; (c) d=0.5L (--Lateral profile, - -thrust line, — intrados line)

In conclusion, it seems that the best portion of masonry adjacent to the window in order to guarantee stability of the architrave and closure of the pseudo arch is d=$2L$ (Fig. 3a). When the portion of masonry surrounding the window is lower, we can expect overcharge over the window.

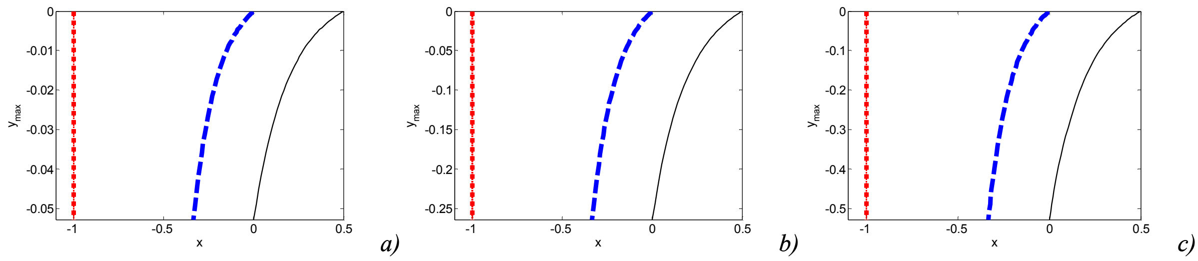


Figure 4: (a) h=0.01L; (b) h*=0.05L; (c) h*=0.1L (--Lateral profile; - -thrust — intrados lines)*

Alternatively, possible solutions can be thought to preserve the architrave by inserting proper resistant *ad hoc* structures whose task is to lighten the load weighting on the window. Such argument is treated in the next paragraph. Figs. 4a-c show some of the results obtained for the optimal value of d (d=$2L$) and different values of V_0 by varying h^*: $0.01L$ ($1cm$), $0.05L$ ($5cm$) and $0.1L$ (10 cm), respectively. As V_0 increases as y_{max} will be higher. All the area below the intrados curve represents the load weighting on the architrave. Generally, if we refer to the load of a single line of bricks over the pseudo arch we can assume a value of h^* around 10 cm ($0.1L$). In such case (Fig. 4c) the height of the pseudo-arch y_{max} is around $0.5L$ (50 cm). For the effect of the pseudo-arch generated, a load of masonry contained in a semi-circle of radius around half span of the window weights the architrave.

Arch Behaviour of Masonry over a Window The second optimal problem concerns the identification of the intrados line of the masonry resistant structure surrounding a window in the case where some horizontal interaction (*thrust*) is expected to be active between the two half-part covering the window span. Above the region where the pseudo-arch is generated there are two symmetrical portions of masonry interacting each other. We can expect that some horizontal thrust originates by such interaction, developing an arch resistant structure. As in the previous case, consider a masonry element of unity depth and let L=1 m and d=$2L$ be the window span and the extension of the masonry support at side of the window, respectively. To identify the *best shape* corresponding to the minimum thickness of the resistant section, an intrados curve needs to be determined in order that the centre of pressure falls on the middle third closest to the geometric lateral profile of the wall, with the result that the neutral axis falls on the intrados line to be determined. In a reference system (x, y), starting at y=0, that is at the height where the resistant section fully covers the window span (Fig. 2), assume an applied weight V_0 and a thrust H applied at point E_0, the middle third of the section closest to the vertical lateral profile; moreover, let x be the thrust line coordinate. This time the forces acting on the element dy are: thrust H and weight V applied at E; weight dw of the element which extends from y to y+dy, applied at centre of mass G; and, finally, reaction applied at point E' at y+dy. Equilibrium with respect to E' then requires that:

$$Hdy - Vdx - (x_{E'} - x_G)dw = 0 \quad (7)$$

in which *dw* is the weight of the infinitesimal element:

$$dw = \tfrac{1}{2} p\left[(L+d) - 6x\right] dy \quad (8)$$

and *p* the specific weight of the material. From Eqs. 7 and 8, assuming that $(x_{E'} - x_G)$ is equal to the same quantity at height *y*, we obtain:

$$V = \left[H - \tfrac{3}{2} p \left(\frac{L+d}{6} - x \right)^2 \right] \frac{dy}{dx} \quad (9)$$

Furthermore, since *V(x, y)* is equal to:

$$V(x, y) = V_0 + \tfrac{1}{2} p \int_0^y \left[(L+d) - 6x\right] dy \quad (10)$$

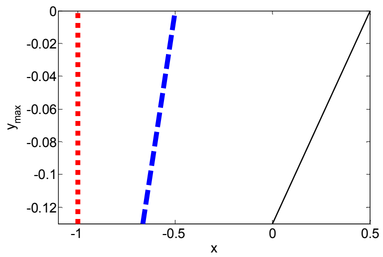
by deriving Eq. 10 with respect to *x* and making it equal to the derivative of Eq. 9, finally we get the differential equation with corresponding boundary conditions:

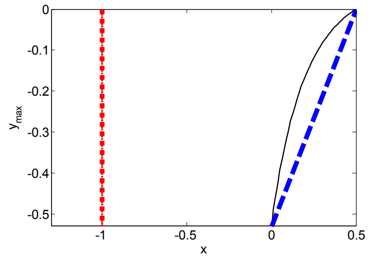
$$\left[H - \frac{3}{2} p \left(\frac{L+d}{6} - x \right)^2 \right] \frac{d^2y}{dx^2} = 0 \quad \text{with: } y(0) = 0; \quad \left.\frac{dy}{dx}\right|_{x=0} = \frac{24V_0}{\left[24H - p(L+d)^2\right]} \quad (11)$$

which depend on the value of applied weight V_0 and thrust *H*. The value of *H* is defined as that guaranteeing the starting of the thrust line at $x = 0$ for $y = 0$, that is: $H = 1/16\, p(L+d)^2$.

Eq. 11 allows us to determine the thrust line and, therefore, the intrados line which corresponds to a completely reactive section, with the centre of pressure at the middle third closest to the vertical lateral profile and neutral axis at the intrados. Starting from $y=0$, the solution of Eq. 11 is a straight line, whose slope depends on the geometry of the masonry surrounding the window and masonry height *h* over the level at $y=0$. The arch fully covers the given span if: $y_{max} = 4hL/(L+d)$, so that y_{max} increases linearly with *h*. In Fig. 5 the shape of the arch generated is shown for $L=1m$, $d=2L$ and $h=0.1L$. The internal profile - *intrados line* - is an inclined line and represents the envelope of the positions of the neutral axes, whereas the thrust line – dashed in figure – is at the medium third of the bearing section; again a vertical lateral profile has been assumed. The region extending from the *ideal* intrados to the vertical lateral profile is the *compression strength domain*. Such behavior connected to the results observed in the previous paragraph, suggests, if a horizontal thrust it is expected, to realize an arch whose height y_{max} is equal to the height of the corresponding pseudo-arch. The arch will have different height y_{max} depending on the height of the masonry above the level $y=0$. The question is to find which is the optimal masonry height *h* for which heights y_{max} of both arch and pseudo-arch coincide. We will always refer to cases where *h* is of the same order of magnitude than *L*. Assuming $d=2L$ it has been seen that y_{max} for the pseudo arch is 0.53 m (Fig. 4c). For the same geometry of the masonry surrounding the window, the optimal arch height h_{opt} will be: $h_{opt} = y_{\max}(L+d)/4L = 0.4L$.

In Fig. 6 arch and pseudo-arch intrados lines corresponding to the same y_{max} of *0.53 m* and $h_{opt}=0.4$ *m* are compared. Such are the optimal proportions which guarantee an overall well functioning of the masonry with windows, ensuring also that the compression strength of the material is utilized at its best. By assuming over the window an arch with these sizes the area below it, which is less then the area below the pseudo-arch, weights the architrave. The height y_{max} of the arch is influenced by the height value *h* of masonry over it as well; such variations are schematically reported in Fig. 7.





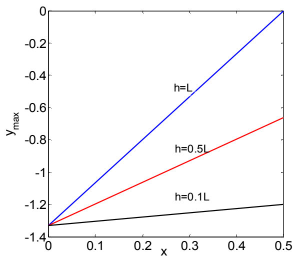


Figure 5: Arch shape. Fig. 6. Arch and pseudo-arch for equal y_{max} Fig. 7. Intrados by varying h

When through *h* the load becomes too high it would seem useful to build *ad hoc* sub-structures to lighten the architrave and redistribute loads on full masonry sub-structures. The portion of masonry next the window influences arch height y_{max} as well; results are reported in *Table 1* for *h=0.1L*, where it can be observed that y_{max} decreases as *d* increases.

Table 1: Variation of y_{max} versus d

y_{max} [m]	0,27	0,20	0,16	0,13	0,11
d	0.5	1	1.5	2	2.5

Conclusion

In a masonry wall with a window, a pseudo arch without needing horizontal thrust originates for the presence of vertical loads. The presence of further masonry over this region will produce an interaction between the two symmetric parts, which originates a horizontal thrust: a natural arch behavior is therefore expected. Optimal proportions of such structures are in this case outlined. If the portion of masonry next the window is large enough (*d*=2*L*), the natural arch conveniently addresses loads in well-defined bearing sections of wall. A load represented by a triangle of masonry of height y_{max} equal to half span of the window weights the architrave. When either the available portion of masonry adjacent the window is small or the load over the arch is quite high - but of the same order of magnitude than *L* and *d* - it is more convenient to build an appropriate arch to lighten the architrave.

References

[1] Giaquinta, M and Giusti, E (1985). "Researches on the equilibrium of masonry structures." *Arch. for Rational Mechanics and Analysis*, 88, 359-392.

[2] Lourenço, P B, Milani, G, Tralli, A, and Zucchini, A (2007). "Analysis of masonry structures: review of and recent trends in homogenization techniques." *J. of Civil Engng*, 34, 1443-1457.

[3] Milani, G, Lourenco, P, and ralli, A (2006). "Homogenised limit analysis of masonry walls, Part I: Failure surfaces." *Computers and Structures*, 84, 166–180.

[4] Milankovicth, M (1910). "Zur statik der massiven widerlager." *Zeitschrift für Mathematik und Physik*, 58, 120-128. Translated in Italian by F. Foce, 2006 (private communication).

[5] Romano, G, and Sacco, E (1987). "Convex Problems in Structural Analysis," in *"Unilateral Problems in Structural Analysis"*, G. Del Piero and F. Maceri Eds., CISM Courses and Lectures, Springer-Verlag, 304, 279-297.

[6] Signorini, A (1925). "Un teorema di esistenza ed unicità nello studio dei materiali poco resistenti a trazione," *Rendiconti Accademia Nazionale Lincei*, 2, 401-406 (in Italian).

[7] Sinopoli, A (2010). "Vaults and pseudo-vaults: Unilaterality and dry friction," in *"Mechanics and Architecture between Epistêmê and Technê"*, Sinopoli A. Ed., *Proc. Int. Conf. in commemoration of E. Benvenuto*, Edizioni di Storia e Letteratura, Rome, 2008, 37-78.

[8] Zucchini, A, and Lourenco, P B (2002). "A micro-mechanical model for the homogenisation of masonry." *Int. J. of Solids and Structures*, 39, 3233–3255.