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Two-step Pushover Analysis of an Ancient Masonry Oil-mill in the Southern Italy

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Abstract Historical masonry buildings located in the Southern Italy are usually built with irregular stones joined with mortar with poor mechanical properties. Therefore, piers and spandrels ultimate resistance is not always well predicted by simplified formulas suggested by codes of practice, which typically are tailored to regular patterns. In this framework, we present a two-step numerical model –within the equivalent frame approach assumption– for the pushover analysis of in-plane loaded historical masonry walls constituted by an irregular assemblage of stones. In Step I, ultimate bending moment-shear force strength domains of piers and spandrels are derived by means of a heterogeneous upper bound FE limit analysis and the results are stored in a database. Assessing the capacity of both piers and spandrels is crucial for correctly predicting the ultimate resistance of masonry walls acted upon by in-plane loads. Heterogeneous limit analysis is particularly suitable for computing failure loads, since it permits a distinct modeling of stones and mortar joints. Appropriate static and kinematic boundary conditions are set to account for the complex interaction of internal forces and deformed shapes of single elements. At Step II, a frame model of the masonry wall is assembled, where piers and spandrels are modeled as elastic Timoshenko beams. At each analysis step it is checked that the internal forces in each structural element are smaller than the failure loads stored in the database created at Step I. If the capacity is exceeded, suitable flexural hinges are introduced at the end of the structural elements. The resistance of the element is then set to zero when a limit chord rotation is exceeded. With the numerical tool developed, a real scale old masonry oil-mill located in the Southern Italy is analyzed in the inelastic range under increasing static loads.

Keywords: Historical masonry, limit analysis, upper Bound, finite element, irregular pattern, equivalent frame approach, two-step modeling

Introduction

Historical masonry buildings, especially in Italy, are usually realized with irregular stones joined through mortar with poor mechanical properties. Therefore, piers and spandrels ultimate resistance is not always well predicted by simplified formulas suggested by codes of practice (2005), which typically are tailored to regular patterns. For this reason, to propose a comprehensive numerical model able to give reliable information on the non linear behaviour of such buildings –but taking at the same time into account properly the actual texture of the walls- seems at present a prohibitive challenge.

In this paper, a novel two step approach for the pushover analysis of historical masonry buildings is presented and applied to a case-study relying in an ancient oil-mill located in the southern Italy and constituted by walls in irregular texture. The model still uses for the global analysis of the masonry wall a simple equivalent frame model (Milani, Beyer and Dazio 2009, Belmouden and Lestuzzi 2009, Magenes and Della 1998) and is able, at the same time, to estimate masonry macroscopic mechanical properties taking into account the disordered disposition of stones. The aim of the paper is to put at disposal to practitioners a very simple tool to be used in ordinary design, but at the same time capable of giving realistic estimations of the load carrying capacity of structural elements resulting from the irregular assemblages of stones with variable and irregular shape.

The tool is based on a two step procedure. In the first step (Step I), ultimate bending moment-shear force resistance of piers and spandrels are derived by means of a heterogeneous upper bound FE limit analysis (Sloan and Kleeman 1995, Milani, Lourenço and Tralli 2006) and the results are stored in a database. With reference to a real case study, piers and spandrels of an existing oil-mill located in Calabria (Southern Italy) are meshed through triangular elements on the base of a precise survey of the actual texture and geometry of each structural element. A large database is collected, considering the behaviour of each spandrel and pier belonging to the structure. A very refined discretization is used in order to take into account as close as possible the effect of the irregular texture and the presence of preferential planes of weakness. A heterogeneous limit analysis is adopted because is suitable for computing failure loads of complex structures with a moderate computational effort. The limit analysis is carried out for the range of expected axial loads in the structural elements and the expected relative rotations and displacements of piers and spandrels – two parameters that were found to affect the strength of unreinforced masonry significantly-. Appropriate static and kinematic boundary conditions are imposed to account for the complex interaction of internal forces and deformed shapes of single elements.

In Step II (macro-level), not reported in this paper -specifically focused on the first step of the procedure proposed-, a frame model of the masonry wall is implemented. In the model, for spandrels and piers, elastic Timoshenko beam elements are used. The strength of the piers and spandrels is defined by the strength domains stored in the database. At each analysis step, the bending moment and shear demands are compared to the respective capacities. If the capacity is exceeded, a rigid-plastic flexural hinge is introduced. The proposed analysis procedure assumes that the demand up to collapse on the piers and spandrels of a masonry wall can be determined by an equivalent frame approach.

Description of the Case Study: the Oil-Mill Building Considered

The building under study (see Figure 1) is an ancient oil-mill built in the early years of the twentieth century and located in the town of Bova Marina (Italy), in the center of a flat area between a small river (San Pasquale) to the west, the Ionian coast to the south and a hill (Agrillei) to the east. The building is composed by a main body with rectangular plan connected perpendicularly to two smaller bodies, giving rise to a overall plant in the shape of "C". The mail body has two stories, whereas the other two have a single storey. Recently, a canopy realized using steel has been added in adherence to the entire eastern side. A traditional building technology was used to erect the oil-mill, which is typical of many constructions still present in the southern Italy. In particular, the building presents a 1.40 meters height basement of thickness 80 cm, realized using large stones and mortar. This portion of the perimeter walls actually coincides with the continuation of the foundation structure above the ground level. The remaining part belonging to the first story and the whole second story are realized with smaller irregular stones, pieces of bricks and mortar. With the aim of regularize the external walls realized with a disordered pattern, small vertical ribs constituted by a single row of solid clay bricks have been built. The first floor is made by small clay bricks vaults supported by old double "T" iron beams, whereas the roof is sustained by a truss-like timber structure, currently very degraded.

The Upper Bound FE Limit Analysis Heterogeneous Discretization

For each pier and spandrel belonging to the building under consideration, strength domains are numerically obtained in terms of ultimate bending moment (M_u), ultimate shear (V_u) and imposed vertical pre-compression (N).

All the numerical models used for limit analysis are based on an upper bound approach based on the kinematic discontinuous formulation originally presented by Sloan and Kleeman (Sloan and Kleeman 1995), which has already been applied successfully to various masonry problems in (Milani, Lourenço and Tralli 2006). The formulation is based on a triangular discretization of the 2D domain and on the introduction of discontinuities of the velocity field along the edges of adjacent triangles. At

each node j, a horizontal velocity u_x^j and a vertical velocity u_y^j are introduced. The resulting velocity field within a triangular element is linear whereas the strain rate field is constant. Across the interfaces a linear velocity jump is assumed and therefore for each interface four unknowns are introduced ($\Delta\mathbf{u} = [\Delta v^1 \ \Delta u^1 \ \Delta v^2 \ \Delta u^2]^T$) representing the normal (Δv^j) and tangential (Δu^j) velocity jumps with respect to the discontinuity direction evaluated at the nodes $j = 1$ and $j = 2$ of the interface.

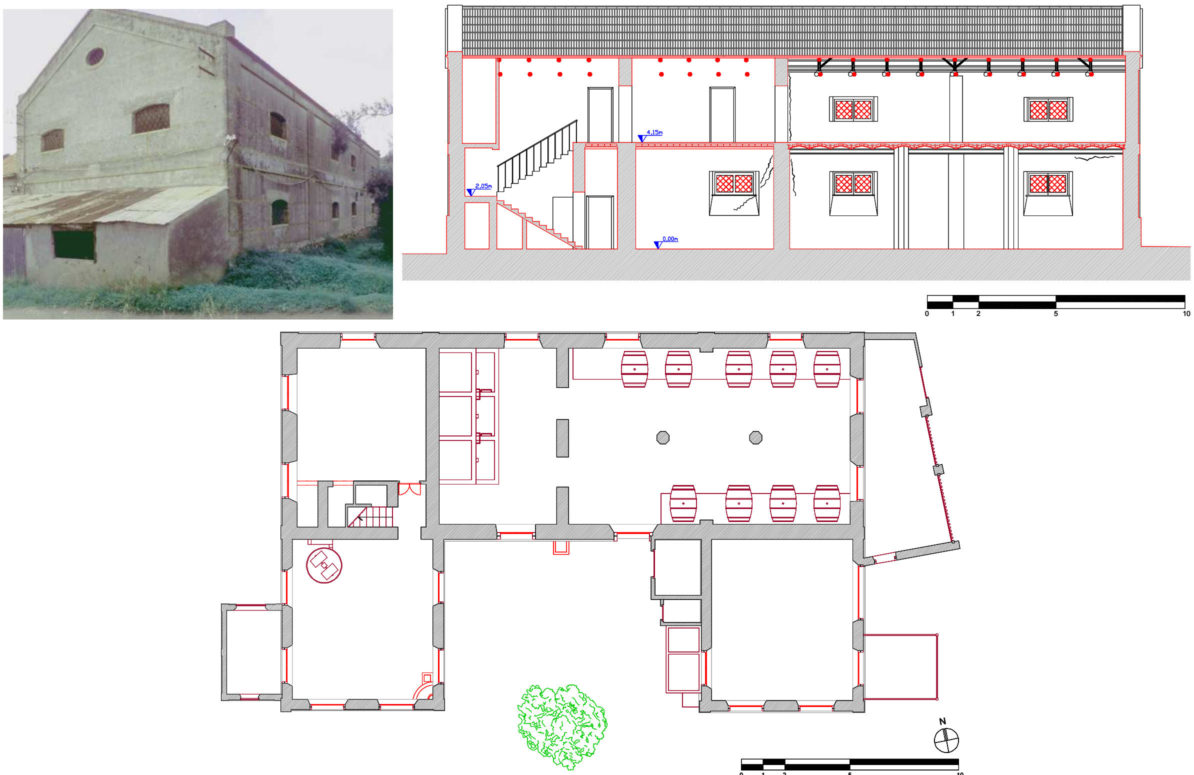

Figure 1: The ancient masonry oil-mill studied

A full description of the heterogeneous FE upper bound limit analysis model used in this paper is given in (Cundari and Milani 2010) and the reader is referred there for further details. Here, it is worth remembering that, from a numerical point of view, the evaluation of the ultimate load bearing capacity of both piers and spandrels can be evaluated solving a suitable linear programming problem, where the objective function consists in the minimization of the total internal power dissipated:

$$\begin{cases} \min\left\{\left(\mathbf{b}_{ass}^{in}\right)^T \dot{\boldsymbol{\lambda}}_E^{ass} + \left(\mathbf{b}_{I,ass}^{in}\right)^T \dot{\boldsymbol{\lambda}}_I^{ass}\right\} \\ \text{such that} \begin{cases} \mathbf{A}^{eq}\mathbf{U} = \mathbf{b}^{eq} \\ \dot{\boldsymbol{\lambda}}_E^{ass} \geq \mathbf{0} \\ \dot{\boldsymbol{\lambda}}_I^{ass} \geq \mathbf{0} \end{cases} \end{cases} \quad (1)$$

where $\mathbf{b}_{ass}^{in}$ and $\mathbf{b}_{I,ass}^{in}$ are the assembled right-hand sides of the inequalities, which determine the linearized failure surface of the material of the continuum and of the interfaces, respectively, $\mathbf{U} = [\mathbf{u} \quad \dot{\boldsymbol{\lambda}}_E^{ass} \quad \Delta\mathbf{u}^{ass} \quad \dot{\boldsymbol{\lambda}}_I^{ass}]$ is the vector of global variables, which collects the vector of assembled nodal velocities ($\mathbf{u}$), the vector of assembled element plastic multiplier rates ($\dot{\boldsymbol{\lambda}}_E^{ass}$), the vector of assembled velocity jumps on interfaces ($\Delta\mathbf{u}^{ass}$), and the vector of assembled interface plastic multiplier rates ($\dot{\boldsymbol{\lambda}}_I^{ass}$); $\mathbf{A}^{eq}$ is the overall constraints matrix and collects velocity boundary conditions, relations between velocity jumps on interfaces and elements velocities, constraints for plastic flow in velocity discontinuities, constraints for plastic flow in continuum and spandrels equality.

Piers Strength Domains

In the context of an equivalent frame approach, a generic pier can be schematically represented by a shear deformable beam with 4 d.o.f. represented by nodal rotation (ϑ_1 and ϑ_2) and displacements perpendicular to the beam axis (u_1 and u_2), see Figure 2. Due to the very low axial deformability of the piers, we suppose that displacements along beam axis are negligible. We consider that an interaction between axial force N and ultimate bending moment M_u, and axial force N and ultimate shear V_u occurs. A rigorous approach for piers would require the determination of the ultimate shear (V_u) and the ultimate bending moment (M_u) failure surfaces taking into account all the possible combinations of the following kinematic/static input variables: (i) the ratio $\rho = \vartheta_2 / \vartheta_1$ between head and foot rotations, (ii) the ratio $\pi = (w_2 - w_1)/(H\vartheta_2)$, where w_2 and w_1 are top and bottom horizontal displacements and H is the pier height and (iii) the applied pre-compression N.

Nevertheless, this approach would require an almost prohibitive computational effort, also in light of the very refined discretizations adopted in this paper to reproduce the actual microstructure of the piers. For this reason, we reasonably assume that ultimate shear and ultimate bending moments are "uncoupled", thus evaluating ultimate shear V_u simply as a function of vertical pre-compression N and keeping $\vartheta_2 = \vartheta_1$=0. Considering the importance of relative head and foot rotations on the evaluation of the ultimate bending moment, M_u is computed as a function of ρ ratio and vertical pre-compression. On the other hand, within a limit analysis framework, velocity field has to be considered instead of displacement and rotation fields. It is therefore necessary to evaluate piers resistance as a function of rotations rates ratio $\rho = \dot{\vartheta}_2 / \dot{\vartheta}_1$.

In this way, the limit analysis procedure proposed furnishes, at fixed pre-compression, two values of the ultimate shear (depending if the pier is loaded from west to east or vice versa) and a series of bending moments associated each one to a single value of the non-dimensional parameter ρ.

In the framework of a pushover analysis, at the end of the iteration i, the axial force $N_{(i)}$ acting on the beam as well as the rotations $\vartheta_{1(i)}$ and $\vartheta_{2(i)}$ at the beam extremes are known. The coefficient $\rho_{(i)} = \vartheta_{2(i)} / \vartheta_{1(i)}$ is defined as the ratio of the rotations at beam extremes at the iteration i and is kept also equal to rotation rates. Given $\rho_{(i)}$ and $N_{(i)}$, corresponding Mu and Vu values are computed from the collected database and compared to the actual bending moment and shear, to determine if the piers strength has been exceeded.

Differently from a standard limit analysis problem, additional kinematic boundary constraints must be imposed on piers to reproduce, in the collapse mechanism, a fixed ratio of rotations (rates) $\rho = \vartheta_2 / \vartheta_1$ of the upper ($\partial\Omega_1$) and lower ($\partial\Omega_2$) boundary, see Figure 2. For spandrels, the same considerations may be repeated, provided that such constraints are applied to the left and right boundaries. In a heterogeneous FE limit analysis, the kinematic constraint on ρ is taken into account by imposing a constraint on the rotational velocities of top and bottom edges ($\partial\Omega_1$ and $\partial\Omega_2$). A full description of the additional constraints to impose to the FE limit analysis model can be found in [2] and they are omitted here for the sake of conciseness.

Here it is worth noting that ρ may range between –Inf and +Inf, since it is possible that $|\vartheta_2| > |\vartheta_1|$. In order to cover all the possibilities that can be encountered (at least theoretically) at structural level and considering that in limit analysis ρ represents a ratio between rotations rates, it may be useful to limit the range of variability of ρ between -1 and 1 and to introduce a further non-dimensional variable ρ' defined as $\rho' = \frac{\dot{\vartheta}_1}{\dot{\vartheta}_2}$ ranging between -1 and 1. By means of the alternative imposition of boundary conditions related to ρ and ρ', all the possibilities that can be encountered in practice may be covered.

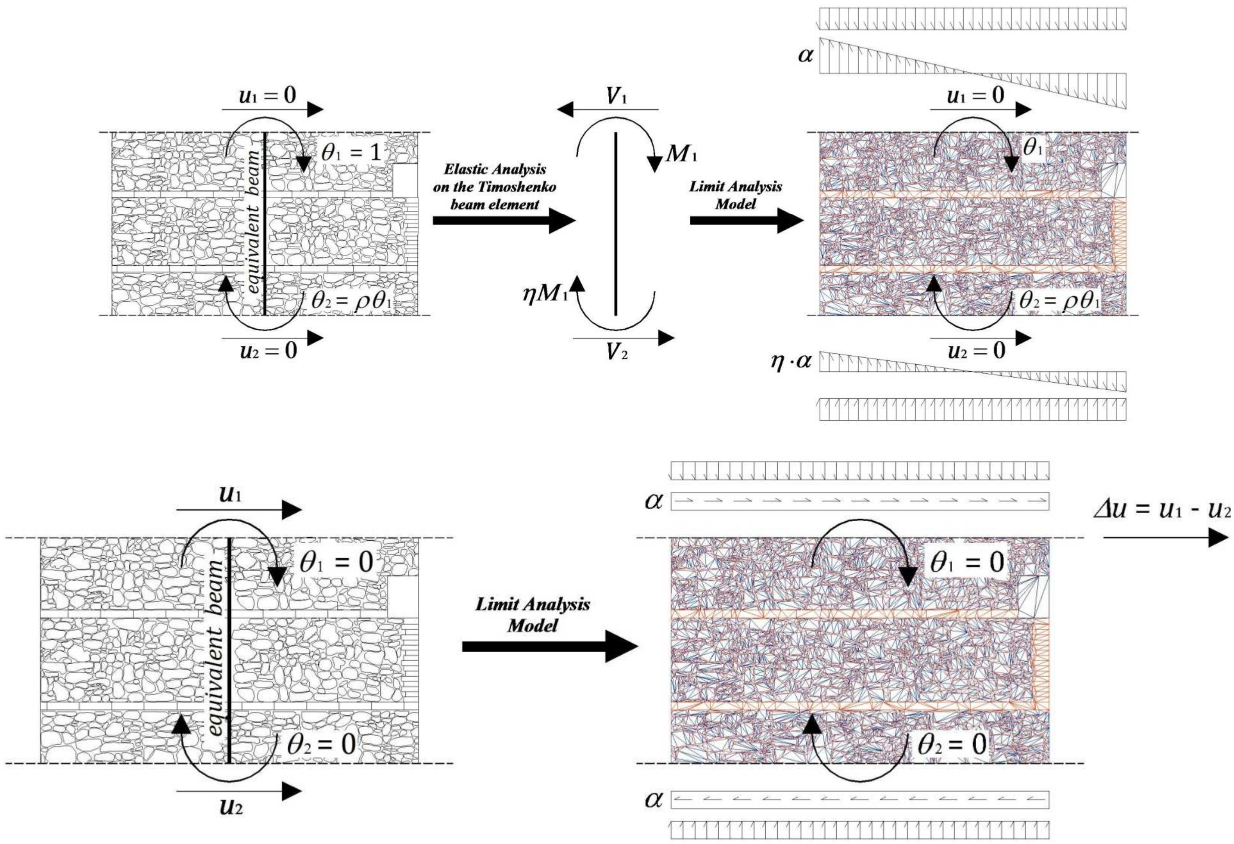

Figure 2: General procedure adopted for the determination of masonry piers strength domains

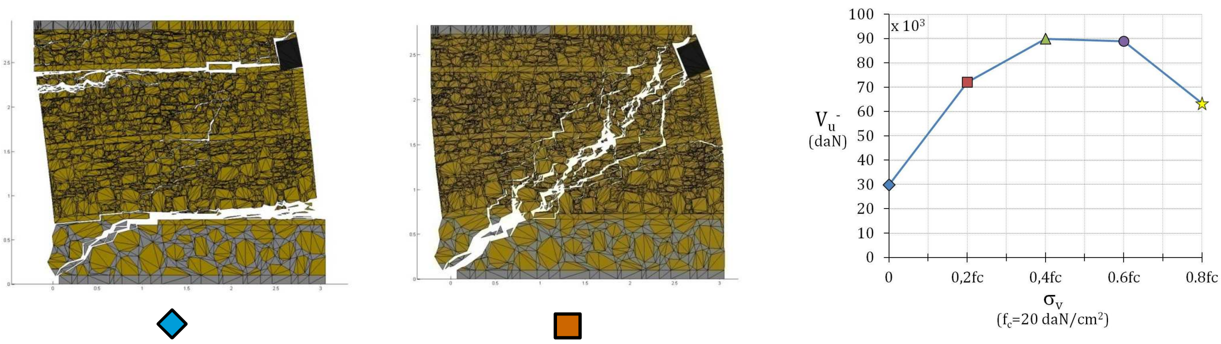

Figure 3: Pier 1, deformed shape at collapse corresponding to Vu at two increasing levels of pre-compression and corresponding shear strength as a function of vertical pre-compression

In Figure 3 and Figure 4, a few results of a massive numerical analysis campaign conducted using a heterogeneous FE limit analysis software and reported in [2] are summarized.

In particular, in Figure 3, two deformed shapes at collapse of one ground floor pier subjected to shear at increasing pre-compression and the corresponding ultimate shear-vertical pre-compression curve are represented. The role played by vertical pre-compression in the change of the failure mechanism is particularly evident. The presence of irregular cracks which zigzag between rigid stones suggests once again that simplified formulas provided by codes of practice may be not reliable and well suited for this typology of masonry.

Figure 4 shows some deformed shapes of the same pier subjected to bending moment. In the simulations, ρ is maintained equal to zero, whereas the vertical pre-compression is increased starting from zero and ending to the 80% of mortar compressive strength. The corresponding M_u-σ_v curves

are also reported in Figure 4 (here σ_v indicates vertical stress acting at the top of the pier). As expected, a moderate vertical pre-compression increases considerably the ultimate bending resistance of the pier, whereas for σ_v exceeding 50% of the compressive strength of the joint the load bearing capacity of the structural element slightly decreases, which seems again in agreement with experimental evidences.

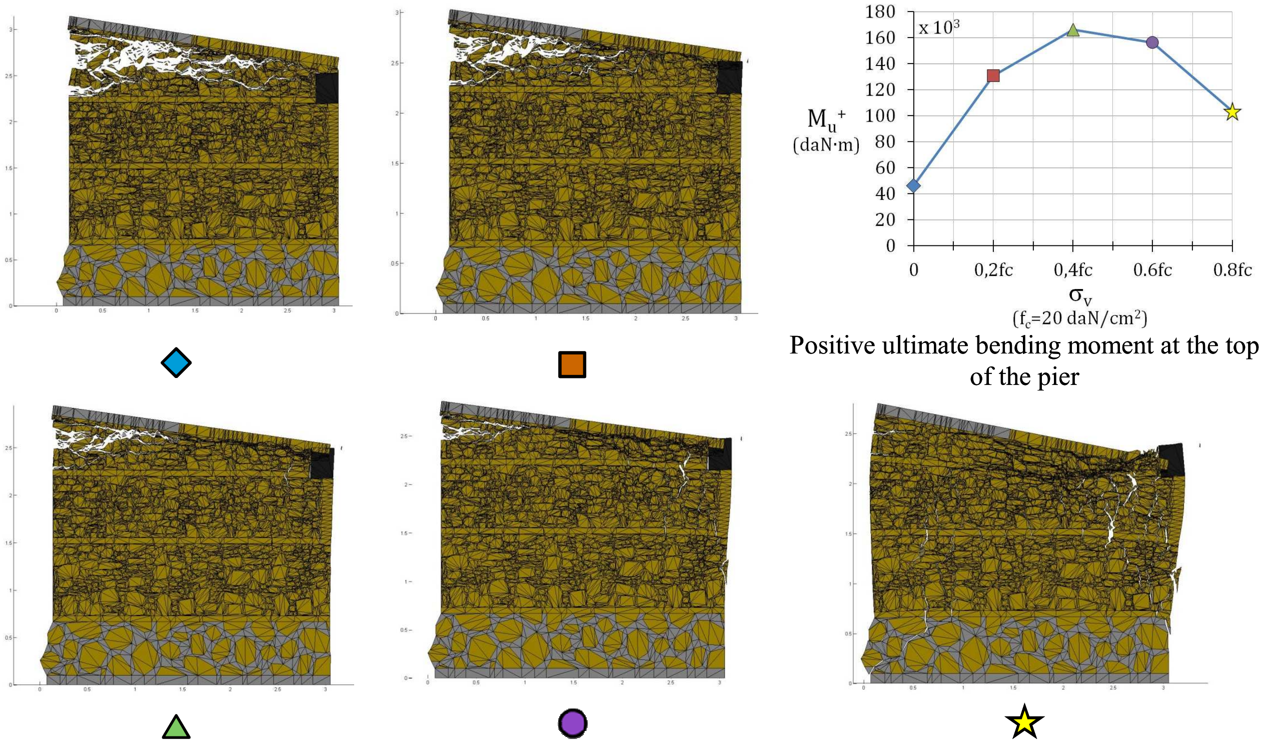

Figure 4: Pier 1, M_u (from left to right) strength domain at different pre-compression levels and corresponding deformed shapes at collapse

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