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Modeling the Static Behaviour of a Double Curvature Brickwork Vault

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Abstract For safety assessment, a double curvature hollow bricks cloister vault with lunettes has been studied. Its geometry, constructive aspects, crack pattern have been surveyed and a 3D finite element analysis has been carried out, the numerical model exploiting the accurate survey of the vault geometry.

Keywords: Cloister vault, hollow bricks, steel, geometrical survey, numerical modelling

Introduction

The study of a double curvature hollow bricks vault - a cloister vault with lunettes - built at the beginning of XX century, has been carried out, in view of the assessment of its bearing capacity. The large structure, ended in 1927, is partially hanged to the timber trusses of the roof and is extremely deformable. As shown in Figs. 2 and 3 it is characterized by a nearly flat central sector, supported by steel beams, buttress walls on the extrados, and hanging devices to connect the vault with the roofing trusses.

Historically, large vaults similar to this have been extensively used in public buildings between the end of the 19th and the beginning of the 20th century, when Italy and Europe experienced a transition from the use of traditional stone and brick masonry structures, to the progressive diffusion of reinforced concrete structures. This phase is characterized by an interesting development of vaults and floors built with a mixed use of steel and hollow bricks, which implies a lower weight of the vaults and consequently the use of steel beams with smaller cross sections. Examples of mixed floor in steel and bricks are presented in Italy by Curioni (Curioni 1868), Misuraca and Boldi (Misuraca and Boldi 1916), Donghi (Donghi 1925).

A certain degree of structural damage since almost 100 years of service life and the new safety prescriptions of the Italian Seismic Code highlights the interest of the research, aimed to the safeguard of a public building belonging to the Italian constructive heritage.

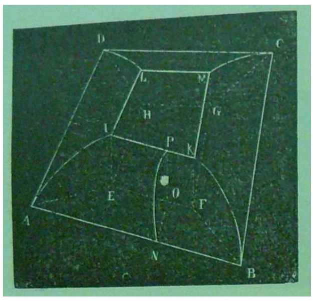


Figure 1: Cloister vault (Donghi, 1925)

Figure 2: Intrados of the vault

Figure 3: Extrados of the vault

Geometrical Survey

Through a careful 3D geometric survey it was possible to reliably trace the geometry of the vault.

Using a laser integrated theodolite some station points were identified and a polygonal of 5 points was used (Fig. 4). Station points were set to collimate points on the entire extrados surface of the vault.

A detailed survey was carried out hitting directly the masonry surface in the relevant points defining the geometry, such as edges and corners. Fig. 5 show the points cloud, where it is possible to see all the elements of the vault: internal perimeter walls, lunettes, their groins and their intersection with the vault, and finally a nearly flat central area. This area consists of nine little shallow double-curvature vaults (Fig. 6), supported by a frame of metal beams. The three-dimensional model built from the points cloud is shown in Fig. 7 in 3d view.

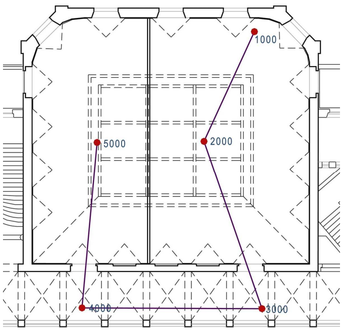


Figure 4: Polygonal line: plan view

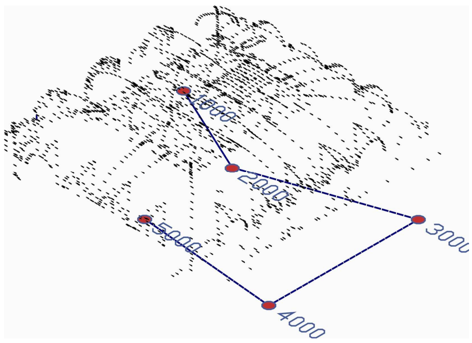


Figure 5: Polygonal line and points cloud: isometric view

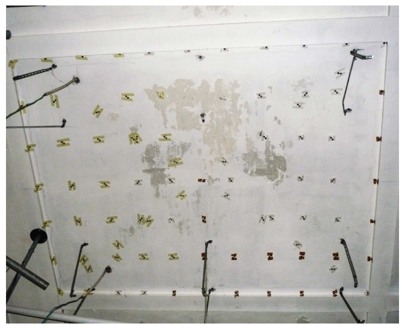


Figure 6: Detailed points on one little shallow vault

According to the phase of restitution of the survey and the three-dimensional reconstruction of the geometry to be carried out subsequently, a different accuracy of point gathering has been adopted depending on the different complexity of the element shape which was surveyed in turn. Since one of the nine little shallow vaults was going to be subsequently analyzed through the numerical modeling, many detailed points were taken on its extrados. These points were set along a reference grid dividing its surface into equal parts, in order to gain a reliable three-dimensional reconstruction and generate a solid 3d mesh (Fig. 10).

The survey campaign with the laser integrated theodolite was completed with a direct survey, some details not being detectable with the automatic tool. After the geometrical survey phase, the restitution phase was carried out through specific software with the aid of points monographs (Fig. 7). It was possible to visualize the points cloud, by which the overall geometry and the details of architectonical elements were identified.

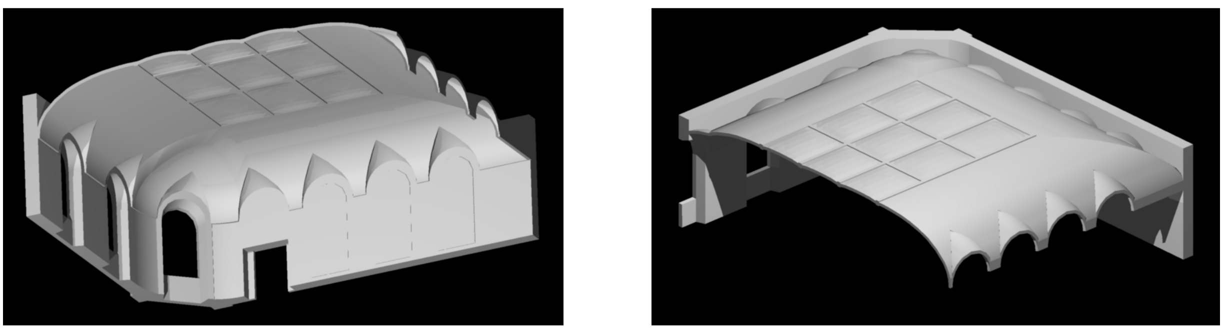

(a) *(b)*

Figure 7: Three-dimensional modelling of the vault, (a) intrados view, (b) extrados view

Geometric Deformation and Survey of the Crack Pattern at Intrados of the Vault

After the geometry, the crack pattern was visually surveyed to highlight the presence of material discontinuities at both sides of the structure. The type of cracks and their direction may help to

qualitatively understand the stress distribution and possible mechanisms of collapse (Binda et al., 2007).
In Fig. 8 the different cracks are reported on the plan view distinguishing between cracks crossing or not crossing the whole vault thickness, with opening greater or smaller than one millimeter. It is possible to see that the cracks are more concentrated on the vault portion characterized by a greater curvature, and especially on the north side, not adjacent to the hallway, where the vault perimeter is polygonal and various diagonal cracks are visible. Other cracks of significant size are located in the opposite south-east and south-west corners, two of them cutting the whole vault thickness: they are in fact visible at the extrados of the vault. These cracks are parallel to those involving the adjacent hallway.

Other cracks can be observed near the groins of the lunettes and along the iron beams supporting the little domes.

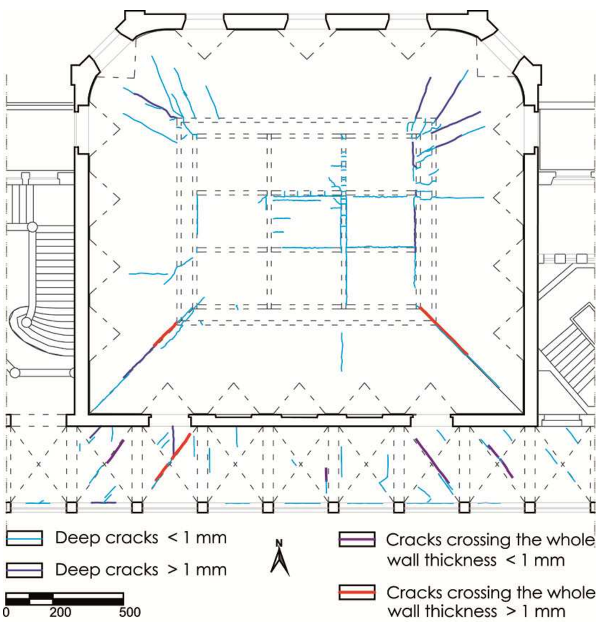


Figure 8: Crack pattern of the vault intrados

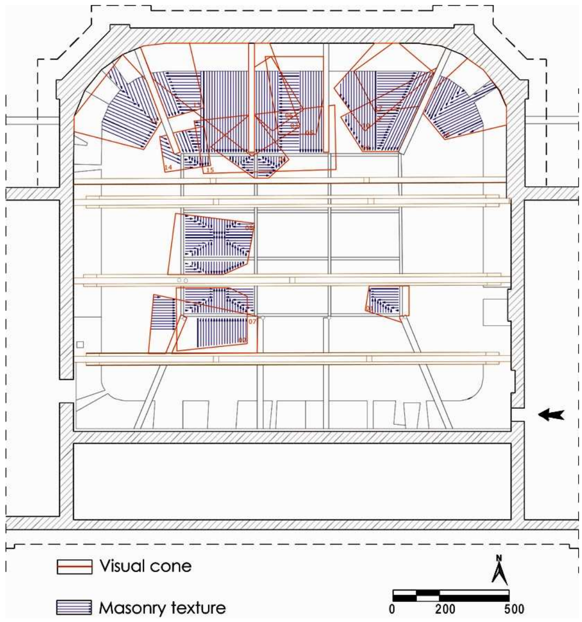


Figure 9: Vault masonry texture

Structural Analyses

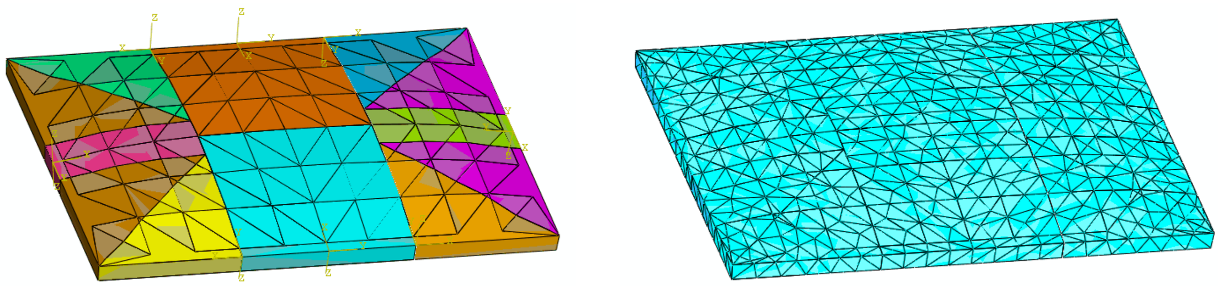
The most delicate part of the vault dealt with in this paper is the central one, consisting of 9 shallow vaults with rectangular plan, supported by a frame of steel beams (Fig. 9). The FE analysis of one of these vaults was carried out to evaluate the stress state under uniform dead and live loads (Boothby et al., 2008). The geometry of the vault model was defined according to the survey described above. Thermography allowed the masonry texture to be defined; a different orientation of the bricks in the different part of the vault was found (Fig. 9). Fig. 10a displays the subdivision of the model into parts: each one corresponds to a different orientation of the bricks, according to Fig. 9. The vault consists of hollow bricks with 2 or 4 holes, mostly laid orthogonally to the vault plan. The main mechanical properties of the bricks were obtained through simple compression tests carried out on a few samples taken from the collapsed part of the roof.

Finite Element Model Fig. 10b shows the FE mesh, consisting of tetrahedral 10-node finite elements with a quadratic modelling of the displacement field. Accordingly, stresses and strains vary linearly across the vault thickness, in agreement with the Kirchhoff-Love's theory for thin shells. The model globally consists of 2239 nodes, with 3 d.o.f.s each, and 1318 FEs.

The structural analyses were carried out assuming material and geometrical linearity. Masonry is modeled as a homogeneous orthotropic material. The values of the (engineering) elastic constants employed in the numerical analyses are listed in Table 1. The starred values were deduced from the

mechanical tests performed; the remaining ones were estimated according to results available in the literature. In Table 1, x is the hole direction (that is, the largest size of the bricks), y is orthogonal to x in the vault base plan, and z is orthogonal to the middle surface of the vault.

For comparison, also the results obtained with an isotropic model will be shown. Isotropy is an assumption often made for simplicity in the analysis of masonry structures, despite the inherent anisotropy of the material. The elastic constants of the equivalent isotropic material are $E = 790$ MPa, $\nu = 0.157$, which are averages of the values recorded in the experiments along different directions.



(a) *(b)*

Figure 10: (a) Geometrical model of the vault; each part corresponds to a different brick orientation; (b) FE mesh

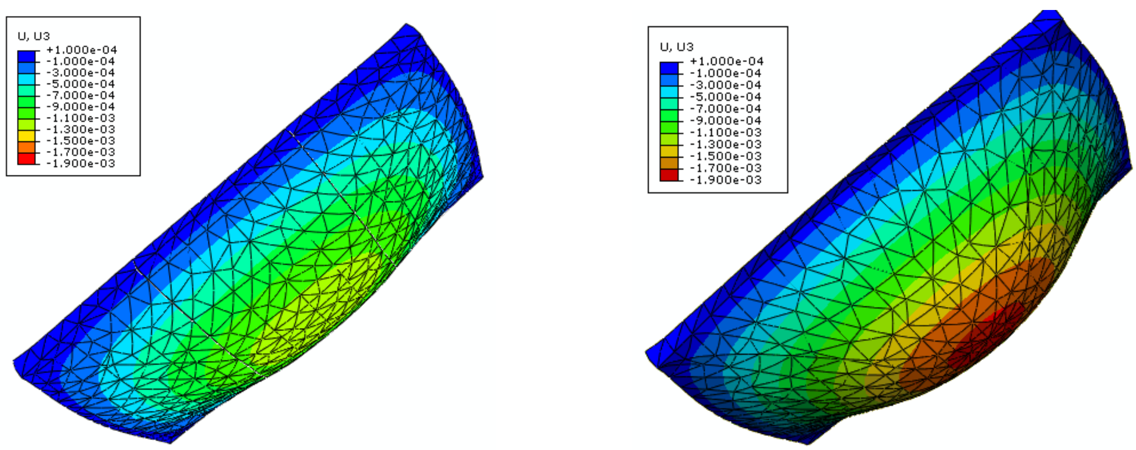
Table 1: Elastic constants employed in the numerical analyses

E_x [MPa]	E_y [MPa]	E_z [MPa]	ν_{xy}	ν_{xz}	ν_{yz}	G_{xy} [MPa]	G_{xz} [MPa]	G_{yz} [MPa]
1940*	610*	610	0.2*	0.2	0.1	200	200	280

The unit weight, estimated through tests, was taken equal to 12 kN/m^3. According to the current regulations, in the analyses this value was increased of a 10%. A uniform live load of 1.50 kN/m^2 was also taken into account, as prescribed for practicable roof decks; for simplicity, this load was assumed to act perpendicularly to the vault extrados.

As far as boundary conditions are concerned, all the lower edges of the vault were assumed to be supported, and the displacements of the upper edges orthogonal to the each edge were constrained to match the type of constraint exerted by the beams on which the vault lies.

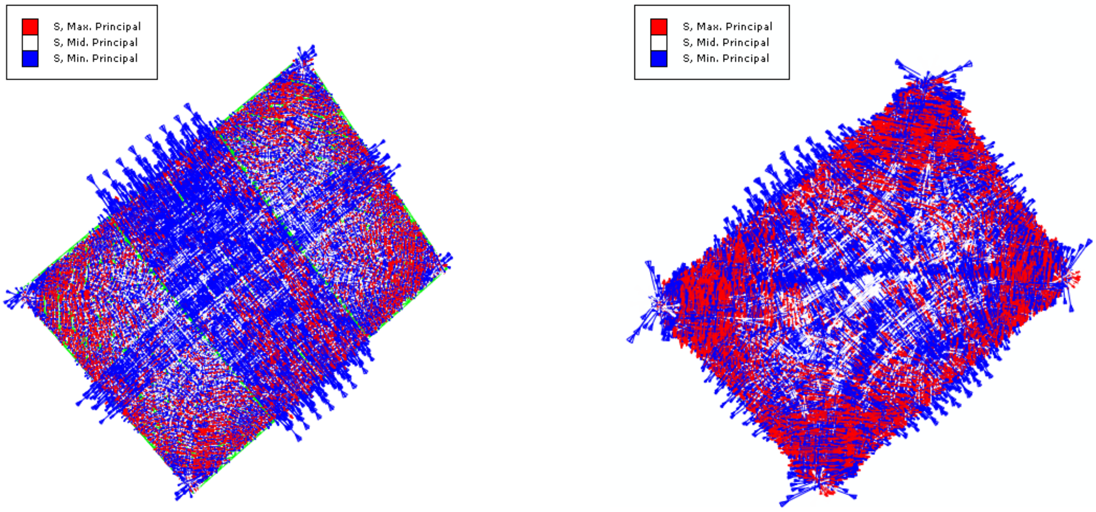
Numerical Results The deformed shape of the vault is shown in Fig. 11a. The maximum vertical displacement is attained at the center of the vault and is of 1.23 mm. Note that an isotropic model would be much more deformable, and a maximum displacement of 1.76 mm would be predicted (Fig.11b).



(a) *(b)*

Figure 11: Deformed shape of the vault (displacement magnification factor = 500) and contour plots of the vertical displacements: (a) orthotropic model; (b) isotropic model

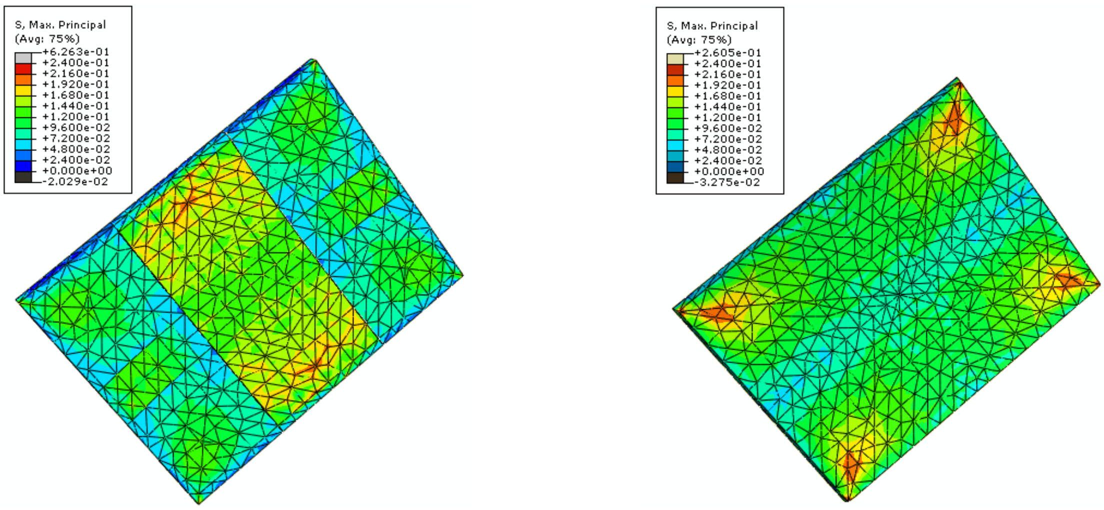
Fig. 12 shows the vector plots of the principal stresses. The difference in static behaviour according to the two material models is apparent. Using the orthotropic model (Fig. 12a), the brick courses parallel to the longest sides and, especially, to the shortest sides of the vault define a sort of arches subjected to compression and bending, with tensile (resp. compressive) stresses at the intrados (resp. extrados). The stress conditions near the corners are more complex, although the extreme principal stresses roughly run at 45° respect to the vault sides. Using the isotropic model (Fig. 12b), the stress distribution is somehow similar to that in a uniformly loaded plates, supported along the edges. In both cases, the weak curvature of the vault allows the weight to be partially born by the membrane forces, and not simply by moments and transverse shear forces as in a flat plate.



(a) *(b)*

Figure 12: Vector plots of the stresses in the vault: (a) orthotropic model; (b) isotropic model

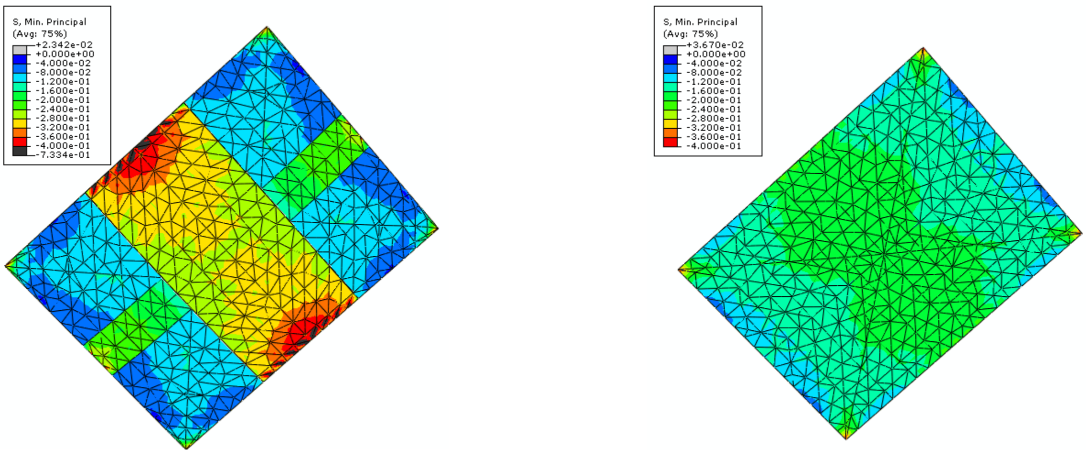
Figs. 13 and 14 show the contour plots of the maximum principal stress at the intrados of the vault and the minimum principal stress at the extrados, respectively. Using a correct orthotropic model (see Figs. 13a and 14a), the highest computed tensile (resp. compressive) stresses are about 0.17 (resp. –0.25) MPa. The most stressed region of the vault is the central band parallel to the shortest sides of the vault, which roughly behaves like a flat arch. The stresses computed according to an isotropic model would be completely different, both in distribution and value (see Figs. 13b and 14b). Although the isotropic model predicts highest peak stresses, it is not conservative compared to the orthotropic one. Comparing Figs. 13a and 14a with Figs. 13b and 14b, respectively, the highest stresses are apparently widespread over a wider region in the orthotropic model rather than in the isotropic model.



(a) *(b)*

Figure 13: Maximum principal stress at the intrados: (a) orthotropic model; (b) isotropic model

The computed compressive stresses are below the material strength experimentally determined, which ranges between 0.9 and 18.8 MPa, depending on the load direction. The tensile strength of the masonry forming the vault was not measured experimentally. Of particular interest is the tensile strength of the mortar-brick interface, which is the weakest component of masonry. According to the literature, this strength is very scattered and roughly ranges between 0.1 and 0.4 MPa. The highest computed tensile stresses might exceed the mortar-brick bond strength, should it fall in the lowest part of the range reported above. On the other hand, in the orthotropic model the highest tensile stresses are nearly parallel to the brick courses in the central part of the vault (see Figs. 13a and 14a) matching the observable crack direction parallel to the iron beams, as shown in Fig. 8, and do not act on the mortar-brick interface.



(a) *(b)*

Figure 14: Minimum principal stress at the extrados: (a) orthotropic model; (b) isotropic model

Conclusions

A double curvature hollow bricks cloister vault built at the beginning of XX century was studied. A 3D geometric survey was produced through a laser integrated theodolite constituting the base for understanding the static behaviour of the vault. Thermo-vision allowed to collect details on the location and connections of metal beams and to reconstruct the brick texture significantly influencing the stress distribution. A 3D numerical model exploited the accurate survey of the vault geometry. The results of the FE analysis of a typical shallow vault element allowed stresses and displacements to be estimated and the position of the maximum stress values to be compared with the crack directions. Also, the material orthotropy was taken into account: its strong influence on the numerical results was pointed out.

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