Structural Changes Effect on Deflect of Tower St. Jiri on Prague Castle

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Abstract The basilica st. Jiri on Prague Castle has two towers and three bodies. The north tower is deflected from vertical about 450mm. During the last measurement it was found, that the walls were diverted too. The reasons and beginning of fault genesis have not been known yet. It can be larger self weight of tower then weight of navy. It leads to different settlements during homogenous foundation conditions. The next reasons are not clear. I investigated: tower weakening by the door opening, influence of inside vault over middle navy, foundation conditions. The results obtained from numerical model show that every mentioned reason contributed to the deflection of the tower.

Keywords: Repair, vault, deflection, tower, analysis model

Introduction

The basilica st. Jiri on Prague Castle is very important historical landmark (Fajman et al. 2010). During years was made lot of changes on it, which left a lot of problems. The most important failure is the north tower deflection from vertical – see Fig. 1. The horizontal forces from heavy vault were specified as the main reason of problem during reconstruction in the late 19th century. In that time the vault in central navy was taken out. The last rebuilding was in sixties 20th century and the tower was stiffened and the foundation was improved. Nowadays is not clear if the way of the last reconstruction was necessary and suitable. Even is unclear if the deflection will go on. This was the reason for the monitoring of the tower. At the same time the numerical models were created, which should give us the answer to the question: “Why the tower is deflected?”

Figure 1: Situation in 2009
Briefly from History

The original church was built in 920. The towers and vaults were added in 1142 after fire. During late 13\textsuperscript{th} and 14\textsuperscript{th} century the door holes through northern tower in ground floor were managed and was realized reconstruction. The north part was changed – the upper vault in side body was taken out and the north wall was rebuilt. In 1541 when the whole Prague Castle area was destroyed by fire the basilica was saved by vaults. The following repair connected body to the close monastery by corridor along north side.

Repairs

We know two documented reconstructions. The radical repair was in the late 19\textsuperscript{th} century. The builder Mach removed the vault in central navy and improved foundation of the south tower. The camber of north tower was measured 0,28 m but the builder established that the main reason (horizontal vault forces) had been resolved and the repair was not urgent.

Later the repair of north tower proceeded in some phases (stages). At first in 1947 the north tower were supported by two new concrete structures (Fig.2). In 1969 bottom of the north-eastern tower part was removed and its foundation was deepened. Then new concrete column was built and was covered by stone.

Technical Parameters

The basilica is consisted from three bodies. The central navy has span length 8 m, south body 2 m and north body 2 – 4,5 m. Length is 40 m and high of central navy is 12,5 m, high of towers is 38 m. The navy vault was barrel with opened lunettes. Ground plan and cross section are in fig. 3.

Material Properties

The basilica is created of stone masonry with various thicknesses from 0,6m to 1,3m. The vault masonry was made of stone with thickness 0,45m. The tower is built from sandwich masonry. The central part is made of mortar with rubble. Interesting think is that the tower is not jointed with basilica by wall, but only by ceiling, roof and vault.
Foundation Condition

The rock top line proceeds in a parallel way with longitudinal axis of basilica - see crosssection in Fig. 3. The rock under boundary wall is weathered and damaged into depth 3m. The footing bottom of north tower is on clay and batt (clay shale) in irregular depth 0 - 0.6 m. The rock is in 3.5m depth. The foundation width is 1.8m. The last reconstruction caused little uncleared situation with used material and depth foundation (1-1.4m concrete foundation).

Computational data (Kuklik et al. 2009)

Contact stress 250kPa deforms layers of soil about thickness $h = 2$ m
- Corresponding material parameters of soil (stiffnesses) are $c_1 = 15$MN/m$^3$, $c_2 = 1$ MN/m,

Contact stress 400kPa deforms all layers of soil up to the rock - $h = 3.5$ m
- Corresponding material parameters of soil (stiffnesses) are $c_1 = 8$MN/m$^3$, $c_2 = 1.5$MN/m,

Degradation of soil is taken into structural analysis as change of soil stiffness under tower. Stiffness is decreased to $c_1 = 1$MN/m$^3$.

Deflection from Vertical of North Tower

Diversion of the north tower is observed long time. Builder Mach measured displacement on weather moulding 0.28m during his reconstruction. Later, the diversion was measured 0.4m during the reconstruction in sixties. The question is if the expanded displacement could be real during such a short time. The difference could be caused by the wrong interpretation of measurement (e.g. the different points were measured). At present time the displacement is 0.45m on the northeast edge of weather moulding and 0.54m on the northwest edge of weather moulding.

It is not clear, from historical observation, if the deformation process is finished or is in progress. The same measurements were made in 2000-2002 but the results did not answer to it. It leaded to the new geodetic surveying. The tower and wall position were measured in 2008. From results it is evident that the tower and walls are diverted, but in various ways. The next outcome shows that the upper half of walls are more deformed then the bottom half which points to the problem with vault horizontal forces. The tower is uniformly deformed as a rigid body which can correspond to the problems with foundation and vault horizontal forces.

The deformations are in Figs. 4 and 5.

![Figure 4: Declinations](image)

We can search sources of tower diversion in some planes. At first it is necessary to focus on historical construction activities. The original church was expanded spontaneously, some walls was founded on the older walls with unclear foundation conditions. The next problem of the similar buildings with non-uniform distribution of weight is differential settlement during homogenous parameters of foundations and subsoil. In this case it leads to different vertical displacements between heavier tower and navy. The serious problem is soil degradation. It is evident from
geological section (0,00-0,80m foundation arenaceous marl quarry, 0,80-3,67m deluvial silty soil, 3,67-8,00m alum slate). Actually, the weathered subsoil reaches to 3,5m of depth.

List of the reasons:
- historical construction activities
- non-uniform distribution weight
- soil degradation
- vault horizontal forces

Analysis Model
For discovering influence of particular effects the computational FE model was made. The model was modified in dependence on research problem. The space FE model is designed. The standard finite element method was used with triangular (Fajman 2002) and rectangular shell (plane-plate or flat) elements and beam elements with six degrees of freedom in a node. Calculations on the structure were made under the geometrical and the material nonlinearity conditions (tensile is not allowed). Incremental Newton-Rawson method was used and the tensile stress was suspended.

Computed Structures and Results
Some models were established to find the influences of non-uniform distribution weight, vault horizontal forces, historical construction activities and soil degradation.
- Original basilica without central vault
- Basilica with central vault
- Basilica with central vault and door openings in ground floor
- Basilica with diversion tower (0,2m) (see Fig. 6)
Last model is subjected to change of soil stiffness under north tower. Stiffness is decreased from $C_1 = 15 \text{ MN/m}^3$ to $C_1 = 8$ and $1 \text{ MN/m}^3$.
Considered loading is self weight, wind load and imposed load.
- Original basilica without central vault

\[ C_1 = 15 \text{ MN/m}^3, \ E = 3 \text{ GPa} \]

<table>
<thead>
<tr>
<th>Self weight</th>
<th>tower eaves</th>
<th>central</th>
<th>Line forces [kN/m] tower foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>-</td>
<td></td>
<td>630 – 720</td>
</tr>
<tr>
<td>wind</td>
<td>-</td>
<td></td>
<td>44 – 55</td>
</tr>
</tbody>
</table>

- Basilica with central vault

\[ C_1 = 15 \text{ MN/m}^3, \ E = 3 \text{ GPa} \]

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</tr>
</thead>
<tbody>
<tr>
<td>71</td>
<td>89</td>
<td></td>
<td>680 – 805</td>
</tr>
<tr>
<td>wind</td>
<td>1</td>
<td></td>
<td>44 – 55</td>
</tr>
</tbody>
</table>

- Basilica with central and vault door openings in ground floor

\[ C_1 = 15 \text{ MN/m}^3, \ E = 3 \text{ GPa} \]

<table>
<thead>
<tr>
<th>Self weight</th>
<th>tower eaves</th>
<th>central</th>
<th>Line forces [kN/m] tower foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>89</td>
<td></td>
<td>685 – 815</td>
</tr>
<tr>
<td>wind</td>
<td>3</td>
<td></td>
<td>44 – 60</td>
</tr>
</tbody>
</table>

- Basilica with diversion tower (see Fig. 6)

\[ C_1 = 15 \text{ MN/m}^3, \ E = 3 \text{ GPa} \]

<table>
<thead>
<tr>
<th>Self weight</th>
<th>tower eaves</th>
<th>central</th>
<th>Line forces [kN/m] tower foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>See Fig. 5.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>115</td>
<td></td>
<td>795 – 930</td>
</tr>
<tr>
<td>wind</td>
<td>5</td>
<td></td>
<td>52 – 65</td>
</tr>
</tbody>
</table>

- Basilica with diversion tower (soil degradation)

\[ C_1 = 15 \text{ MN/m}^3, \text{ under tower } C_1 = 8 \text{ MN/m}^3, \ E = 3 \text{ GPa} \]

<table>
<thead>
<tr>
<th>Self weight</th>
<th>tower eaves</th>
<th>central</th>
<th>Line forces [kN/m] tower foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>122</td>
<td>119</td>
<td></td>
<td>765 – 880</td>
</tr>
<tr>
<td>wind</td>
<td>19</td>
<td>6</td>
<td>42 – 55</td>
</tr>
</tbody>
</table>

- Basilica with diversion tower (soil degradation)

\[ C_1 = 15 \text{ MN/m}^3, \text{ under tower } C_1 = 1 \text{ MN/m}^3, \ E = 3 \text{ GPa} \]

<table>
<thead>
<tr>
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<th>tower eaves</th>
<th>central</th>
<th>Line forces [kN/m] tower foundation</th>
</tr>
</thead>
<tbody>
<tr>
<td>285</td>
<td>128</td>
<td></td>
<td>625 – 690</td>
</tr>
<tr>
<td>wind</td>
<td>40</td>
<td>8</td>
<td>32 – 40</td>
</tr>
</tbody>
</table>
Conclusions

- Historical construction activities
  - door openings significantly increases compression stress in wall from 0.6 MPa to 4.2 MPa, but not deflection,
  - central vault expands of tower horizontal displacement less then 20%.
- It was verified, that non-uniform distribution weight leads to the tower horizontal deflection.
- Imperfections of walls (second order effect) have a similar impact as horizontal forces of vault.
- Change of stiffness foundation has the biggest impact to the horizontal displacement at all.
- The final displacement when all effects are considered together is bigger then sum separated effects. It means that structure is subjected to geometrically and materially nonlinear behaviour.

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References