Statistical Time Series Analysis of Crack Movements at Bayon Main Tower, Cambodia

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Abstract Based on the past monitoring data of crack movements and various weather conditions by JASA (Japanese Government Team of Safeguarding Angkor), we quantitatively examine about the effects of each weather conditions to each crack movements at Bayon main tower. Then, we applied the time series analysis using a state-space representation in the examination. In the model of the state-space representation, the factors of crack movement are assumed as temperature, wind velocity and rainfall. Those quantitative examinations of crack movements will be necessary for the planning of reinforcement and restoration at Bayon main tower.

Keywords: State-space model, time series analysis, crack movement, masonry, angkor monument

Introduction

Bayon temple, Angkor Tom in Cambodia was constructed at the latter half of 12 th century. The main tower of Bayon is 31 m high tower structure of sandstones by the dry masonry. Around the main tower, there are 8 sub towers. The top of main tower lost the original symmetric proportion, caused by the collapses in long years. In 1930s, at the main tower, EFEO (Ecole française d'Extrême-Orient) conducted a conservation and restoration activities such as concrete repair, re-construction and iron cramp. However, after the repair by EFEO, other new cracks and gaps have proceeded. Now, the repairs are severely damaged and second repairing should be necessary. From 1997, as the part of JASA’s restoration work, Iwasaki and Fukuda started the monitoring of the crack movement at the severely cracked site of the top of the main tower\textsuperscript{1}. In this paper, using those monitoring data, we examine about the effects of each weather conditions to each crack movements by applying the time series analysis using a state-space representation to Bayon main tower.

Figure 1: Bayon Temple

Figure 2: Location of Crack
Outline of a State-Space Model in a Time Series Analysis

Kitagawa and Matsumoto (1996) proposed a state-space model with regression covariates for the analysis of the underground water level. In this paper, we applied the state-space model with regression covariates to the crack movement of masonry structure. As the factors of crack movement in Bayon main tower, Iwasaki and Fukuda reported the solar radiation, wind velocity, rainfall based on their monitored data of crack movements and various weather conditions. So, based on the above research, we analyzed the crack movements by the model of a temperature, wind velocity and rainfall. The model is

\[ y_n = t_n + P_n + W_n + R_n + \epsilon_n \]  

(1)

where \( y_n, t_n, P_n, W_n, R_n \) and \( \epsilon_n \) are the observed crack movement, trend, temperature, wind velocity, rainfall effect components and the observation noise. Then each components are represented as

\[ t_n = t_{n-1} + v_n, \quad v_n \sim N(0, \tau^2) \]  

(2)

\[ y_n = t_n + w_n, \quad w_n \sim N(0, \sigma^2) \]  

(3)

\[ P_n = \sum_{i=0}^{l} a_i p_{n-i}, \quad W_n = \sum_{i=0}^{m} b_i w_{n-i}, \quad R_n = \sum_{i=0}^{k} c_i R_{n-i} + \sum_{i=0}^{k} d_i r_{n-i} \]  

(4)

where \( p_n, w_n \) and \( r_n \) are the observed temperature, wind velocity and rainfall. And, \( a_i, b_i, c_i \) and \( d_i \) are the coefficients of each observed components. And, \( l, m \) and \( k \) are the model orders of each components. The above equations are represented as a state-space representation as

\[ x_n = F x_{n-1} + M r_{n-1} + G v_n \]  

(5)

\[ y_n = H x_n + w_n \]  

(6)

\[ x_n = \left( t_n, a_0, \ldots, a_l, b_0, \ldots, b_m, R_n, \hat{R}_{n+1/n-1}, \ldots, \hat{R}_{n+k/n-1} \right) \]  

(7)

\[ H_n = (1, p_n, \ldots, p_{n-l}, w_n, \ldots, w_{n-m}, 1, 0, \ldots, 0) \]  

(8)

\[ F = \begin{bmatrix} 1 \\ I_{l+1} \\ I_{m+1} \\ c_1 & 1 \\ c_2 & 0 & \ddots \\ \vdots & \ddots & 1 \\ c_k & 0 \\ \end{bmatrix}, \quad G = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ d_1 \\ \vdots \\ d_{k-1} \\ d_k \end{bmatrix}, \quad M = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \]  

(9)
\( \tau^2, \sigma^2, c_i \) and \( d_i \) are estimated in the algorithm of Kalman filter by maximizing the log-likelihood of the model. And the model orders of \( l, m \) and \( k \) are estimated by minimizing AIC (Akaike Information Criterion) parameter.

**Time Series Analysis of Crack Movement at Bayon Main Tower**

**Outline of analysis** In this paper, we conducted a time series analysis at 5 crack sites of NE2, NE3, NE4, NW1 and NW2 in Bayon main tower as Fig. 2. And, the period for the analysis is from September in 2006 to August in 2007. Fig. 3 shows the weather monitoring data of the temperature, the wind velocity and the rainfall. Each weather monitoring data are sampled at 30 minutes intervals, but the crack movements are sampled at 3 hours intervals. So, using the weather monitoring data of 30 minutes interval, we set the weather data of 3 hours interval for the analysis as follows: The temperature is set by linear interpolating, the wind velocity is set by the maximum value in 3 hours and the rainfall is set by the sum of 3 hours.

As the process of the analysis, if we try the analysis of all the weather monitoring data at once, it takes a huge calculation time. So, we applied the method of Kitagawa et al. The method is that the temperature effect and the wind velocity effect are estimated as the first process, after that the rainfall effect is estimated by fixing the order of the temperature and wind velocity effect model.

In this paper, we aimed at basic studies of applying the time series analysis to a crack movement in a masonry structure. Then, we divided one year data to per month data and compared the model orders and ranges of each effects by per month. By the above process, we examined about the effect of each weather conditions to each crack movements.

**Result of the Time Series Analysis** The results are shown in Fig. 4, 5 and 6. First, Fig. 4 shows the each estimated time series data divided to the estimated trend, temperature effect, wind velocity effect and rainfall effect which are combined with the results by per month to one year. As the problem, on December at NE3 and August at NW2, a divergence in the calculation process is occurred, so we could not estimate the each effect. And, Fig. 5 and 6 shows the transition of estimated model orders and the range which is the difference value between the maximum and minimum value by per month in each effects. From those results, we examined as follows.

- About the temperature effect, it is relatively resolved well. From the transition of model orders in Fig. 5, we can estimate that the degree of the effect of temperature to crack movements is relatively large. In Fig. 6, a maximum value of the ranges of the temperature effect are about 0.02 mm at NE2, 0.05 mm at NE3, 0.03 mm at NE4, 0.08 mm at NW1 and 0.03 mm at NW2. At the crack site which the range of movement is relatively small such as NE4, the movement by per month is comparatively caused by the temperature.

- About the wind velocity effect, at NW1 on December and NW2 on May, the effects have a slight value, but, altogether, it is not resolved well. It is because that this analysis is separated by per month and the maximum wind velocity is 14 m/sec in the period. The stronger wind velocity is sampled in the other year, and in the stronger windy condition, there is a high potential to occur a sudden plastic movement and the effectiveness of wind will be higher. Additionally, to consider the direction of wind to the model will be necessary.

- About the rainfall effect, it is smaller than our original assumption. It is because the longer term analysis and harder rainy condition will be necessary to detect the rainfall effect well. Additionally, the model considered the time lag between the time of rain and crack movement will be appropriate.

- About the change of the trend in a year or longer term, the reason of the change is not quantitatively detected. The analysis of longer years will be necessary for it.
Figure 3: Monitoring Data of Temperature, Wind Velocity and Rainfall

Figure 4: Estimated Data of Trend, Temperature Effect, Wind Velocity Effect and Rainfall

Figure 4: Estimated Data of Trend, Temperature Effect, Wind Velocity Effect and Rainfall
Figure 5: Monthly Transition of Model Orders  
Figure 6: Monthly Range of Each Components
Conclusions
In this paper, using monitoring data of crack movements, temperature, wind velocity, rainfall, we quantitatively examined about the effects of each weather conditions to each crack movements by applying the time series analysis using a state-space representation to Bayon main tower. As the next study to reflect the range of crack movement caused by each weather conditions to the planning of reinforcement, it is necessary to conduct the analysis in a longer term. Additionally, it is also necessary to study the other models of analysis for improving the accuracy.

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References