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Structural Effects of Brick Arrangement and Span Length on Mid-pointed Arches

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Abstract One of the most important, valuable and remarkable elments of Persian architecture is brick masonry arch. Structural behaviour of Persian brick masonry arches has not been studied in details. Any investigation into their characteristics can be beneficial for maintenance, retrofit, restoration and reconstruction of such arches. The effect of a brick arrangement in the fabric of arches, such as Roman and barrel arrangements, on structural behaviour of brick masonry arches has been a serious controversy among architects and structural engineers for many years. In this study, micro-modelling finite element technique has been used to analyse mid-pointed arches with two different brick arrangements, i.e. Roman and barrel arrangements, under static weight load using the finite element method. Analyses have been carried out and obtained results have been discussed to describe the effect of brick arrangement on structural behaviour of analysed arches with three different span lengths.

Keywords: mid-pointed arch, Roman, barrel, brick, mortar, tensile stress, compressive stress

Introduction

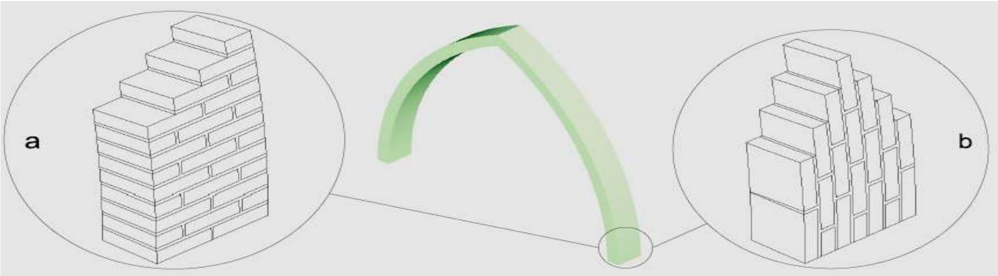
Structural masonry is encountered persistent prejudices, its brittleness, inability to resist earthquakes, and the dependence of its performance on quality and workmanship. Today, the preceding effort for the masonry researchers is to rationalise the engineering design of structural masonry. The key to validate, extend and improve existing design methods is an integrated experimental/numerical research programme. At the present stage of knowledge, numerical simulations are fundamental to provide insight into the structural behaviour and support the derivation of rational design rules. Numerical methods capable of predicting the behaviour of the structures, in the both linear and non-linear stages, from cracks and degradation to complete loss of strength make it possible to fully understand failure mechanisms, reliably assess structural safety and control the serviceability limit states (Lourenco 1996).

For masonry structural members, the linear elastic analysis gives acceptable outputs when the case is handled under low compressive stresses. It is more convenient to perform elastic calculations initially for the analysis before protection and restoration of historical structures (Korany 2003).

Bricklaying Methods in the Arch

Roman Method This kind of arches is made by laying stones or bricks in horizontal courses with their beds radiating from the center, as illustrated in Fig. 1(a) (Zomarshidi 1988).

Barrel Method Another method of building arches and vaults is to lay the masonry units in a series of rings side by side so that the arch or vault grows longitudinally from one end to the other. Construction begins by laying units on a slight lean against an end wall until a complete inclined arch is laid. An additional layer of thin mud bricks can then be added. Hence, the completed arch was literally a series of arched single brick courses constructed side by side, as shown in Fig. 1(b) (Zomarshidi 1988, Kashani 1987).



(a) (b)

Figure 1: Bricklaying methods in an arch: (a) Roman method; (b)Barrel method

Structural Modelling

In this study, mid-Pointed arches with both Roman and barrel bricklaying methods with three different span lengths have been modeled (Jafari 2005).

1-3-Mechanical and Dimensional Properties Each arch consists of square clay bricks and clay-gypsum mortar. The mechanical properties of materials are determined from the available sources shown in Table 1 (IBMBC 1986, Alkass et al. 1989). The thickness of mortar in the barrel arch intrados is 3 mm and in the Roman arch intrados is 10 mm, but it is relatively increased in the arch extrados because of the arch curvature. Mid-pointed arches are studied with three different span lengths of 4, 5 and 6 m. The width and thickness of arches are 41 cm and 21 cm, respectively (Jafari 2005).

Table 1: Material properties

Material properties	*Clay brick*	*Clay-gypsum mortar*
Dimension (cm)	20×20×5	-
Water absorption (%)	23	-
Bulk density (kg/m^3)	1330	1600
Modulus of elasticity (MPa)	5300	1300
Poisson's ratio	0.17	0.17
Tensile strength (MPa)	0.53	0.25
Compressive strength (MPa)	5.3	1
Coefficient of thermal expansion (m/m/°C)	0.6×10^{-5}	0.6×10^{-5}
Coefficient of friction of mortar joints	-	0.6
Tensile bond strength of mortar joints (MPa)	-	0.13
Shear bond strength (cohesion) of joints (MPa)	-	0.3

Technical Properties Micro-modelling finite element technique considering different elements for brick and mortar was used for linear modelling of the arches. Brick and mortar modelling were carried out using the structural 3D element. This SOLID element is defined by eight nodes and the isotropic material properties, and is capable of cracking in tension and crushing in compression that is very suitable to model the brittle and quasi-brittle materials. Contact elements were used to simulate brick/mortar interface. Meshing was carried out by trial and error finally led to the dimension of elements with optimum size of $5\times5\times7\,\mathrm{cm}^3$ for the analysis integration. All of arches were analysed under self-weight loading. Also, boundary conditions in skewbacks include 3 degrees of freedom: rotation in x, y and z direction (Jafari 2005).

Analysis

Numerical results include maximum displacements and maximum tensile and compressive stresses resulted from the 1st and 3rd principal stresses compared with allowable values (Jafari 2005). Obtained results are shown in Tables 2-7. In the Tables, σ , *t*, *c*, *Bri,Mor, B, R, max, all,* and *D* stand for stress, tensile, compressive, brick, mortar, barrel, Roman, maximum , allowable, and displacement, respectively.

Table 2: Numerical results for brick in Roman mid-pointed arch

Span Length (m)	$D_{(max)\,R}$ (m)	$\sigma^{t}_{Bri\,(max)\,R}$ (N/m^2)	$\sigma^{c}_{Bri\,(max)\,R}$ (N/m^2)	$\frac{\sigma^{t}_{Bri(max)R}}{\sigma^{t}_{Bri(all)}}$	$\frac{\sigma^{c}_{Bri(max)R}}{\sigma^{c}_{Bri(all)}}$	$\frac{\sigma^{t}_{Bri(max)R}}{\sigma^{c}_{Bri(max)R}}$
4	0.13×10^{-3}	135670	-265958	0.25	0.05	0.51
5	0.31×10^{-3}	231551	-392859	0.43	0.07	0.58
6	0.62×10^{-3}	351517	-543799	0.66	0.10	0.64

Table 3: Numerical results for mortar in Roman mid-pointed arch

Span Length (m)	$D_{(max)\,R}$ (m)	$\sigma^{t}_{Mor(max)\,R}$ (N/m^2)	$\sigma^{c}_{Mor(max)\,R}$ (N/m^2)	$\frac{\sigma^{t}_{Mor(max)R}}{\sigma^{t}_{Mor(all)}}$	$\frac{\sigma^{c}_{Mor(max)R}}{\sigma^{c}_{Mor(all)}}$	$\frac{\sigma^{t}_{Mor(max)R}}{\sigma^{c}_{Mor(max)R}}$
4	0.13×10^{-3}	109198	-216601	0.43	0.21	0.50
5	0.31×10^{-3}	191767	-325066	0.76	0.32	0.58
6	0.62×10^{-3}	295887	-454969	1.18	0.45	0.65

Table 4: Numerical results for brick in barrel mid-pointed arch

Span Length (m)	$D_{(max)\,B}$ (m)	$\sigma^{t}_{Bri\,(max)\,B}$ (N/m^2)	$\sigma^{c}_{Bri\,(max)\,B}$ (N/m^2)	$\frac{\sigma^{t}_{Bri(max)B}}{\sigma^{t}_{Bri(all)}}$	$\frac{\sigma^{c}_{Bri(max)B}}{\sigma^{c}_{Bri(all)}}$	$\frac{\sigma^{t}_{Bri(max)B}}{\sigma^{c}_{Bri(max)B}}$
4	0.10×10^{-3}	132018	-295105	0.25	0.05	0.44
5	0.24×10^{-3}	225612	-431215	0.42	0.08	0.52
6	0.48×10^{-3}	344219	-592168	0.65	0.11	0.58

Table 5: Numerical results for mortar in barrel mid-pointed arch

Span Length (m)	$D_{(max)\,B}$ (m)	$\sigma^{t}_{Mor(max)\,B}$ (N/m^2)	$\sigma^{c}_{Mor(max)\,B}$ (N/m^2)	$\frac{\sigma^{t}_{Mor(max)B}}{\sigma^{t}_{Mor(all)}}$	$\frac{\sigma^{c}_{Mor(max)B}}{\sigma^{c}_{Mor(all)}}$	$\frac{\sigma^{t}_{Mor(max)B}}{\sigma^{c}_{Mor(max)B}}$
4	0.10×10^{-3}	82545	-166420	0.33	0.16	0.49
5	0.24×10^{-3}	154797	-267257	0.61	0.26	0.57
6	0.48×10^{-3}	246923	-392234	0.98	0.39	0.62

Table 6: Mortar/brick maximum stress ratio in both Roman and barrel cases

Span Length (m)	$\frac{\sigma^{t}_{Mor(max)R}}{\sigma^{t}_{Bri(max)R}}$	$\frac{\sigma^{t}_{Mor(max)B}}{\sigma^{t}_{Bri(max)B}}$	$\frac{\sigma^{c}_{Mor(max)R}}{\sigma^{c}_{Bri(max)R}}$	$\frac{\sigma^{c}_{Mor(max)B}}{\sigma^{c}_{Bri(max)B}}$
4	0.80	0.62	0.81	0.56
5	0.82	0.68	0.82	0.50
6	0.84	0.71	0.83	0.66

Table 7: Roman/barrel arches maximum displacements and stresses ratios

Span Length (m)	$\frac{D_{(max)R}}{D_{(max)B}}$	$\frac{\sigma^{t}_{Bri(max)R}}{\sigma^{t}_{Bri(max)B}}$	$\frac{\sigma^{c}_{Bri(max)R}}{\sigma^{c}_{Bri(max)B}}$	$\frac{\sigma^{t}_{Mor(max)R}}{\sigma^{t}_{Mor(max)B}}$	$\frac{\sigma^{c}_{Mor(max)R}}{\sigma^{c}_{Mor(max)B}}$
4	1.28	1.02	0.91	1.32	1.30
5	1.28	1.02	0.91	1.23	1.21
6	1.29	1.02	0.91	1.19	1.15

The following results are obtained.

1. Tensile and compressive stresses in brick are more than the same stresses in mortar. Stresses and displacements increase as the span length increases.

2. Tensile/compressive stress ratio in brick and mortar increases commensurate to the increase of the span length in both Roman and barrel arches. In the other word, the rate of tensile stress development is more than that of the compressive stress.

3. Mortar/brick maximum stresses ratio generally increase as the span length increases in both Roman and barrel arches.

4. Mortar/brick maximum stresses ratio increases in the Roman arches are more in comparison with the barrel arches.

5. Maximum tensile and compressive stresses in mortar are more in Roman arches than barrel arches.

Stress Distribution in Mid-Pointed Arches

According to experimental tests on masonry materials, tensile stresses play an important and critical role in the masonry arches (Toker and Unay 2004, Drysdale et al. 1994, Brick masonry arches introduction 1995). Therefore, tensile stress distribution has been studied in the mid-pointed arches. Tensile zones have been evaluated under the "1st principal stress". A numerical evaluation and report processing in all of the 6 arches lead to the following results (Figures 2 -7):

- Four zones of tension can be seen in every half arch.
- The tensile stress diminishes from zone 1 to zone 4, therefore, zone1 has been allocated with maximum tension and other zones have been allocated with tensile stresses smaller than 30% of the maximum tensile stress. In this case, the values of tensile stresses in zones 2, 3 and 4, have been evaluated in all of arches in details and the results are summarised as follows:
    - Zone 2: tensile stress smaller than 30% of maximum tensile stress
    - Zone 3: tensile stress smaller than 25% of tensile stress
    - Zone 4: tensile stress smaller than 15% of tensile stress
- Maximum tensile stress is located at impost inside and maximum compressive stress is located at impost outside under the skewback of mid-pointed arches. Also, maximum displacement is located at the centre of the haunch (centre of zone 2) of the arch (Jafari 2005).
- The possibility of cracking in the arch under larger loads will be decreased from zone 1 to zone 4.
- As a hypothesis, violent concentration of tensile stress in impost inside depends on the kind of boundary conditions in skewbacks. Then tensile stress concentration and potential cracking would be transferred to upper parts of the arch if skewbacks and imposts were anchored in effective abutments.

The location of tensile zones has been shown by angle of rotation of the zone centre relative to the spring line in the Figures.

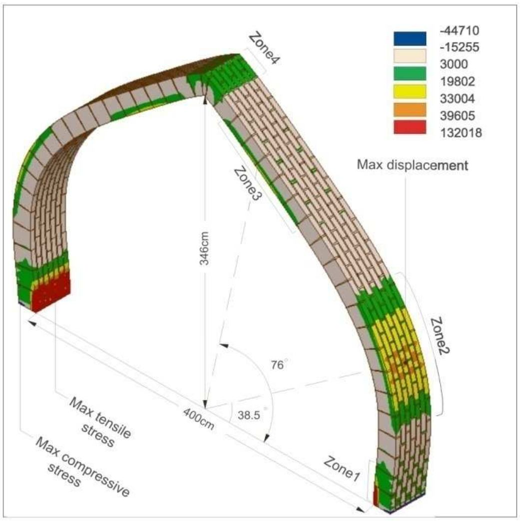


Figure 2: Stress distribution in barrel mid-pointed arch with a span of 4 m

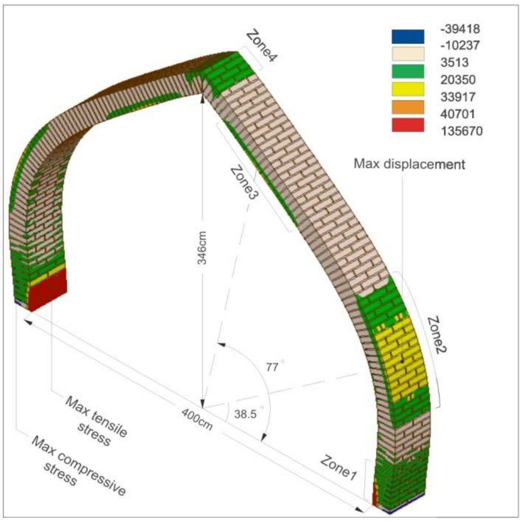


Figure 3: Stress distribution in Roman mid-pointed arch with a span of 4 m

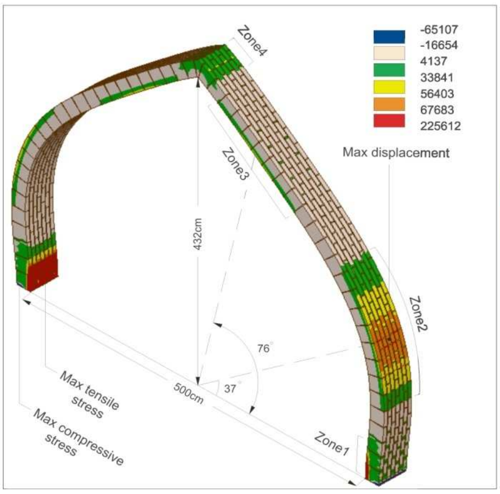


Figure 4: Stress distribution in barrel mid-pointed arch with a span of 5 m

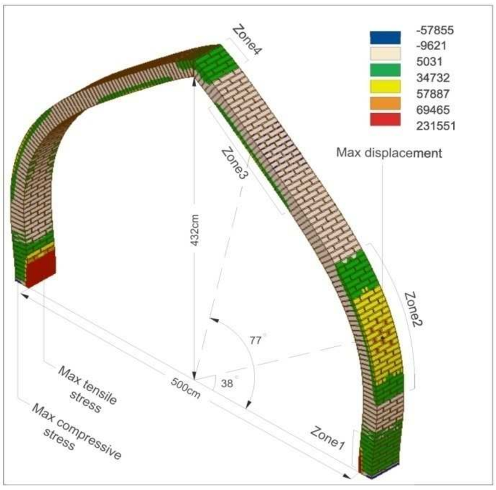


Figure 5: Stress distribution in Roman mid-pointed arch with a span of 5 m

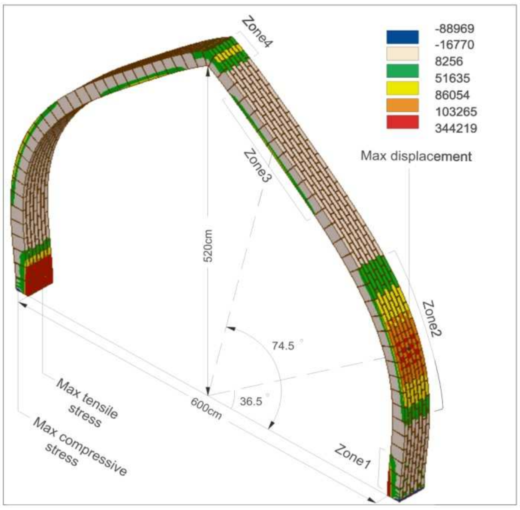


Figure 6: Stress distribution in Roman mid-pointed arch with a span of 6 m

Figure 7: Stress distribution in barrel mid-pointed arch with a span of 6 m

Conclusion

The effect of brick arrangement on structural behaviour of Roman and barrel mid-pointed arches of three different spans has been studied. Roman/barrel maximum tensile and compressive ratios in brick is constant when the span length increases, but these ratios in mortar. Maximum displacement increases in the Roman arches in comparison with the barrel arches. But Roman/barrel arch maximum displacement ratio is constant for all span lengths. Tensile zones, except zone 1 (impost inside), have been allocated with tensile stress smaller than 30% of maximum tensile stress under self-weight loading in all of the mid-pointed arches. A comprehensive comparison between the Roman and barrel mid-pointed arches in 3 different span lengths indicates that from structural viewpoint the behaviour of the barrel arches is more satisfactory than the Roman ones, in particular in arches with larger spans.

Acknowledgement

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References

[1] Lourenco, P B (1996). “Computational strategies for masonry structures.” Ph.D thesis, Delft University press, Netherlands.

[2] Korany, Y (2003). “Mechanics and modeling of URM structures.” International short course on architectural and structural design of masonry, Dresden University of Technology, Dresden, Germany.

[3] Zomarshidi, H (1988). “Vault and arch in Persian architecture.” *Keyhan publications*, Tehran, Iran, First edition, 152–162.

[4] Kashani, Q J (1987). “Vault and arch Resource.” *Soroush publications*, Tehran, Iran, First edition, 81–103.

[5] Jafari, F (2005). “Structural effect of brick arrangement in brick masonry arches.” Msc. Thesis, University of Najaf Abad, Esfahan, Iran.

[6] Alkass, R, Hadi, M, Khalil, N, and Ali, F (1989). “Ancient Babylonian bricks, their technological characterization and simulation,” in *Proceedings of First international conference on structural repairs and maintenance of historical buildings, computational mechanics publications. Southampton, UK*, 71-80.

[7] Toker, S, and Unay, A I (2004). “Mathematical modeling and finite element analysis of masonry arch bridges.” *G.U. journal of science*, 17(2), 129-139.

[8] Drysdale, R G, Hamid, A A, and Baker, L R (1994). “*Masonry structures behaviour and design*.” Prentice Hall, New Jersey, USA.

[9] Brick masonry arches introduction, (1995). Technical notes on brick construction, Brick industry association, Virginia, USA, revised 31.

[10] IBMBC Iranian brick masonry building code (1986). Sharif University of Technology press, Tehran, Iran.