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Kinematic Stability of Masonry Arches

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Abstract To quantify the safety of masonry arches and vaults using limit-analysis, various types of safety factors have been devised. The most well-known were introduced by Heyman: a "static factor of safety" (or load factor) assessing how vulnerable the structure is to increases in the living loads and a "geometrical factor of safety" assessing how critical the thickness of the structure is for its stability. In non seismic areas, one of the main risks of total or partial collapse of arches and vaults is excessive displacement of the supports (following walls or soil deformation). Limit analysis technique can be used to analyse this risk, quantifying movements permitted before collapse and evolution of the thrust on the supports. This analysis can be combined with pathological investigations and displacement monitoring to study the evolution of the risk and define a "kinematic factor of safety". A software program was developed (a) to compute domains of stability for particular mechanisms of deformation, (b) to study possibility of transitions between mechanisms during deformation and (c) to interactively study the influence of movements of the supports on thrust and stability. Scaled physical models are used to validate the limit analysis approach, using an experimental rig where horizontal and vertical displacements are controlled by computer. A high-speed camera is used to study transition between mechanisms. Finally it is referred to techniques integrating this kinematic approach into a more general probabilistic approach, taking into account various uncertainties in the structure (shape, thickness, loads, movements).

Keywords: Masonry, arches, limit analysis, large displacements, safety assessment

Introduction

The idea that collapse of arches may result from large displacements of the abutments is clearly not new. References can -for instance- be found in Danisy (Frézier 1737-39) in the 18th century or in Viollet-le-Duc (Viollet-le-Duc 1854-1868) in the 19th century. Quantitative analysis of those displacements is, on the other hand, a much newer subject. Some basic results were presented by Beranek (Beranek 1989) but only on the influence of the displacements on the thrust in arches. Heyman in his classic papers on the analysis of masonry structures does not say much more (Heyman 1966 and others). In the 1990s, experiments and a first formulation of the problem were proposed by researchers of the University of Florence (Briccoli-Bati 1997). More recently, Smars (Smars 2000) and Ochsendorf (Ochsendorf 2002, 2006) study more systematically the influence of large displacements on the stability of arches and present techniques to compute stability zones.

Nowadays, the standard tool used by engineers for the analysis of structures is FEA software. As many other researchers, the author is convinced of the interest to use limit analysis for the analysis of masonry structures (for reasons very clearly exposed by Mainstone (Mainstone 1997) and Heyman in various occasions). The program of limit analysis is not looking for stress distributions but only for ultimate conditions. It tackles the problem of main interest to structural engineers: controlling safety levels.

Limit analysis has nevertheless limitations. Some of them are arguably inherent to the technique but others can be lifted. Elsewhere, the influence of friction and of the low (but not null) tension resistance of mortars on the stability of arches was discussed (Smars 2000, 2008). In the present paper, the question of the influence of displacements is presented. Limit analysis cannot provide alone all answers to it but with the help of other investigation techniques, useful results can be produced. The structures under investigation are existing structures and their deformations can be measured directly in the field. In the case of an evolution of the deformations, it is much more effective to use the results of a structural monitoring to control the long-term "displacement" aspect of

the problem than to use mathematical models. Limit analysis software can then be used to provide safety estimates related to those displacements. Mathematical model, displacement monitoring and pathological investigations can provide estimates of the safety factors in time. This approach is, in our opinion, very coherent with the recent recommendations of ISCARSAH (ICOMOS).

Mathematical Model

Problem under Investigation. Let us consider an arch made of rigid blocks (voussoirs), infinitely resistant to compression, in contacts through bed joints without tension resistance, with infinite compression resistance and with a high friction preventing sliding. This is the classic arch of Kooharian/Heyman (Heyman 1966). Such an arch can only deform by forming hinges.

The question is then to predict how much the abutments of an arch can move before collapse. Related questions are: how does the thrust evolve? what is the mechanism of this deformation?

Kinematic Analysis. An arch of n blocks has $n+1$ joints. Hinges can potentially form in each joint, either around the extrados edge, either around the intrados edge. The arch being three times statically indeterminate, its deformation can lead to the formation of one, two or three hinges, making a mechanism allowing displacement of the abutments. The number m of potential rotation mechanisms is:

$$m = 2\binom{n+1}{1} + 2^2\binom{n+1}{2} + 2^3\binom{n+1}{3} = \sum_{k=1}^{3} 2^k \binom{n+1}{k} \qquad (1)$$

This number grows very fast. For an arch formed of 16 blocks, m=6018. Of course not all of them will have the same importance but more than one could imagine in first instance. In the following, a special notation will be used to designate a particular mechanism. Joints are numbered starting from the left. The extrados edge of joint i is called $+i$ and the intrados edge $-i$. Both are potential hinges. Mechanisms are designated by the edges where hinges form. Fig.1 shows an example of a three hinge mechanism: *-4+8+17*.

The domain of stability of a mechanism is defined as the area accessible by the point of the left abutment corresponding to *-1* if the right abutment is considered fix.

As voussoirs cannot interpenetrate, rotations can only occur in one direction, corresponding to the opening of the hinge. In Fig.1, the right support (at +17) is considered fixed. If the hinge at +17 opens and hinge at +8 remains closed, the locus of displacement of -4 will be the arc segment r17 and the displacement of +1, will follow r17' (a translated copy of r17). Similarly, if +17 remains closed and +8 opens, edge -4 will move along r8 and point -1 along r8'. The sector comprised between r8' and r17' therefore defines all the points accessible by -1.

Static analysis. Static conditions also limit possible movements. To be stable, a resistance line has to fit entirely in the shape of the arch (Heyman 1966). The statically admissible domain corresponding to a particular mechanism is the subset of the kinematic domain for which this condition can be achieved. Fig.1 shows an example of such a domain. The domain G was computed numerically using a specially designed software: Calipous (Calipous, Fig.3) [the numerical analysis of arches by limit analysis is best described by Livesley (Livesley 1978, 1992)]. It can be seen that the border of the domain sometimes result of kinematic conditions (Fig.1 b and d) and sometimes of static conditions (Fig.1 a, c and e).

Reaching the border of the stability domain of a mechanism does not necessarily lead to collapse. In some circumstances, the active mechanism can change and the arch continues to deform. Fig.2 can be used to illustrate the process. Three domains are drawn, corresponding to the mechanism -4+9-14, -4+9+17 and -4+8+17 (also presented in Fig.1). The transitions between mechanisms are of two types: *smooth transitions* and *brutal transitions*.

Smooth transitions happen when a hinge is closing (kinematic border). In such a circumstance, the pressure line is not anymore unique (three passage points are required to fix it). The thrust in the arch can change (increase in the passage from *-4+9-14* to *-4+9+17* presented in Fig.2) until it touches another edge of the arch, provoking another hinge to form (in Fig.2, *-4* closes and *+17* opens). During

smooth transitions, the variation of the potential energy of the arch is continuous, but the variation of the thrust is fast, which can possibly present a danger.

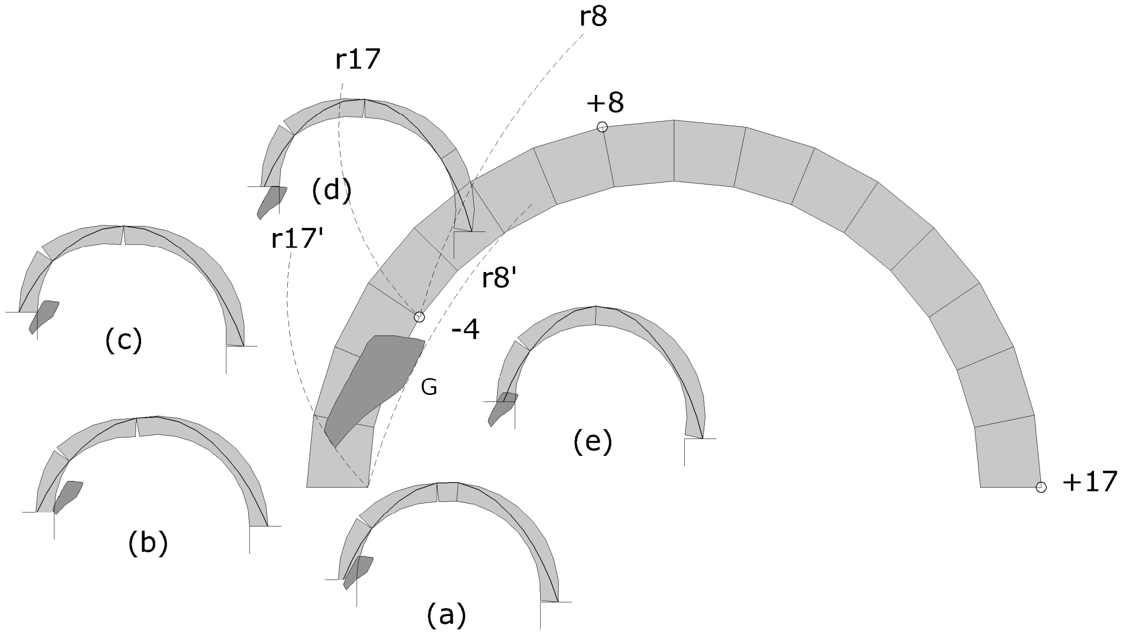

Figure 1: Stability domain G for the mechanism -4+8+17. The border of the domain is defined by various limit conditions: (a) resistance line touches +9: transition or collapse, (b) +17 is closed, (c) resistance line touches +1: collapse, (d) resistance line touches -13: collapse, (e) +8 is closed

Brutal transitions happen when the pressure line is touching a new edge. This lead to the formation of a fourth hinge. The new configuration is not stable and the arch will experience a dynamic transition (arch segments will acquire kinetic energy). This often lead to collapse (in Fig.2, at the top of the *-4+9+17* domain, a new hinge forms at *+1* and the arch collapses). It is nevertheless also possible that during the process, one of the existing hinges closes (in Fig.5, hinge +9 closes and hinge +8 opens). When the number of hinges is reduced to 3, the arch is potentially reaching a new domain of stability and ready to resist further displacements. It is also possible that the momentum is to important for the arch to resist and consequently collapses (as in Fig.5 where just after hinge *+9* closes, a new hinge forms in *-13* and the arch collapse. In other circumstances the arch can also resist, transforming the kinetic energy in heat (as it was witnessed in experiments very similar to the one presented in Fig.5). A key factor will be the difference of potential energy between the two configurations (i.e. the maximum kinetic energy that the arch segments can acquire). During brutal transitions, both thrust and potential energy experience fast transitions.

A figure with (many) more mechanisms that can result from the deformation of such an arch was presented elsewhere (Smars 2000).

Physical Models

A semicircular arch of 16 identical prismatic voussoirs was made and tested. The arch has a span of 300mm. Voussoirs are in stainless steel (in order to keep an accurate shape and resist numerous experiments). Their bed joints have a square section of 30x30 mm^2. The friction between the beds is increased by covering them by a thin medical tape (~75µm). The angle between their faces is 180°/16=11.25°. This is a symmetric arch; it has no keystone and obviously, results are influenced by the stereotomy.

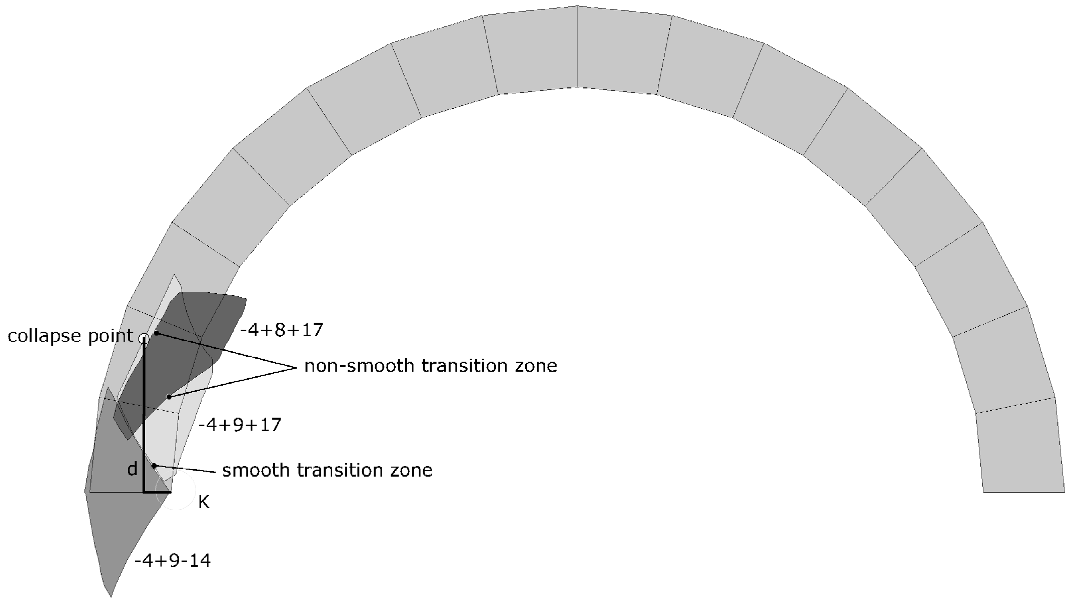

Figure 2: Stability zones corresponding to three possible rotation mechanisms: -4+9-14, -4+9+17 and -4+8+17

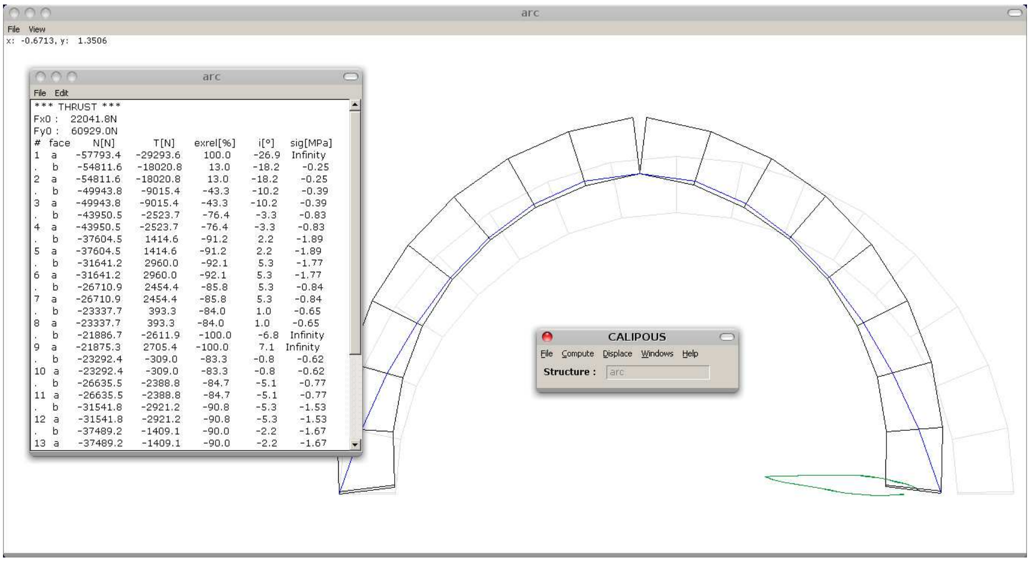

Figure 3: Calipous. Program used to compute the stability domains of a mechanism (here +1-9+17) of arches of arbitrary shapes.
Numerical results (normal and tangential force, eccentricity...) are output in a text window

Testing Rig. A experimental rig (Fig.4) was build to test the resistance of this (and other) arch(es) to large displacements. The right abutment of the arch is fixed. The left abutment can be displaced horizontally and vertically by two stepper motors. The rig is connected to a computer and a software was designed to drive the system (in C++ and Tcl/Tk). The arches are deformed slowly (0.4mm/s). In future this same rig will be used to investigate multi-ring arches and the influence of mortar.

Behaviour was monitored using a high speed camera (Casio Exilim Pro Ex-F1) from undeformed state up to collapse (512×384 pixels^2, 300 frames/s). Mechanisms are observed and monitored as they develop. Provisions were taken to allow measurements.

The program of investigation is still proceeding. The formation of mechanism, the transition between mechanisms and collapse was observed and documented.

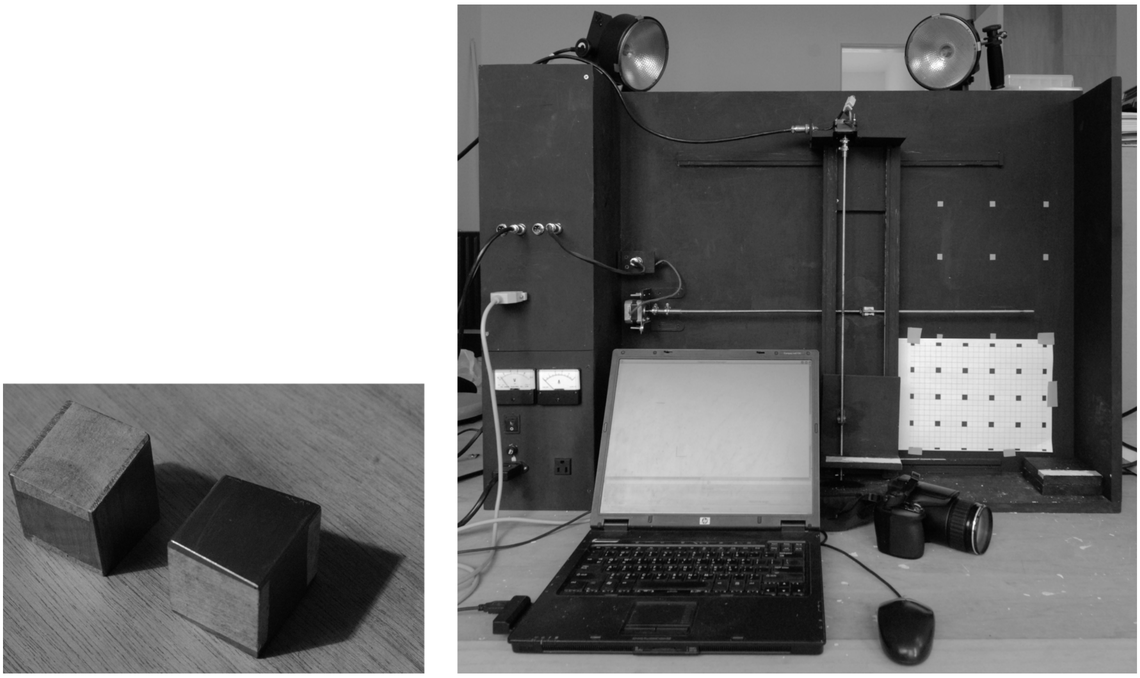

Figure 4: Left. Voussoirs of the arch used for the experiments.
Right. Experimental rig, controlling computer and high speed camera

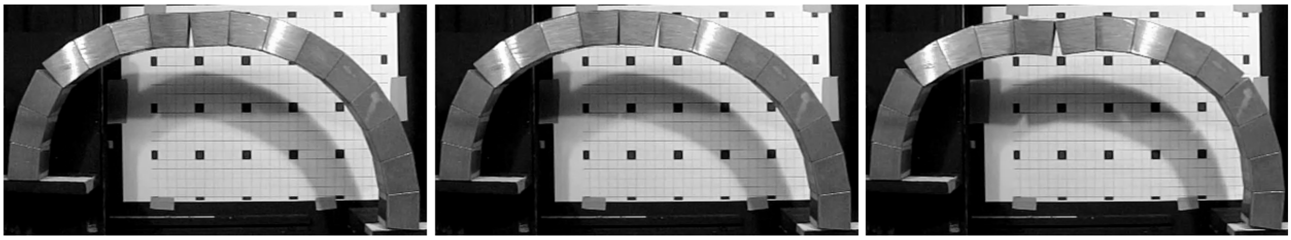

Figure 5: Hinges forming in a 16 block arch. Deformation history: opening of 10mm, followed by a vertical displacement up to collapse. (a) hinges form in -4+9+17;
(b) just after first image. hinge +9 closes and is replaced by hinge +8;
(c) the arch does not resist the impulsion following hinge transition, a new hinge is formed in -13 and the arch collapses

Conclusions

Taking into account large displacements complicates very much the assessment of the stability of masonry arches. It is nevertheless possible to investigate the effects of such displacements using limit analysis techniques. Experiments are in good qualitative agreement with the mathematical model presented but from a quantitative point of view, significant spread were observed in the results. This is hardly a surprise. It was shown elsewhere (Smars 2000, Schueremans 2001a, 2001b) that a limit analysis approach can also be used to deal with such variations, from a good understanding of their cause. The results of the experimental program will no doubt bring new insights to the understanding of the deformation process.

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