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A Discrete Element for Modeling Masonry Vaults

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Abstract The assessment of the seismic response of historical masonry buildings represents a subject of considerable importance but, at the same time, of very difficult task. Refined finite element numerical models, able to predict the non-linear dynamic mechanical behavior and the degradation of the masonry media, require sophisticated constitutive law and a huge computational cost that makes these methods nowadays not suitable for practical application. In the past many authors developed simplified or alternative methodologies that, with a reduced computational effort, should be able to provide numerical results that can be considered sufficiently accurate for engineering practice purposes. However most of these methods are based on simplified hypotheses that make these approaches inappropriate for monumental buildings. In this paper a three dimensional discrete element model, able to predict the nonlinear behaviour of masonry shell elements, is presented as an extension of a previously introduced spatial discrete-element conceived for the simulation of both the in-plane and the out-of-plane behavior of masonry plane elements. The new macro-element enriches a larger computational framework, based on macro-element approach, devoted to the numerical simulation of the seismic behaviour of historical masonry structures.

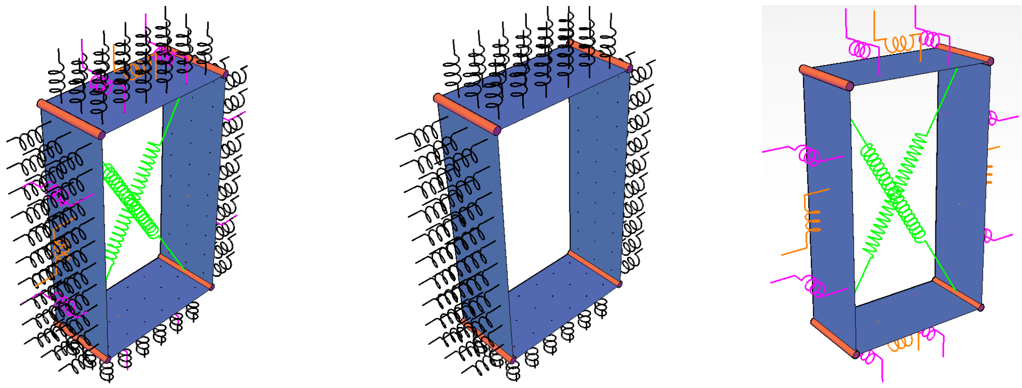
Keywords: Historical masonry building, nonlinear numerical method, macro-element, discrete element method, masonry vaults

Introduction

The problem of the evaluation of the seismic vulnerability of an existing historical masonry building represents a subject of high practical relevance but, at the same time, of very difficult task. In order to estimate the seismic vulnerability of an existing historical building and to ascertain if a structure requires a seismic upgrade, a structural engineer needs simple and efficient numerical tools whose complexity and computational demand must be appropriate for the practical engineering purposes. However, the simulation of the nonlinear dynamic behaviour of a monumental masonry building still represents a very challenging problem which rigorously requires the use of computationally expensive nonlinear finite element models and, above all, very expertise judgments. For this reason, in recent years, many authors developed simplified methodologies that, with a reduced computational effort with respect to a nonlinear finite element simulation, should be able to simulate the nonlinear seismic behaviour of masonry buildings and to provide numerical results that can be considered sufficiently accurate for engineering practice purposes. However most of these methods are based on simplified hypotheses that in many case are not suitable for modeling the seismic behaviour of historical buildings. In this paper a three-dimensional discrete-element, derived by an upgrade of plane and spatial macro-elements previously introduced by the same authors (Caliò and Pantò 2005, Caliò and Pantò 2008), is presented. The basic element, developed for the simulation of the in-plane response is constituted by an articulated quadrilateral with four rigid edges and four hinged vertices connected by two diagonal nonlinear springs; each of the rigid edges can be connected to other elements by means of discrete distributions of nonlinear springs with limited tension strength. This plane discrete element has been applied for the simulation of the nonlinear behaviour of masonry buildings in which the masonry walls are subjected to in plane forces without taking into account the out-of-plane response of masonry walls that is ignored. In order to overcome this important restriction, the plane macro-element has been upgraded by introducing a third dimension and the needed additional degrees-of-freedom with the aim to describe the out-of-plane kinematics of masonry walls (Caliò and Pantò 2008). Nevertheless, in order to simulate the seismic response o monumental masonry buildings, in many cases it is also necessary to model the nonlinear behaviour of structural elements with a curved geometry, such as arches, vaults, domes whose role is fundamental both in the local and global behavior of the buildings. In this paper starting from the three dimensional macro-element proposed in (Caliò and Pantò 2008) a new discrete element able to describe the nonlinear behaviour of shell masonry structures is introduced. The proposed three-dimensional macro-element will enrich a larger computational framework, based on the macro-element approach, conceived for the evaluation of the seismic resistance of historical masonry structures both in static and dynamic context.

The Proposed Model

The proposed macro-element is an extension, to the case of shell structures, of a three-dimensional rectangular macro-element proposed for the simulation of the seismic response of masonry buildings (Caliò and Pantò 2005, Caliò and Pantò 2008), Fig. 1. This rectangular element has been conceived in order to simulate the mechanical behaviour of masonry walls when subjected both to in-plane and out-of-plane loading. It is represented by a simple mechanical scheme consisting in a rectangular quadrilateral with rigid edges and hinged vertices connected by diagonal springs which simulate the in-plane shear deformability of the corresponding represented masonry macro-portion. The quadrilateral can interact with other elements along each rigid layer edge by means of a discrete distribution of nonlinear springs with limited tension strength. Each interface includes springs orthogonal to the rigid layer, which govern the in plane and out of plane flexural behaviour of the element, and springs parallel to the layer edges able to simulate the in-plane and out of plane sliding and torsional behaviour. The kinematics of the element is governed by 7 degrees-of-freedom able to describe both the rigid body motions and the shear deformability of the base element.



(a) *(b)* *(c)*

Figure 1: (a) the rectangular macro-element 'rect'. (b) the rect with the representation of the orthogonal NLinks for the simulation flexural behaviour; (c) the rect with the representation of the transversal and diagonal NLinks for the simulation of shear and torsional behaviour

The discrete element conceived to model shell masonry elements represents an extension of the rectangular element and is constituted by an articulated quadrilateral whose geometry is not regular in order to allow the meshing of a generic curved surface with macro-element. Each macro-element is still characterised by four rigid layer edges whose orientation and dimension is associated to the shape of the element and to the thickness of the portion of modelled masonry that is represented. In Fig. 2, it is shown how a portion of a dome can be meshed by means of the proposed macro-element approach. A diagonal spring simulates the shear deformation of the quadrilateral in its own medium plane, while spatial interfaces (Fig. 3) govern the interaction with the adjacent elements or the external supports. These interfaces are in general skew relative to the medium plane of the element, and their movements are ruled by a discrete number of non linear springs. Each quadrilateral is defined by the geometric coordinates of his vertices, the four normal vectors to the surface and the thickness in these points (Fig. 4).

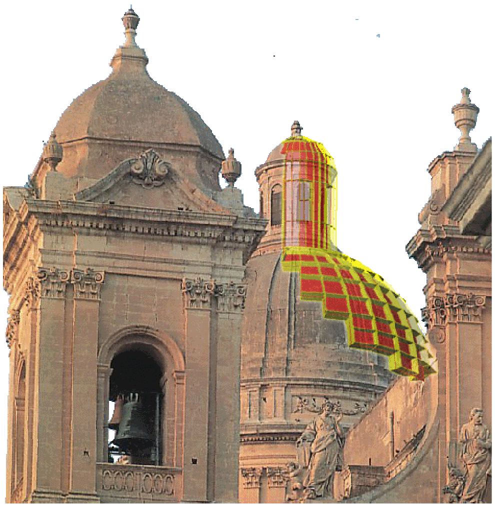


Figure 2: Macro-element meshing of a dome

If the geometric data and the mechanical characteristics of the masonry media are known the properties of the equivalent macro-element, corresponding to the part of the masonry portion modelled, can be established through a fibre calibration procedure summarised in the following paragraphs.

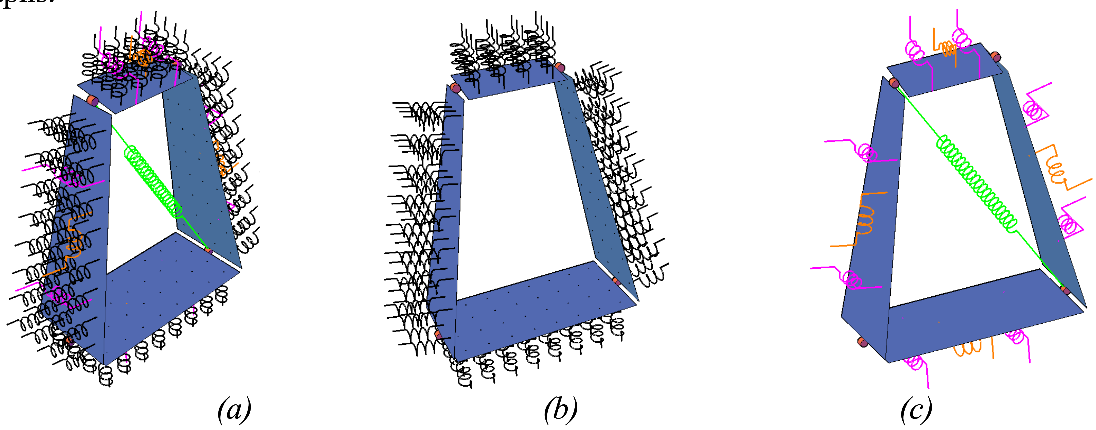


Figure 3: (a) the quadrilater macro-element 'quad'. (b) the quad with the representation of the orthogonal NLinks for the simulation of membrane and flexural behaviour; (c) the quad with the representation of the transversal and diagonal NLinks for the simulation of shear and torsional behaviour

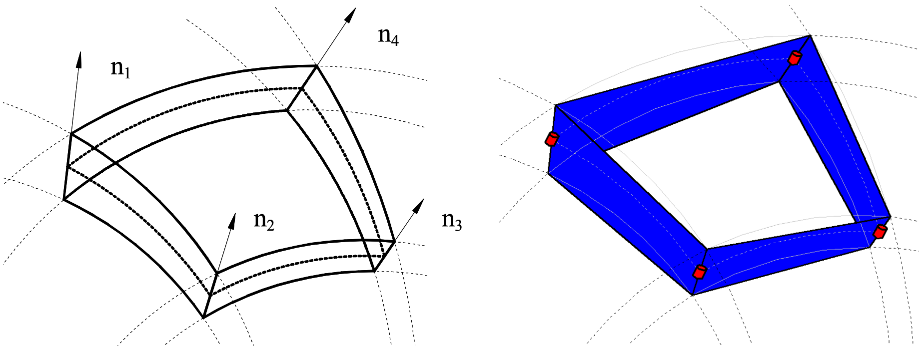


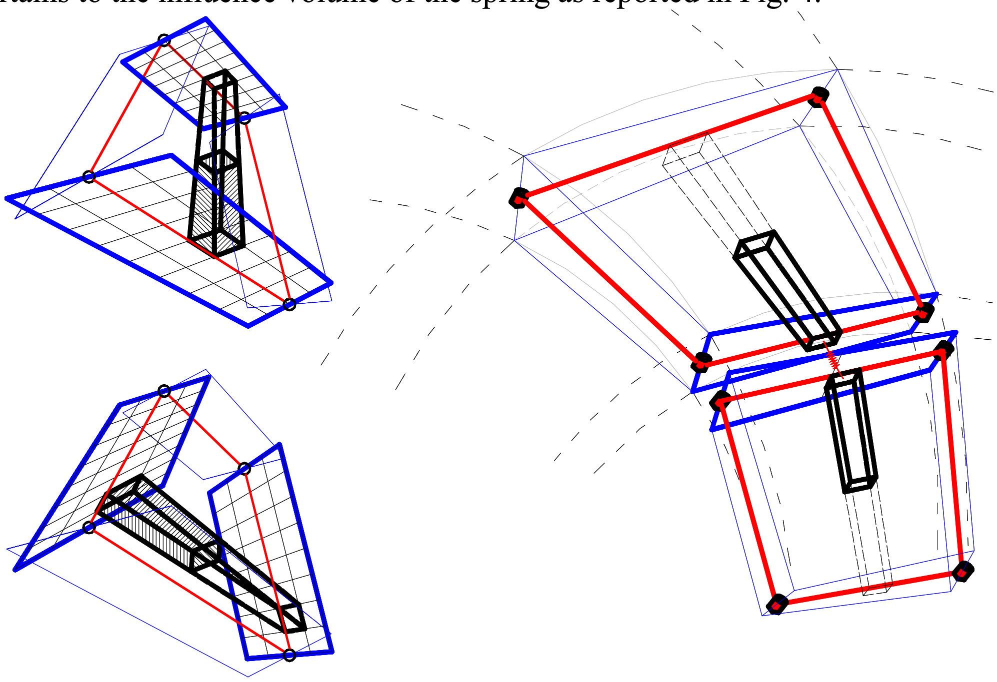
Figure 4: Masonry portion and its equivalent mechanical model without the NLink representations

Calibration of the Nonlinear Links

The calibration procedure summarised in the following is based on the assumption that the behaviour of a continuously curved surface can be adequately represented by flat elements, under this assumption bending and in-plane forces cause independent deformations. Rigorously, in the division of an arbitrary shell into flat elements triangular elements should be used, however shells with general cylindrical shapes can be well represented by quadrilateral flat elements. For the sake of conciseness, here only the quadrilateral shape element is described. The element must be representative of the corresponding finite portion of the shell cut out by plane sections which are located to the edges of the irregular quadrilateral and whose orientations and thicknesses are associated to the actual represented shell (Fig. 4).

The membrane and the bending behaviour of the element is governed by the nonlinear links orthogonal to the rigid layer edges (Fig. 3b) while three additional transversal springs controls both the in-plane and out-of plane sliding shear and the twisting of two adjacent layers (Fig. 3c). Finally a single diagonal nonlinear spring governs the in-plane shear nonlinear behaviour of the quadrilateral element (Fig. 3c).

The orthogonal NLinks that govern the membrane and flexural response are calibrated by means of a fibre modelling approach. Namely each spring inherits the elasto-plastic behaviour of the masonry fibre that pertains to the influence volume of the spring as reported in Fig. 4.



(a) *(b)*

Figure 5: (a) fiber semi-volumes and corresponding nonlinear springs. (b) NLink as a results of two nonlinear springs in series of two adjacent elements

The calibration procedures of each nonlinear link are based on the mechanical characterization of masonry and the geometric properties of the elements. Masonry, considered as a continuous homogeneous solid, will be modelled considering different constitutive laws for each fundamental behaviour: membrane, bending, shear, sliding, and torque. From a mechanical point of view the interfaces govern membrane, bending, sliding and torsional behaviour while the in-plane shear deformability of the element is controlled by the internal diagonal spring.

Orthogonal springs

The discrete model has been thought of as a fibre model, and each element of the mechanical scheme simulates the behavior of a corresponding fibre. Each orthogonal spring is calibrated in order to be equivalent to the corresponding portion of masonry that represents, Fig. 5. The calibration procedure consists of two phases. In the first one the mechanical properties of two springs, which correspond to the fibers of each of the two elements connected by the interface, are calculated, Fig 5a. In the second phase the two springs in series will be condensed in a single equivalent nonlinear spring, Fig 5b. Each of the two springs can be calibrated assuming an equivalence between the discrete model subject to an axial load along the fiber axis and a homogeneous elasto-plastic beam with a variable section.

Sliding springs

The transversal springs govern the in-plane and the out-of-plane sliding of the shell element according to a Mohr-Coulomb law. The calibration of the springs is performed on the basis of the mechanical properties of masonry, according to the corresponding influence area. The in-plane transversal spring is rigid-plastic and governs the in-plane sliding of the element while the other two parallel transversal springs are elasto-plastic and control the out of plane sliding and the torsional behaviour between two adjacent elements. The elastic stiffness of these springs is associated the shear stiffness of the continuous element while the distance between the two springs is evaluated by imposing the equivalence, in terms of torque, with the corresponding continuous model.

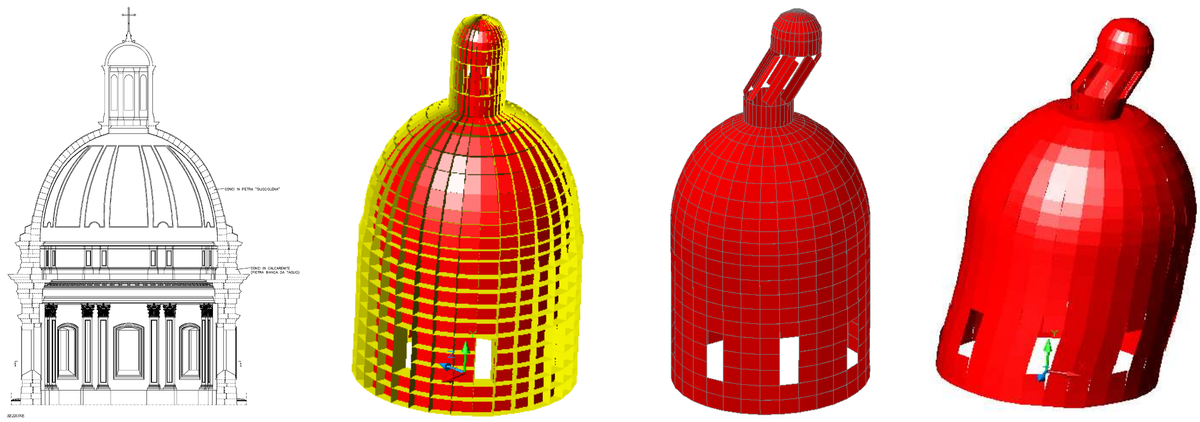
Diagonal spring

In the elastic range the diagonal shear spring is calibrated by imposing an energy equivalence with a continuous reference elastic model. The yielding forces are associated to the reaching of the limits of tensile or compressive stresses in the reference model.

Further details on the calibration approach can be found in (Cannizzaro 2010).

Numerical Applications

The application reported in the following is relative to the dome of the Cathedral of Noto in the eastern part of Sicily. The Cathedral was built after the strong earthquake which hit the eastern Sicily in 1693, its plan is a three nave basilica structure with a dome resting on a tall drum supported by four large arches at the centre of the transept (Tringali, Benedictis and La Rosa, et al 2003). On March 13 1996 the central part of the right lateral nave collapsed together with a large part of the dome. The application here reported is relative to a push-over analysis of the original dome subjected to mass-proportional horizontal loading. The mechanical characteristics of the dome, reported in table 1, and its geometry have been derived by the referenced papers (Binda, Baronio and Tedeschi, et al 2003, Binda, L, Saisi, Tiraboschi and Valle, et al 2003, Tringali, Benedictis and La Rosa, et al 2003).



(a) (b) (c) (d)

Figure 6: (a) Section of the dome (figure from reference [9]); (b) The macro-element discretization; (c) the deformed configuration in the condition of incipient collapse considering an unreinforced masonry lantern; (d) the deformed configuration in the condition of incipient collapse considering a linear elastic behaviour of the lantern

The dome has been modeled by means of 824 macro-elements 792 of which are quadrilateral and 32 are triangular. The latter are necessary for the modeling of the upper part of the lantern. The whole model is described by 5736 degrees of freedom considering that the dome has been assumed fixed at the base of the drum and the triangular elements are characterized by six degrees of freedom only.

Table 1. Mechanical characteristics of the dome derived from [1]

E Young modulus [kN/cm^2]	G Shear modulus [kN/cm^2]	σ_t Tensile strength [kN/cm^2]	σ_c Compressive strength [kN/cm^2]	w Unit weight [kN/m^3]
90	37.5	0.005	0.0716	17

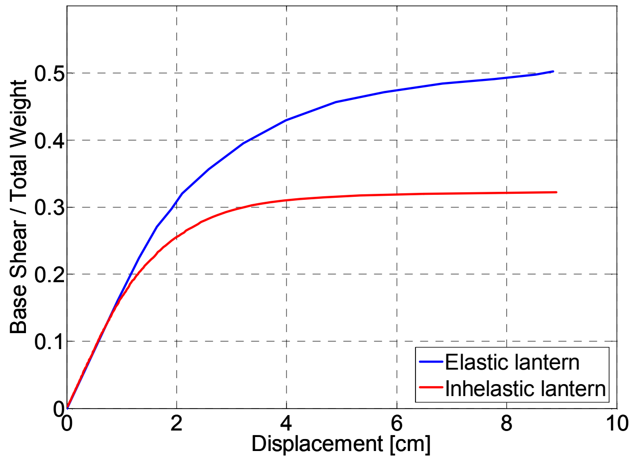


Figure 7: Push-over analysis curve

Two different models have been analyzed, in the first the whole dome is characterized by an unreinforced masonry structure, in the second model it has been assumed that the lantern has been reinforced by steel elements in the columns in order to avoid its premature collapse. In the first model the collapse is concentrate at the lantern level while the other parts of the dome still exhibit a quasi-elastic behaviour, in the second model the collapse in characterized by the cracking of the dome in the meridian directions and the shear damage of the piers in the drum (Fig. 6). Fig. 7 reports the push-over curve of the dome expressed as the base shear normalized with respect to the total weight (10886 kN) of the dome as a function of a target point corresponding to the top of the dome for both the considered models.

References

[1] Binda, L, Baronio, G, Tedeschi, C, and Tiraboschi, C (2003). “Characterisation of the materials used in the construction of the Noto Cathedral.” *Construction and Building Materials*, 17, 557-571.

[2] Binda, L, Saisi, A, Tiraboschi, C, Valle, S, Colla, C,and Forde, M (2003) “Application of sonic and radar tests on the piers and walls of the Cathedral of Noto.” *Construction and Building Materials* 17, 613-627.

[3] Caliò, I, Marletta, M, and Pantò, B (2005). “A simplified model for the evaluation of the seismic behaviour of masonry buildings,” in *Proc. of Tenth International Conference on Civil, Structural and Environmental Engineering Computing*, August 30 – September 2, Rome, Italy.

[4] Caliò, I, Marletta, M, and Pantò, B (2008). “A discrete element approach for the evaluation of the seismic response of masonry buildings,” in *Proc. of 14th World Conference of Earthquake,* October 12 -17, Beijing, China.

[5] Cannizzaro, F (2010). “*The seismic behavior of historical buildings: a macro-element approach*.” Univ. of Catania, PhD Thesis in Structural Engineering – Cicle XXIII.

[6] Tringali, S, De Benedictis, R, La Rosa, R, Russo, C, Bramante, A, Gavarini, C,Valente, G, Ceradini, V, Tocci, C, Tobriner, S, Maugeri, M, Binda, L, and Baronio, G (2003). “The Reconstruction of the Cathedral of Noto.” *Construction and Building Materials*, 17, 573-578.