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Small-Scale Models for Testing Masonry Structures

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Abstract Experiments may be used to verify numerical and analytical results, but large-scale model testing is associated with high costs and lengthy set-up times. In contrast, small-scale model testing is inexpensive, non-invasive, and easy to replicate over several trials. This paper proposes a new method of masonry model generation using three-dimensional printing technology. Small-scale models are created as an assemblage of individual blocks representing the original structure's geometry and stereotomy. Two model domes are tested to collapse due to outward support displacements, and experimental data from these tests is compared with analytical predictions. Results of these experiments provide a strong understanding of the mechanics of actual masonry structures and can be used to demonstrate the structural capacity of masonry structures with extensive cracking. Challenges for this work, such as imperfections in the model geometry and construction problems, are also addressed. This experimental method can provide a low-cost alternative for the collapse analysis of complex masonry structures, the safety of which depends primarily on stability rather than material strength.

Keywords: Small-scale testing, unreinforced masonry, limit analysis, historic structures, 3D printing

Introduction

Interest in the safety of historic monuments has created a need for accurate, low-cost methods for determining the collapse limits of unreinforced masonry structures. Significant work has been done to understand and assess the failure modes of individual structural elements such as arches, vaults and buttresses. However, to further understand three-dimensional collapse mechanisms in masonry, new approaches are necessary. Computational methods have made significant advances for the analysis of historic masonry structures, yet they require significant assumptions about material properties and other modeling parameters. Ideally, numerical methods can be combined with empirical methods to provide a more robust analysis. In this paper, a new method of small-scale structural model generation using rapid prototyping technology is proposed.

Small-scale model testing is a cost-effective method for assessing safety compared to the invasive testing of monuments or full scale experimental testing. In particular, the use of scale models is a valuable method for determining collapse conditions in historic masonry structures, which are usually governed by stability rather than material strength (Heyman 1995; Huerta 2006). Furthermore, physical models provide an observation of collapse modes that may not have been detected with analytical or numerical models, such as combined hinging and sliding. Finally, physical experiments provide a benchmark against which future numerical modeling techniques can be compared.

Medieval designers used scale models to help them solve problems of construction as well as to prove the integrity of their designs. Danyzy's experiments with plaster models in 1732 show examples of collapse mechanisms for masonry arches and buttresses (Frézier 1737). Bland (1862) presents a series of experiments using weights to load model arches built of wooden voussoirs and piers made of wooden bricks. Vicat (1832) conducted scale model experiments to investigate the use of masonry piers as the support system for suspension bridges. More recently, Boothby (2001) summarized work combining experimental and analytical methods to determine the collapse conditions for model masonry arches. Orduña and Lourenço (2003) use experimental results from Royles and Hendry (1991) on scaled models of masonry arch bridges and from Oliveira (2000) on shear walls to validate a limit analysis method for assessing unreinforced masonry structures.

Ochsendorf (2002; 2004) used small-scale models to determine the overturning capacity of buttresses and the displacement capacity of arches. More recently, DeJong et al. (2008) analyzed the collapse modes of masonry arches under base acceleration and compared the results with model experiments. These studies demonstrate the benefits of comparing the collapse limits of physical models with analytical and numerical methods.

Although the use of small-scale models in structural analysis of masonry structures has shown some potential, a more efficient method of generating accurate small-scale models would facilitate their use. Three-dimensional printing (3DP) of blocks can provide such a method. A Z-Corporation ZPrinter 310 Plus is used to produce small-scale block models for determining the collapse limits of masonry structures (Z-Corp. 2010). The printer extracts detailed information from a CAD file of the model to create each block. This is a relatively new technology which provides accuracy and flexibility in generating models at low costs (Dimitrov et al. 2006). Despite the myriad applications of 3D printed models, they have not been used for testing and analysis of masonry structures.

Methodology

The first step in creating a structural model using 3DP technology is to consider the desired stereotomy of the model. A primary difference between the conventional use of 3DP technology and its use as a structural tool is that instead of creating one complete part, numerous blocks are created with the intention of assembling them as a whole. It is important to consider the phenomenon to be measured, as well as the block pattern and size of the original structure. The location of interfaces between blocks can greatly affect the behavior of the model, making this a crucial step in the process of creating a structural model. Though it would be possible to print a monolithic three-dimensional structure like a masonry vault in a single piece, similar to the continuous photo-elastic models produced by Rauch and Mark (1967), such models are generally not useful for determining the safety limits of historic structures. For the present study, the authors chose to divide the masonry structure into a series of individual blocks, which represent the coursing of the masonry in the structure of interest. It is not necessary for each individual brick or stone to be represented by a 3D printed block. For example, a brick vault can be modeled as a series of larger blocks, where each printed block can represent a conglomerate of 10-20 bricks or more.

Both cost and size considerations are also important in creating the model. The time required and the cost are functions of the volume to be printed. The standard Z-Corporation material costs approximately $0.13 per cubic centimeter. This means that any change in scale will result in a cubic change in both time and cost, and it is therefore cost-effective to create models which are less than one meter in total size. The choice of model material will also impact the final cost of the model as well as the desired material properties of the model. In the case of equilibrium analysis using only self-weight, the coefficient of friction is the only relevant material property, as the internal stresses in the masonry are orders of magnitude below the crushing stress of the material.

Once the model block geometry (or stereotomy in the case of a stone vault), scale, and material have been selected, the printing process can begin. First, CAD files are created which contain the individual blocks arranged to fit within the dimensions of the printer bed. Depending on the size of the model, it may be necessary to print the model in multiple batches. The files are then sent to the printer, providing the necessary information for printing. After the printing process is complete, the pieces must be excavated from the printer. Depending on the material, different post-processing steps are necessary. All models for the present study were printed with Z-Corporation material, excavated, baked for one hour at 200 °C, and then coated with two coats of polyurethane to provide the blocks with sufficient friction and to minimize damage to the block corners during testing.

Two small-scale model domes were created from individual printed blocks with varying thickness to radius ratios (Fig. 1). Dome 1 has a thickness of 17.3 mm, which is 10% of its centerline radius (17.3 cm) and is composed of 145 blocks. Dome 2 has a thickness of 32.8 mm, which is 20% of its centerline radius (16.6 cm) and is composed of 137 blocks. Each dome is built with seven rows of 12° blocks and a central cap of 12°. Each dome was printed in the span of approximately one week, and required seven print batches.

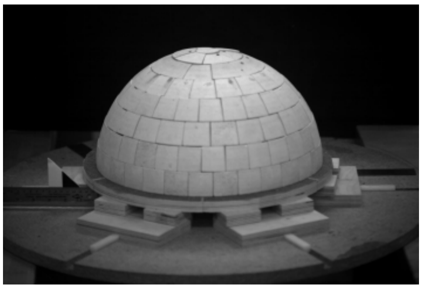


Figure 1: (a) Dome 1

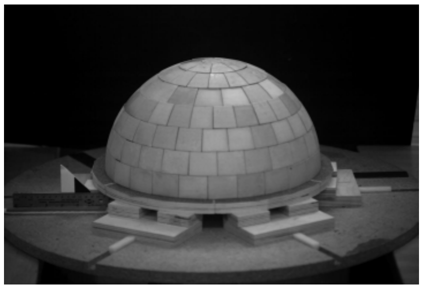


Figure 1: (b) Dome 2

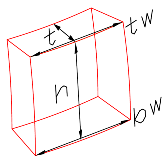
Figure 2: (a) Dome 1 blocks

Figure 2: (b) Dome 2 blocks

Different printers and materials produce varying degrees of geometrical accuracy. Additionally, there is some error associated with the coating process, as it was done manually, which creates a risk of inconsistency in the thickness of the coating (Fig. 2). This variation in thickness combined with the overall increase in block dimensions from coating can create tolerance issues when building the complete model. To determine the variation in the geometry of the printed blocks, measurements of each block's height [h], thickness [t], bottom width [bw], and top width [tw] were recorded. Table 1 quantifies the average percent error between the computer prescribed block dimensions and the actual printed block dimensions after the coating process. The table confirms that the margin of error caused by the printing and coating process is less than 4% on average, and is typically on the order of 0.5 mm. More accurate printing is possible, but this accuracy is sufficient for the current study.

Table 1: Average Percent Error in 3D Printed Blocks

Dome	*Total No. of Blocks*	*Average % Error*			
		Height, h	*Thickness, t*	*Bottom Width, bw*	*Top Width, tw*
1	145	1.7	3.5	0.69	2.3
2	137	1.7	1.9	0.89	1.5



Once a scale block model is completed, three primary types of testing can be applied to induce collapse: 1) load testing; 2) support displacements; and 3) base accelerations. While our research group has applied all three types of testing to scale models in recent years, this paper focuses solely on applied support displacements. In the case of arches, vaults and domes, differential settlements and leaning supports can lead to significant support displacements, which can eventually cause collapse. For historic masonry vaults and domes where the self-weight is the dominant loading, support displacements over time represent a substantial threat to stability (Ochsendorf 2006).

An experimental apparatus with spreading supports was constructed to investigate the increase in dome span before collapse. Six pie-shaped wooden wedges were designed to move outward symmetrically, simulating the effect of leaning buttresses. To measure the outward spread of the supports, a metric ruler was attached horizontally on the wooden base. Furthermore, to prevent sliding of the base blocks on the supports, the friction on the top face of each support was increased with the use of sandpaper glued to the wooden surface. Numerous friction tests were completed and showed that the coefficient of friction for the coated blocks had an average value of 0.7, which is typical for stone according to Rankine (1858).

Each dome was manually constructed for all tests. Additionally, each test was filmed using a high-speed digital video camera, recording up to 100 frames per second, permitting careful observation of the collapse mechanism. The moment of collapse was determined using video footage to identify the outward rotation of the base blocks about an extrados hinge. To protect the blocks from damage during collapse, a protective layer was placed on the ground under the dome.

Depending on the scale of the model, small imperfections in the dimensions of individual blocks can create a lack of connectivity between the blocks and can prevent the structure from fully developing the expected compressive force paths. In addition, the edges of the individual blocks were slightly rounded due to repeated testing. This corner rounding can affect the model by reducing the effective thickness of the structure being tested. A previous investigation by DeJong (2009) using model arches made of autoclaved aerated concrete blocks demonstrated this phenomenon. Naturally, there is a trade-off between choosing a larger scale model that would not be as sensitive to block imperfections, but then the ease of using such models is compromised. Moreover, because historical masonry domes have imperfect geometries, the experiments are broadly representative of the behavior of actual masonry structures.

Results

Zessin et al. (2010) conducted experiments to investigate the collapse of hemispherical domes on supports spreading radially outward using the small-scale 3DP models described above (Fig. 5). Each dome was tested to collapse eleven times, revealing both the failure mechanism of the dome as well as the amount of span increase that the dome can accommodate before failure. These experiments served to confirm simplified two-dimensional analyses, as well as to highlight more complex three-dimensional phenomena that cannot be captured with simplified analytical methods.

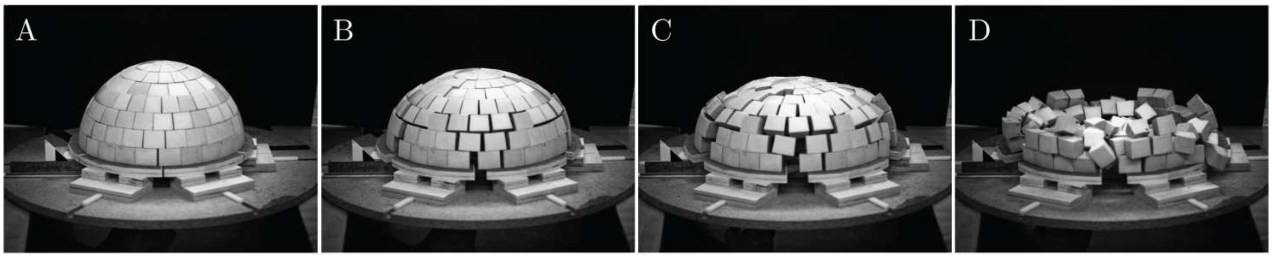


Figure 3: Still images taken during collapse of Dome 2 with high-speed video

In the experiment, the supports were slowly spread outwards, initiating a series of radial cracks in the hemispherical dome (Fig. 3A). Hinge rings formed in the horizontal courses of the dome and the crown of the dome descended to accommodate the span increase (Figs. 3B-C). The collapse mechanism consisted of an extrados hinging ring around the central cap, with an intrados hinging ring above the second course of blocks. Failure occurred when an additional extrados hinge formed at the base of the dome, leading to an outward rotation of the base blocks (Fig. 3D). Some sliding between blocks was observed in order to accommodate these extreme support movements, however, the dominant collapse mode was always observed to be the formation of a symmetrical six-hinge mechanism, confirming two-dimensional theoretical predictions by Zessin et. al. (2010) (Fig. 4).

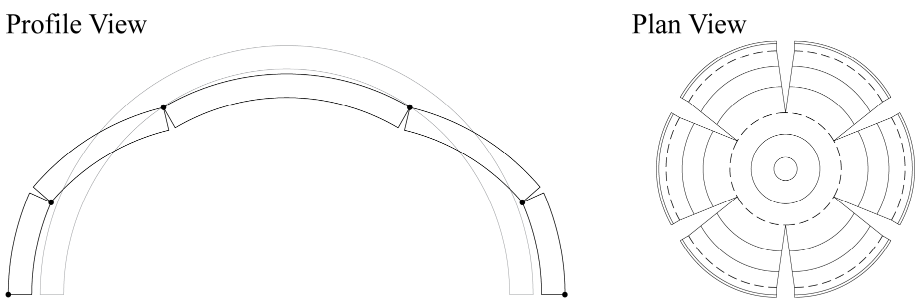


Figure 4: Symmetrical six-hinge collapse mechanism of dome on spreading supports

The theory predicted larger support movements than those measured experimentally (Table 2). While Dome 2 failed earlier than predicted, the average span increase to failure was within 1% of the predicted value. Dome 1 failed significantly earlier than expected, with the average span increase to failure occurring at just over 70% of the predicted value. For both domes the quality of construction varied between tests and was difficult to quantify; however the imperfection of construction appeared to be more pronounced in Dome 1. This discrepancy is attributed to the 17.3 mm thickness of the dome, in which small imperfections (on the order of 0.5 mm) in the dimensions of individual blocks led to a significant decrease in the effective thickness of the dome. This reduction in thickness caused collapse to occur earlier than predicted by analysis.

Table 2: Comparison of Results for the Dome on Spreading Supports

Dome	*% Span Increase to Failure*	
	Analysis	*Experiment*
1	14.3	10.4 ± 0.01
2	32.1	31.9 ± 0.02

For example, consider a hemispherical dome of 5 m median radius with a constant thickness of 500 mm (thickness ratio of 10%). It would not be uncommon for the supporting piers of such a dome to move 100 mm due to foundation movements or pier deflections over time. This movement results in a total span increase of 200 mm over the interior span of 9.5 m, corresponding to a 2% span increase. To accommodate this increase, radial cracking would be expected to occur. These experiments demonstrate that despite such cracks, the dome would exist in a stable equilibrium. A 2% span increase is still well within the 14.3% theoretical spreading limit. The supporting walls would need to deform substantially more to cause failure. Such small movements are not cause for concern.

Conclusion

The results of the model dome experiments confirmed the accuracy of the analytical collapse predictions. The models provided inexpensive and invaluable observations of the 3D collapse mechanisms of masonry domes, which serve as a valuable supplement to analytical predictions. The efficiency of the testing procedure relative to traditional methods of model generation also allows for repeated experiments in a short period of time. These repeated experiments were possible due to the durability of the individual blocks to withstand multiple collapses. All of these factors combined with the relative low cost make the use of small-scale models a viable option for use in analyzing historic masonry structures.

Because small-scale 3D-printed models are relatively new for structural testing, some difficulties arose during the experiments. The lack of a reliable construction method for the domes created some minor geometric inaccuracies, but building the models manually still resulted in an acceptable quality dome for use in the experiments. Additionally, the small imperfections from the printing and coating process along with minor damage to the blocks from repeated testing created tolerance issues. A more precise method for coating the blocks could resolve this issue. Furthermore, limits could be placed on tolerance to ensure that imperfect or damaged blocks are replaced. Despite these difficulties, the advantages of this method merit further development and validation.

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