

Non Linear Response of Masonry Wall Structures Subjected to Cyclic and Dynamic Loading

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Abstract A method for non-linear dynamic analysis of wall masonry structures is presented. The method takes advantage of a Generalized Matrix Formulation (GMF) for the serviceability and ultimate analysis of structures composed of arches and/or masonry walls, in which open and solid walls are described as equivalent frame systems. This formulation has been complemented with a cyclic constitutive model and an algorithm for the integration of the equation of motion, resulting in a numerically efficient method for non-linear analysis in time domain of complex masonry systems.

Keywords: Shear masonry wall, seismic analysis, non-linear time domain analysis, extended frame formulation, flexibility formulation

Introduction

Most of the strategies normally used to characterize the seismic capacity of masonry structures (such as conventional pushover analysis) are based on the application of a set of static equivalent forces in the frame of an instantaneous analysis. These approaches neglect significant aspects of the real seismic response, as in particular the influence of the cyclic earthquake loading effects on the amount of damage experienced by the structure. However, a more accurate non-linear dynamic analysis in the time domain, as a way to account for damage under cyclic effects, is often limited or even prevented by its high computational costs.

A new application for the analysis of masonry systems consisting of arches and/or walls is here presented with the purpose of offering an efficient and yet accurate approach to non-linear analysis in the time domain. The method (denoted Generalized Matrix Formulation, GMF), based on a generalization of conventional matrix methods for the analysis of spatial framed structures, is characterized by its large numerical efficiency and versatility in the modelling of a wide typology of masonry structures. Applications of this formulation to static analysis of masonry structures have been already presented by Molins and Roca (1998) and Roca et al. (2005). Although the formulation was firstly intended for skeletal masonry structures, it was later adapted to the description of load bearing wall systems by means of in-built techniques for the treatment of 2D members as equivalent frame systems.

The extension to dynamic analysis in the time domain here presented includes two novel features, namely (1) a constitutive equation to model the cyclic response of masonry structures, including the case of large amplitude oscillations, and (2) a technique for the integration in the time domain of the motion equations in the frame of the GMF approach. The first aspect (1) has been solved through the formulation of a constitutive equation which takes into account the effect of repeated and large amplitude cycles in compression, tension and mixed stress paths. The constitutive equation has been adjusted by comparison with cyclic loading experiments carried out on un-reinforced concrete and masonry specimens by other authors. The second aspect (2) is satisfied by a Generalized- α algorithm combined with the Newton-Raphson's integration scheme.

The ability of the method to efficiently characterize the dynamic response of masonry structures in the time-domain is illustrated by its application to the analysis of a masonry scale building tested on a shaking table by Tomažević and Weiss (1994).

Generalized Matrix Formulation for Frame and Wall Structures

The Generalized Matrix Formulation was first derived by Marí (1985) for the analysis of reinforced and prestressed concrete frame structures with curved and variable cross-section members. It consists of a hybrid formulation in which the internal forces are described by means of an equation directly derived from equilibrium considerations. Due to it, the formulation is in principle exact, regardless of the number of elements used to model the entire structure, and its accuracy is only dependent on the integration rules used across the element sections and length. A complex structure may be modelled by describing each actual linear member (beam, arch or pillar) with a single two-node GMF element, thus resulting in large numerical efficiency.

Starting from the basic formulation, a set of extensions was implemented to allow static and dynamic analysis of masonry structures composed of arches and walls. Firstly, convenient models were adopted to describe the response of unreinforced masonry (Molins and Roca, 1998). Secondly, a consistent mass matrix (together with other necessary improvements) was derived to allow vibration modal analysis (Molins et al. 1998) taking into account the exact distribution of mass and stiffness along the structural members. Thirdly, an approach was implemented to describe solid or open walls (i.e., walls connected by lintels or arches) as equivalent systems of linear members. For that purpose, a set of special devices was included to represent the shear deformation of the walls in an accurate way.

In its initial configuration, a constitutive model including an elasto-plastic equation for masonry in compression and shear and an elasto-fragile one in tension was implemented. In spite of the simplicity of this approach, the formulation showed significant capacity to localize damage. The method afforded the simulation of unstable mechanisms, as those predicted by limit analysis, thanks to the fact that sections experiencing damage behaved almost as true hinges due to the high concentration of flexibility and curvature (Molins and Roca, 1998).

Fig. 1 provides a set of applications illustrating the ability of GMF to model a wide set of structural types, including skeletal curved structures (Fig. 1,a,b), masonry bridges (Fig. 1,c) and wall systems (Fig. 1,d). The blank region in Fig. 1a represents cracking in tension under gravity forces. Fig. 1c shows the shapes obtained for the first two vibration modes of an ancient masonry multi-arch bridge.

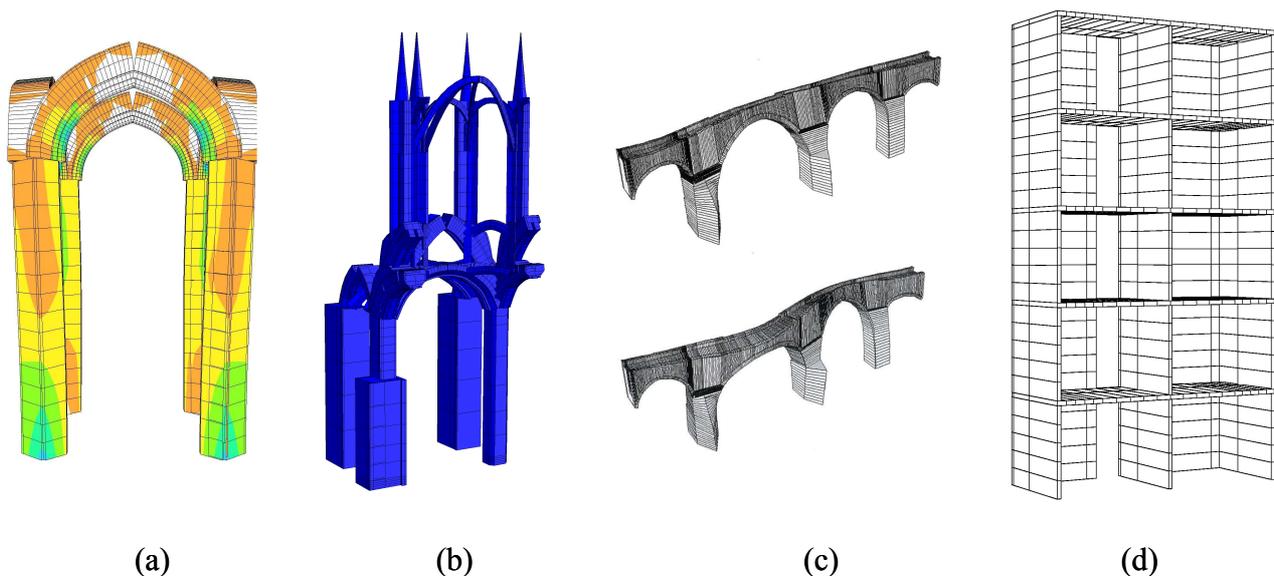


Figure 1: Examples of structures modelled using GMF. From left to right: Arch system; cimbório structure; masonry bridge (mode analysis); masonry wall system

Proposed Cyclic Constitutive Model

A first requirement of any approach oriented to non-linear dynamic analysis is found in a constitutive equation taking realistically into account the cyclic response of the material for large amplitude oscillations. In turn, this requires the modelling of aspects such as (1) the energy consumption through the hysteretic loops, (2) the accumulation of irreversible deformation along the load cycles and (3) the reduction of the stiffness along the unloading and re-loading branches. Moreover, these aspects are to be considered for load cycles occurring in compression, in tension or in a combined way.

For that purpose, a uniaxial cyclic model firstly derived by Sima et al. (2008) for concrete structures has been adopted and modified to allow its application to masonry structures (Fig. 2). The model is formulated in terms of compression and tension damage parameters. The rest of the parameters determining the cyclic response (such as the stiffness of the un-loading and the re-loading branches and the strain-plastic strain ratio) are defined as a function of the basic damage ones.

The constitutive model has been adapted by fitting the necessary material parameters through linear regression. Different available experimental results (in particular, Naraine and Sinha's, 1989) have been considered to carry out this adjustment.

The non linear shear behaviour is modelled by considering that the different portions of the section subjected to tension collaborate to resist the shear stresses depending on the level of tensile damage accumulated; tensile damage equal to 0 (intact material) means that the portion collaborates in the shear response, while tensile damage equal 1 (full damage) means that this portion doesn't collaborate in the shear response) with a linear interpolation between the two extreme values

Strategy for Integration in Time Domain

Choosing an adequate integration scheme becomes crucial in the analysis of complex structures. Desirable characteristics, referred to by Hilber and Hughes (1978), include unconditional stability, accuracy and the introduction of algorithmic damping. The latter allows controllable numerical dissipation over the higher frequency modes, which, in turn, is needed to avoid spurious oscillations due to excitation of spatially unresolved modes caused by the discretization. A basic difficulty in designing these kind of algorithms is in providing high-frequency dissipation without introducing excessive algorithmic damping over the important low frequency modes. Moreover, second-order accuracy and stability properties need to be granted also for the non-linear case.

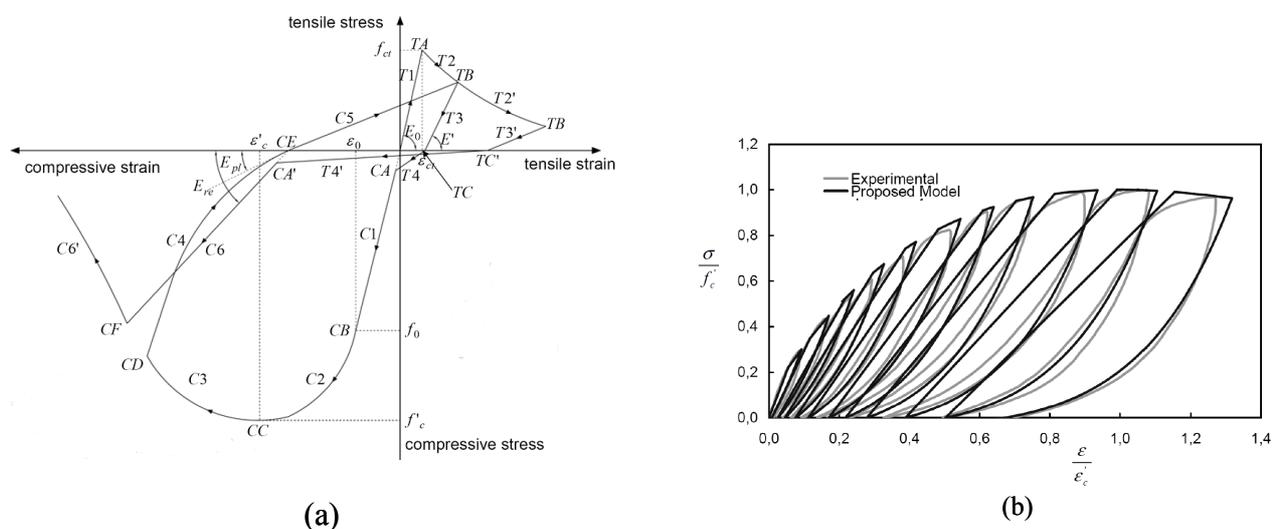


Figure 2. (a) Overall description of constitutive model and parameters. (b) Comparison between numerical curve and Naraine and Sinha's (1989) experimental results after adjustment

Taking all these considerations into account, the so called Generalized- α algorithm proposed by Chung and Hulbert (1993) has been selected as the optimal approach. Its implementation within the GMF has been done by a Newton-Raphson procedure which, for each iteration, involves the determination of an effective stiffness matrix taking into account the tangent stiffness together with inertial and viscous damping effects on the structure. Sima (2010) provides a detailed description of the resulting procedure.

Example of Application

The performance of the method proposed is shown through its application to the study of a 1/5th scale model tested under a simulated ground motion by Tomažević and Weiss (1994).

The model, representing an unreinforced masonry two-story residential building, included a set of perimeter walls and a central cross-wall arranged as described in Fig. 3. The floors were made of reinforced concrete ribbed slabs. The walls consisted of lightweight ceramic perforated blocks. According to the authors, the masonry used for the construction of the model had compressive strength of 6.33 MPa, Young's modulus of 6,450 MPa and tensile strength of 0.40 MPa. In order to meet the requirements of similitude in mass distribution and vertical stresses in the load bearing walls, concrete blocks were fixed to the floor slabs (300 Kg mass at each floor level) and additional vertical stresses at the load bearing walls were introduced by means of prestressed steel ropes at every corner of the model, each providing a force of 12 KN, fixed to the top slab and anchored into the foundation.

The structure was subjected to a series of ground shaking simulations corresponding to the north-south component of the earthquake acceleration record of the Montenegro earthquake of 1979, with a peak ground acceleration of 0.43g. The intensity of the shaking was controlled by adjusting the maximum amplitude of the shaking table displacement. The latter was obtained by numerical integration of the earthquake accelerogram scaled according to the laws of similitude. The building was subjected to a set of runs, characterized by different maximum accelerations, of which those labelled R43 to R49 occurred beyond the elastic range of the structure. The latter, producing a peak acceleration of 2.61g, led the building to collapse.

In order to carry out the analysis, a tangent modulus $G=1400$ MPa and fracture energy $G_f=120$ J/m² were assumed. The model included 123 two-node elements, of which 27 and 96 were used to describe the walls and the slabs respectively. The wall elements were divided longitudinally into 13 integration sections.

Vibration Modal Analysis

An eigenvalue analysis carried out on the undamaged structure yielded a first natural frequency of 13.74 Hz, which is very similar to the value measured experimentally (13.81 Hz) before the application of the ground shakings. The agreement between the numerical and experimental values shows the ability of the model to accurately simulate the flexibility of the wall structure.

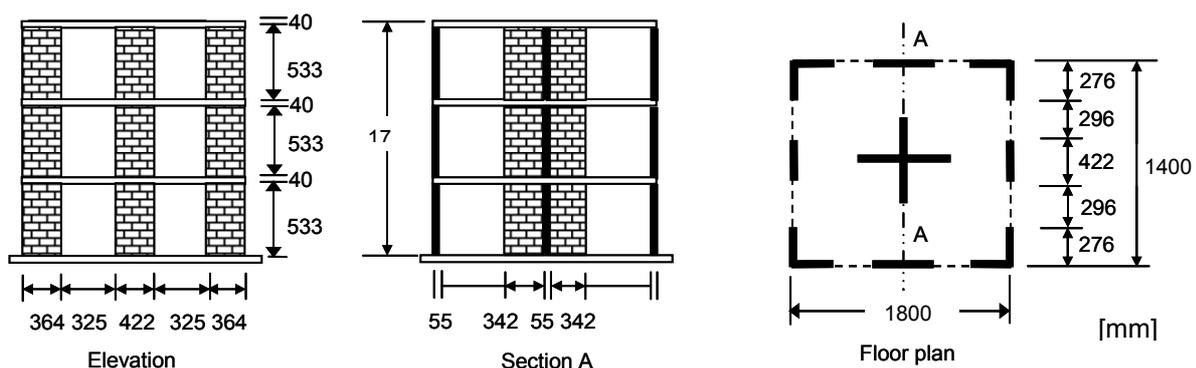


Figure 3: Geometry of unreinforced scale building (based on Tomažević and Weiss, 1994)

Pushover Analysis

As part of their detailed study, the authors of the experiment provide the base shear with the first story drift hysteresis envelope. As a first approach, a pushover analysis is carried out and the resulting ultimate load is compared with the capacity shown by the experimental hysteresis envelope (Fig. 4a). For that purpose, the static equivalent forces have been distributed according to the shape of the 1st mode of vibration, resulting in percentages of the total horizontal force of 21.7%, 36.7% and 41.6% for the 1st, 2nd and 3rd floors respectively.

The maximum horizontal force determined by the pushover analysis, of 23.1 kN, is very close to the maximum base shear measured in the experiment during test R49 (Fig. 4a). However, this experimental base shear appeared for a significantly large lateral drift. Another numerical estimation done using the GMF formulation in combination with a simpler model in tension, consisting of a perfectly brittle equation, produced an ultimate force of 16.5 kN. This value is close to Tomažević's (1994) analytical estimation of 17.1 kN, based on limit analysis, and it is also close to the maximum base shear obtained for tests R44-R47. The difference between the two numerical calculations can be attributed to the fact that the first one considers finite (non-null) fracture energy.

Time Domain Analysis

The test identified as R47 has been modelled herein in order to assess the capability of the proposed method for time domain analysis. The shaking table motion, in this case, was characterized by duration of 5.5 s and maximum acceleration of 1.10g.

The time history analysis has been performed for a time step of 0.006 s and generalized- α method's parameters $\alpha_m = -0.05$, $\alpha_f = 0.1$, $\gamma = 0.308$ and $\beta = 0.61$. These values were adjusted to optimize the agreement with the experimental response. As can be seen in Fig. 4b, good overall agreement with the test results was obtained with acceptable deviations in the amplitudes and the frequencies. The difference between numerical and experimental maximum drifts was of 25%, 3% and 8% for the 1st, 2nd and 3rd floor respectively. The maximum base shear and the maximum lateral drift at the first floor were of 20.3 kN and 4.5 mm respectively. Both values are close to the experimental results.

A Fourier analysis carried out on the numerical drift history of the top floor yielded a final frequency of 3.45 Hz for the 1st vibration mode. Although this value is lower than the corresponding experimental measure (4.99 Hz), it provides a similar estimation of the large impact of the damage caused by the shaking motion. The reduction of the frequency is of 75% and 64% for the experiment and the present numerical analysis respectively.

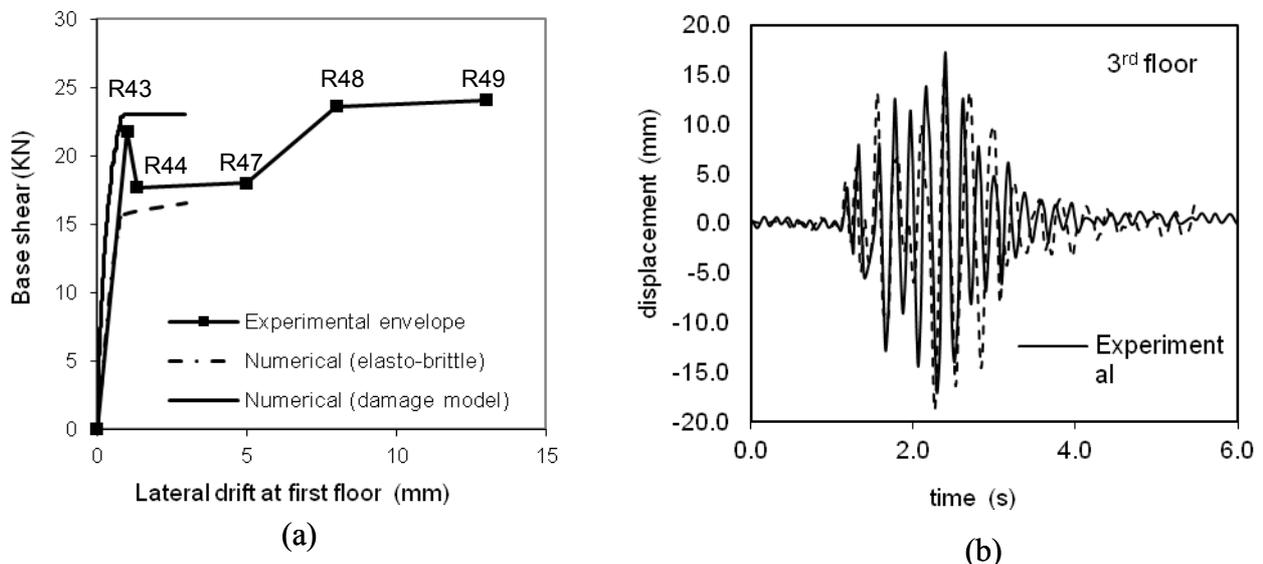


Figure 4. Comparison of pushover curves with Tomažević and Weiss' (1994) hysteresis envelope (a); Comparison of experimental and numerical lateral drift history for the 3rd floor (b)

According to Tomažević and Weiss (1994), cracking was first seen at the bottom of the peripheral walls due to separation and rocking motion. At the ultimate condition, horizontal cracks developed at the joints between most walls and floor slabs. Parts of the inner cross wall failed in shear whereas the middle peripheral walls failed in shear with crushing of blocks in compression. The numerical analysis has predicted a distribution of damage consistent with the experimental one at the ultimate condition. It includes the full cracking (full tensile damage) of the corner walls, while the middle peripheral and inner walls are only partially damaged in tension. Severe compressive damage is observed in the inner cross wall, while the middle peripheral show only moderate compression damage levels.

Conclusions

A method for non-linear dynamic analysis in the time-domain has been developed as a further extension of the Generalized Matrix Formulation for masonry structures. The method includes a constitutive uniaxial cyclic equation in combination with a procedure for the integration of the equations of motion. The method is applicable to masonry structures including linear members (arches, pillars) or walls, the latter being modelled as equivalent frame systems.

The example presented, consisting of the simulation of the effects of a large-amplitude ground motion on a three-story wall building, has shown the applicability of the method to the analysis of complex masonry structures and its ability to provide realistic results on the dynamic response.

The proposed approach is characterized by large computer efficiency thanks to the limited number of degrees of freedom that are needed to model large structures. Entire structural members (such as wall panels) can be modelled using a very limited set of two-node linear elements.

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