Structural Analysis of Historical Constructions – Jerzy Jasieńko (ed)
© 2012 DWE, Wrocław, Poland, ISSN 0860-2395, ISBN 978-83-7125-216-7

SEISMIC CAPACITY OF THEGHIRLANDINA TOWER IN MODENA, ITALY

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ABSTRACT

This paper presents a simplified vulnerability assessment method for masonry tower staking into account the soil-structure interaction and masonry limit analysis method. A model to determine the plane of fracture that defines the kinematic blocks of an overturning mechanism is proposed, based on simple equilibrium conditions and is applied to the Ghirlandina Tower in Modena.
According to the Italian codes, towers are classified as a macroelement characterized by peculiar collapse mechanisms. For towers a slight variation in mechanism geometry implies relevant variations in collapse multiplier values; this is mainly due to the importance of mass and height in these structures. Hence a correct definition of mechanism geometry is very important.
The assessment is based on a*in situ* investigation defining a detailed soil profile and the relevant mechanical parameters, aimed at modelling the soil-structure interaction during both static long-term life and during expected seismic events. Parameters were used to run a frequency analysis (to determine seismic demand), in which different stiffness values for soil restraint were used, corresponding to different hypothesis about soil behaviour.
Limit analysis method was then applied to identify the collapse mechanisms geometry, under the assumption that masonry is a no-tension material, hence at the limit of overturning, a part of the masonry will remain attached to the base and a stress-free diagonal surface of fracture will form.
These analyses allowed for comparing of the limit loads related to the collapse of the fractured masonry with those related to the overturning of the tower as a whole, the horizontal rotation axis being located at the interface foundation-subsoil, establishing a hierarchy of mechanisms.

Keywords: *Medieval towers, Collapse mechanisms, Seismic assessment*

1. INTRODUCTION

1.1. Introduction

The Ghirlandina tower is the ancient bell tower of the Cathedral of Modena, both form part of the UNESCO site of Piazza Grande. The Ghirlandina tower is a square based (side: 10.8 m) structure, 87 m high; the structure has a hollow cross section, thicker on the corner because of the presence of four masonry pillars as shown in figure 1; in the inner part, an open stair runs along the tower from the base to the higher part where the belfry and the spire roof complete the structure.
The tower is characterized by a tall and slender spire built on its top and preciously decorated (*ghirlandina*), defining its slender architectural appearance.
The masonry diaphragms built in the tower are: the vault on the first floor, the floor of the Torresani cell and the vault above the belfry (the deck instead is a timber structure). At ground level, two masonry arches connect the tower with the adjacent cathedral.

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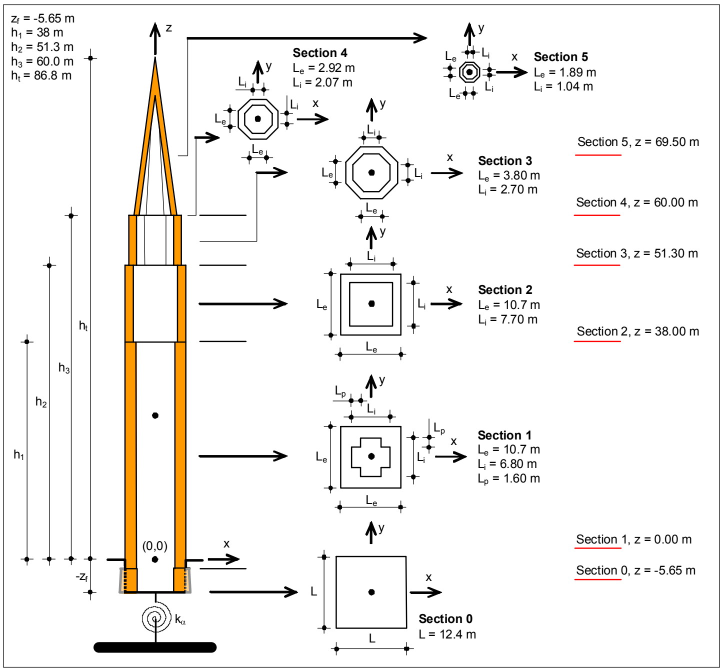

Fig. 1 Geometrical model of the tower

The tower's verticality has been corrected several times during different phases of construction; it is in fact possible to observe, along the façades, segments of variable leaning as signs of corrections for settlement problems. The tower today presents a visible leaning, in particular on the S-W corner where the masonry arches connect the tower to the cathedral.

The construction of a tower is an event of great structural effort, and its leaning in fact hides some problems related to the interaction with the foundation soil, that are sometimes not so evident at first glance. It's important to remember that during the first stage of construction the tower could not have been so far from a bearing capacity collapse, due to a lack of soil strength, and could have safely survived thanks to some delay or interruption in the building process. Therefore, it is of paramount interest to get a full description of the tower's history and to highlight the construction stages and rest periods that allowed the foundation soil to improve its strength, during the consolidation process under constant external load, and that allowed the tower to be successfully finished. The archaeological survey [1] identifies four different construction phases of the tower, a complete chronology, including all the events determining structural effects (fires, earthquakes...) can be found in [2].

At the same time, by investigating the tower's reaction if perturbed by any external action this paper deals with the stability of equilibrium, the danger of a leaning instability as related to the lack of stiffness of the soil. Moving from these arguments, the objective of this paper is to provide a picture of the subsoil conditions (namely soil profile, groundwater conditions and relevant mechanical parameters) that have a major role in defining the seismic behaviour, including soil-structure interaction, and to present the most relevant potential collapse mechanisms arising from expected seismic actions.

2. GEOTECHNICAL ASPECTS

2.1. Soil profile

Since 1980 the town council of Modena has promoted studies and investigations related to the subsidence of the Modena alluvial plain [3-7]. In addition to these studies, it is relevant to outline that the Modena alluvial plain is characterized by a unique abundance of archaeological sites, and that the related interest in this promoted research on the Quaternary sedimentation of the Modena plain [8-10].

Finally, in order to define a detailed soil profile and the relevant mechanical parameters aimed at investigating the behaviour of the tower in relation to subsoil conditions, a rather comprehensive site investigation was planned for September 2007 and December 2008 as described by [11].

By referring to Figure 2, the soil profile down to the investigated depth of 80 m is composed of a first horizon of medium to high plasticity inorganic clays, with an abundance of *laminae* of sands and peat, only millimetres thick. The upper portion of this horizon (Figure 2), whose thickness ranges from 5 to

7 meters, is known as Modena Unit and is linked to the flooding events (of post-Roman age) produced by minor streams (T. Fossa-Cerca).

The subsequent under laying horizons, ranging in depth from 22 to 54 m, represent the result of a complete transgressive-regressive cycle: the fine-grained sediments, belonging to the horizon known as Niviano Unit, were deposited during the penultimate interglacial cycle, and the superimposed coarse-grained materials, belonging to the Vignola Unit, are linked to transport activities of the Secchia River.

A second horizon of coarse-grained materials is encountered at depths ranging from 54 to 63 m, and thereafter a fine-grained materials horizon is found down to a depth of 78 m, here again characterized by a diffuse presence of *laminae* of sand.

2.2. Investigation on seismic parameters

Seismic actions to be considered at a specific site are usually described in terms of peak ground acceleration a_g, being associated to a rigid soil formation and to free-field conditions, and to the elastic response spectrum $S_e(T)$. There is also considerable evidence, both theoretical and experimental, that earthquake waves are affected by soil condition and topography, so that seismic waves may be modified (both in amplitude and frequency content) as they pass from the rigid basement to the soil surface. This phenomenon, known as soil amplification, requires specific site studies, or may be based on lumped parameters. One of these parameters is the shear waves velocity V_{s30}, characterizing the upper 30 m thick horizon.

For this reason the geotechnical activity was complemented with the execution of cross-hole tests (figure 3). In this respect, it must be remembered that shallow seismic exploration tests of soils represent an important class of field tests because of their non-invasive character. This allows for preservation of the initial structure of soil deposits as well as the influence of all diagenetic phenomena contributing to a stiffer mechanical response. Therefore, the cross-hole test represents one of the most reliable methods of determining the shear modulus at small strain amplitude. Based on the results referred to in Figure 3, a relevant shear wave velocity $V_{s,30}$ equal to 192 m/s was deduced, allowing the subsoil to be classified into class C, according to [12, 13].

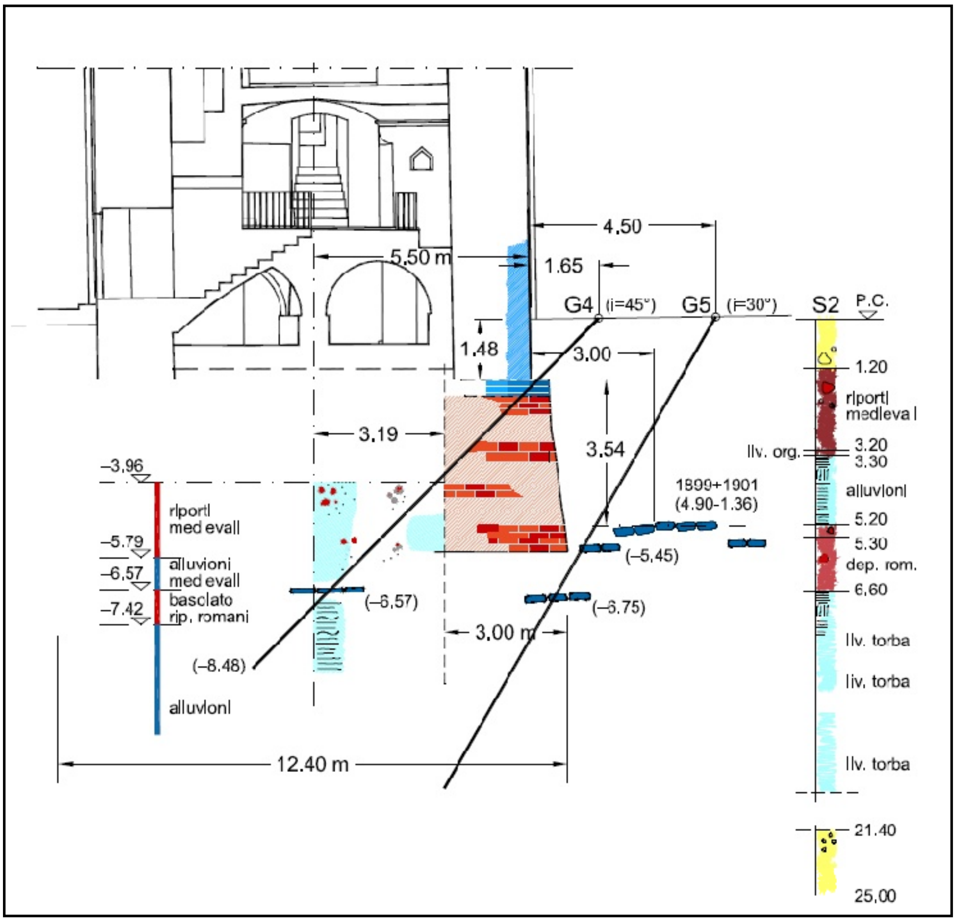

Fig. 2 Details of soil profile and foundation of Ghirlandina Tower [11]

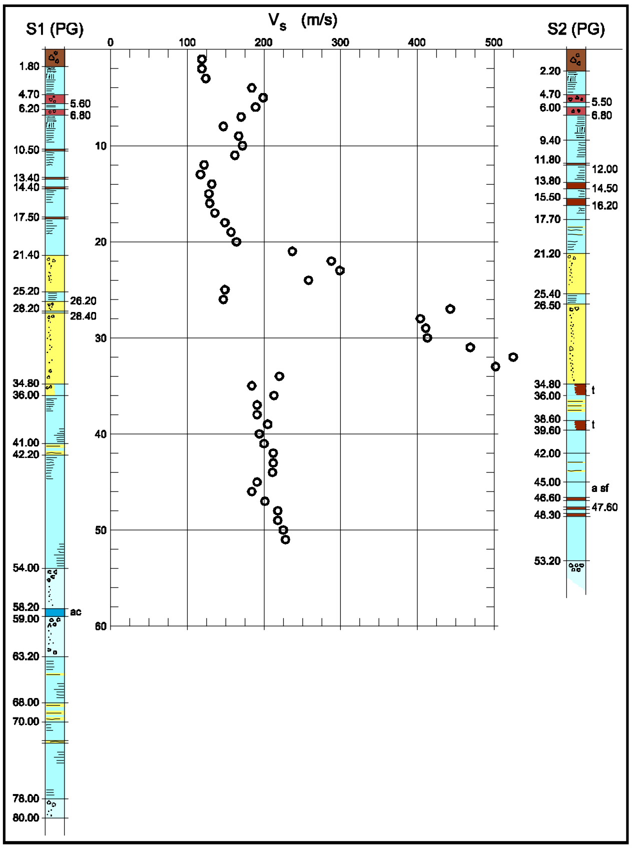

Fig. 3 Cross-hole tests

2.3. Soil-structure interaction

Seismic analysis is strongly dependent on the soil response, and in order to model it we rely on the so-called macro-element approach [14-20]. This approach is aimed at representing soil response in terms of generalized forces and related displacement components, i.e. a formulation suitable for soil-structure interaction, moving from advanced hardening plasticity, in order to account for the irreversible and non-linear soil behaviour. As a consequence of such non-linearity, in the present analysis two assumptions regarding the rotational stiffness were used.

(a) Moving from the shear wave velocity equal to v_s=125 m/s, a small-strain shear modulus has been deduced:

$$G_0 = r\, v_s^2 = 1800 \cdot 125^2 = 28 \text{ MPa} \tag{1}$$

This value refers to free field conditions, so that it has been corrected in order to account for the stress level induced by the tower (G_0 = 44 MPa). Finally, taking into account the strain level, the operational value was estimated to be equal to G_{epl} = 7.26 MPa, the corresponding rotational stiffness being

$$K_\alpha = \frac{3.6Gb^3}{1-\upsilon} = 1715.96 \cdot G = 1245788 \; t \cdot m \tag{2}$$

where $2b$ = 12.40 m is the foundation width and nis the Poisson ratio.

This value was further increased in order to account for the foundation depth (d/b = 5.65/6.2 = 0.91; d/D = 1) [21], the related coefficient being:

$$f_D = \left\{ 1 + 1.26\frac{d}{b}\left[1 + \frac{d}{b}\left(\frac{D}{d}\right)^{0.2}\sqrt{\frac{b}{l}} \right] \right\} = 3.19 \tag{3}$$

where:

$2b$ and $2l$ are the foundation sides;

D is the founding depth;

D is the fraction of D that contributes to the constraint (here assumed equal to D).
By using this coefficient a corrected stiffness was obtained equal to:

$$K_\alpha = \frac{3.6Gb^3}{1-\upsilon} \cdot 3.19 = 3.97 \cdot 10^6 \ \ t \cdot m \tag{4}$$

that, because of the considered strain level, is appropriate for strong seismic motion.
(b) In addition, an upper bound value was estimated by using the elastic shear modulus, moving from the assumption that soil behaviour could still be dominated by an elastic response due to creep hardening:

$$K_\alpha = \frac{3.6Gb^3}{1-\upsilon} \cdot 3.19 = 24 \cdot 10^6 \ \ t \cdot m \tag{5}$$

This value is appropriate for low intensity seismic events.

3. SEISMIC DEMAND

3.1. Natural frequencies of the tower

Referring to figure 4 in which the tower is considered to be made of elastic material and soil-structure interaction is modelled as an equivalent elastic spring with rotational stiffness K_a it's clear that the elastic properties of the system (masonry Young modulus and spring stiffness K_a), determining its natural frequencies, modify the seismic action in terms of spectral acceleration.
According to limit analysis, described in the following paragraphs, the tower will be assumed to be a rigid element fixed on elastic restraint; nevertheless, to assess the reliability of the rigidity assumption, natural frequencies were first determined for different values of masonry Young modulus and elastic soil restraint.
Natural frequencies of the tower were determined by FEM analysis on an elastic model made of beam elements (based on geometry as described in figure 1).
Geotechnical analysis gives two different stiffness values for the elastic spring, corresponding to different assumptions on soil behaviour; a simple parametric analysis was performed to evaluate the effect of the two stiffness values on seismic action while also considering the influence of the masonry Young modulus. This parameter was varied, ranging between 2000 MPa and 4000 MPa; the tower's natural frequencies were then calculated for five different stiffness values (between 0.5 10^8 kNm and 2.5 10^8 kNm) of the spring representing the soil restraint [22].
The results of the analysis are shown in figure 5 for I mode.

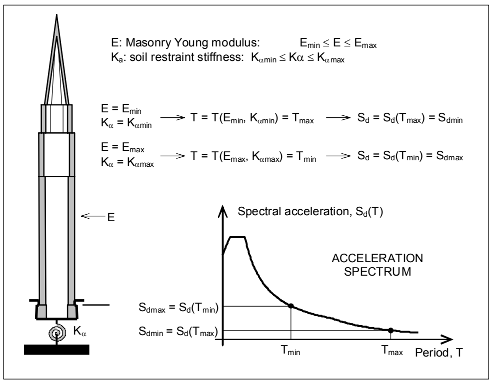

Fig. 4 Expected spectral acceleration depending on elastic properties of the system

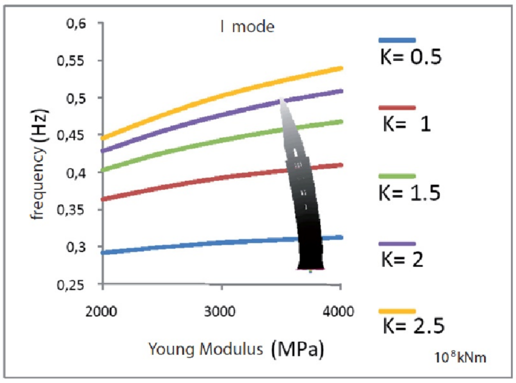

Fig. 5 Natural frequencies of I mode depending on elastic properties of the system

Results corresponding to the rigid behaviour assumption of masonry are instead calculated as:

$$T = 2\pi\sqrt{\frac{I_t}{K}} \tag{6}$$

I_t being the mass moment of inertia about a horizontal axis through the centroid of the base cross section and K the soil-structure restraint stiffness.
Hence the corresponding period and frequencies result in:

$$T_{0,max} = 3{,}437 \text{ s}; \; f_{0,min} = 0{,}29 \text{ Hz for } K_{min}$$
$$T_{0,min} = 1.398 \text{ s}; \; f_{0,max} = 0{,}71 \text{ Hz for } K_{max}$$

These values are similar to those obtained when considering the deformation of the tower, especially in the case of the lower value of K_α

3.2. Spectral acceleration

The influence of Young modulus values depends on the value assumed for soil stiffness, being negligible in the case of lower values of K and more pronounced in the case of higher soil stiffness. Spectral acceleration values obtained for hypothesis a) and b) (see previous paragraph) assuming a rigid behaviour of the masonry tower, are illustrated on the design spectrum [12] in figure 6.

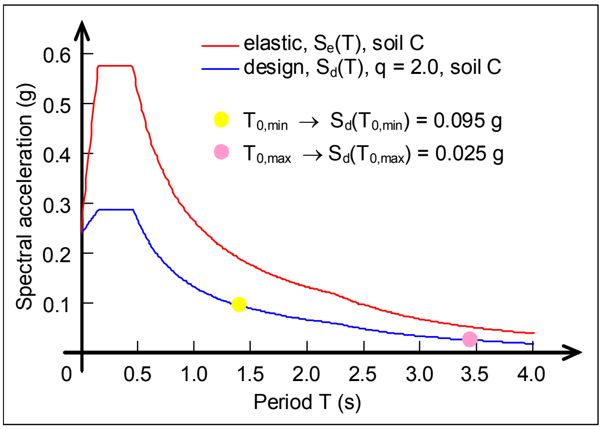

Fig. 6 Spectral accelerations corresponding to assumptions a) and b) on soil-structure interaction

4. SEISMIC CAPACITY

Seismic capacity of the tower was calculated according to Italian code prescription [12] following the collapse mechanisms approach. Mechanisms involving an "opening" of the structure along the existing vertical cracks were not included because in order to guarantee a unitary behaviour of the tower a preventive intervention with tie-rods holding together opposite walls was made.
The collapse mechanisms are identified by the geometry of the overturning block and by the position of hinges. Hence, in this analysis the following conditions (as in a plane problem) were considered:

- overturning block defined by a diagonal fracture surface,
- overturning block defined by a horizontal fracture surface,
- rotational hinge at the edge of cross section (assumption of infinitive compressive strength of the material),
- rotational hinge at a calculated distance from the section edge (assumption of finite compressive strength of the material).

Results were then compared and a reliability assessment was performed. The simplified geometrical model considered is shown in figure 1, where the main discontinuities of the cross section are included; windows and openings are ignored and a hollow base cross-section is assumed. The foundation area is assumed to be a full squared section of 12.40 m width. In fact, besides a collapse mechanism for an overturning at base level (meaning at the level of the ground, at height z = 0 m in figure 12), a conservative evaluation for the overturning at foundation level (meaning at foundation soil level, z = -5 m) was also performed, taking into account the soil properties.
When considering short-term perturbations (earthquakes or wind effects), failure mechanisms are explored with reference to undrained soil conditions. For this reason, the bearing capacity has been evaluated in terms of total stress and assumed as: q_{lim} = 0.714 MPa [11].
Masonry compressive strength, considering the results of sonic test and the heterogeneity of calculated velocities, is assumed as 3 MPa.
Leaning of the tower (1 degree) has been taken into account considering the effective position of the centre of mass.

4.1. Overturning block geometry

In the case of a tower structure, the overturning of the upper part on the base usually represents the weakest mechanism.

The most unfavourable hinge position can be easily determined; instead the identification of the overturning block geometry is a more complex task.

A simplified geometry with a straight horizontal fracture surface is inconsistent with the evidence of real collapse mechanisms, which have occurred during past earthquakes (figure 7).

The presence of a diagonal fracture surface is justified by the fact that masonry is a unilateral material able to resist high compressive stresses but with feeble tensile strength. As a consequence at the limit of overturning, a part of the masonry will remain attached to the base and a stress-free diagonal surface of fracture will form [17].

Hence, a method able to include this property, even in a simplified assessment, such as the collapse mechanisms analysis, proves very important.

In the present paper the fracture line is identified by the position of the neutral axes of the cross sections. According to simple elastic theory, when the line of thrust falls outside the section kern a stress-free zone will develop (figure 8).

Fig. 7 Bell tower of San Martino – Resiutta (Udine) damaged by Friuli earthquake in 1976

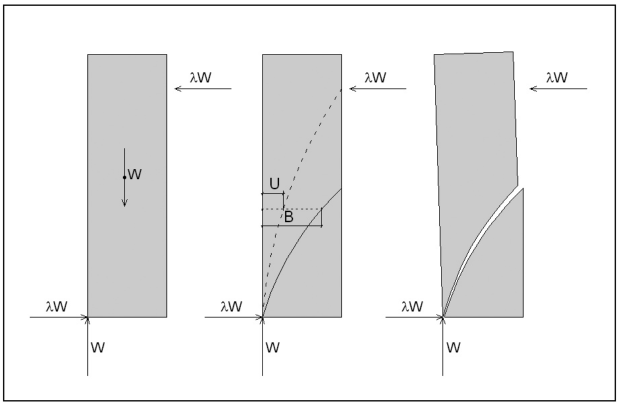

Fig. 8 Position of line of thrust respect to section kern and fracture developing

In the case of rectangular cross sections the depth L of the neutral axis and the depth *U* of the thrust line satisfy the relation *U = B/3*. In general the limit distance value *U* must be calculated in function of *B* for each tower cross section.

An example of this procedure is given in [23] where the method is applied to the analysis of buttresses.

Considering a masonry tower and simplifying the problem into a plane problem, with reference to figure 11, a differential equation is searched whose solution is the curve of fracture $z = z(l)$, z being a vertical reference axis with the origin at the base.

A tower of height h_t, is considered; on the tower the dead load and a horizontal load with distribution proportional to the mass are applied. The horizontal load is considered high enough to produce the cross section partialization between $z = 0$ and $z = h_{fp}$. The following assumptions are made:

- null masonry tensile strength (no-tension material),
- elastic behaviour of masonry in compression,
- only the masonry in compression is involved in the collapse mechanism,
- cross-section is constant where the fracture develops.

Hence, the fracture will form in each cross-section when the line of thrust reaches the edge of the section kern.

The distance of section kern from the external edge, in case of squared cross-section of side L_e, is:

$$U(l) = \frac{L_e - l}{3} \tag{7}$$

In case of hollow squared cross-sections the function must be preliminarily calculated as:

$$U(l) = L_g(l) - \frac{\rho^2(l)}{L_e - l - L_g(l)} \tag{8}$$

where:

$$\rho(l) = \sqrt{\frac{J(l)}{A(l)}}$$

$\rho(l)$being the radius of gyration of the uncracked section, $L_g(l)$ is the distance of the section centroid to the compressed edge; $A(l)$ and $J(l)$ are respectively the area of the compressed part of the cross section and its moment of inertia, being L_i and L_e defined in figure 11.
Distance from the edge, normalized with respect to the uncracked length of the section is:

$$d(l)=\frac{U(l)}{L_e-l} \tag{9}$$

An examples is shown in figure 9 for two different values of ratio L_e/L_i

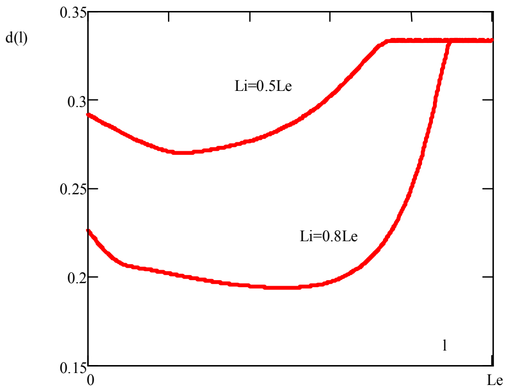

Fig. 9 Values of equation *d(l)* for different values of ratio L_e/L_i

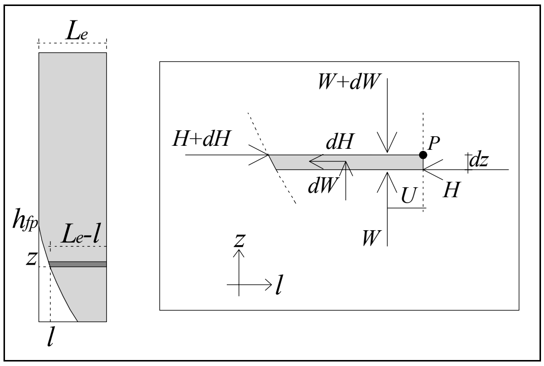

Fig. 10 Elementary slice of the tower in the fractured zone

The equation of the fracture is determined based on the equilibrium conditions of an elementary slice of the tower in the fractured zone (figure 10). It is identified by the differential equation [24]

$$\frac{dz}{dl} = W(l,z(l)) \cdot \frac{d(U(l))}{\{[U(l)-L_g(l)]\cdot A(l)\cdot \gamma_m + H(l,z)\}} \tag{10}$$

Where $W(l,z(l))=W_0(h_{fp})+W_c(l,z(l))$ is the weight of the tower above the abscissa z, g_m is the masonry density.

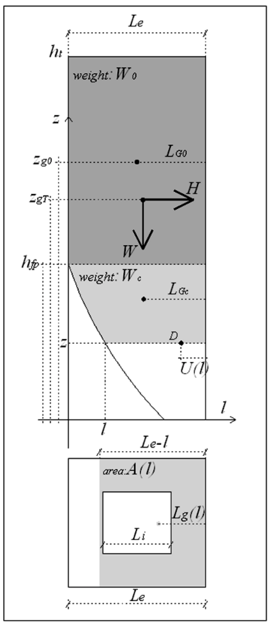

Fig. 11 Geometrical model of the tower

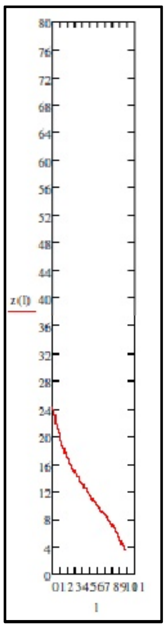

Fig. 12 Fracture plot under assumption of infinite masonry compressive strength

Hence $H(l, z)$can be defined as:

$$H(l,z(l)) = \frac{W(l,z(l))\cdot[L_{Gt}(l,z(l)) - U(l)\cdot(L_e - l)]}{z_{Gt}(l,z(l))} \tag{11}$$

Where $L_{Gt}(l,z(l))$ and $z_{Gt}(l,z(l))$ are the coordinates of the overturning block centroid.
The fracture line is the solution of differential equation (19) with boundary condition $z(0) = h_{fp}$,. This equation was solved via a numerical *ODE solver* that uses the *Runge-Kutta* method in the fourth order increment approximation, obtaining a family of fracture curves $z(l)$ varying with parameter h_{fp}. In a first solution step, the algorithm performs a do-loop on the h_{fp} parameter until the fracture curve reaches the external edge of the section that corresponds to an assumption of infinite masonry compressive strength (figure 12). In a second step the curve of fracture has been determined by imposing the attainment of the ultimate resisting moment at the base section considering a finite value of masonry compressive strength, which defines the final h_{fp} in the iterative scheme.
Having once determined the h_{fp} and the corresponding fracture line, the collapse multiplier l can be obtained as the ratio between the horizontal force and the dead load of the overturning part.

4.2. Cracked tower overturning

The fracture line is defined by equation (10) and it was evaluated in cases of:

- rotation at base level and masonry infinite compressive strength,
- rotation at base level and masonry finite compressive strength,
- rotation at soil foundation level and soil finite compressive strength.

4.2.1. Base level

In the first case, the fracture line defined by (10), intercepts the edge of the base cross section; the corresponding horizontal load collapse multiplier is: $l = 0.143$.
In the second case, the masonry compressive strength is assumed f_m = 3 MPa, the horizontal load collapse multiplier is $l = 0.127$

4.2.2. Foundation level

Evaluating the overturning at foundation level (-5 m from the base level) the bearing capacity of soil must be taken into account, considering that at the overturning limit condition the normal stress on foundation level is uniform and equal to the strength of the soil-foundation system.

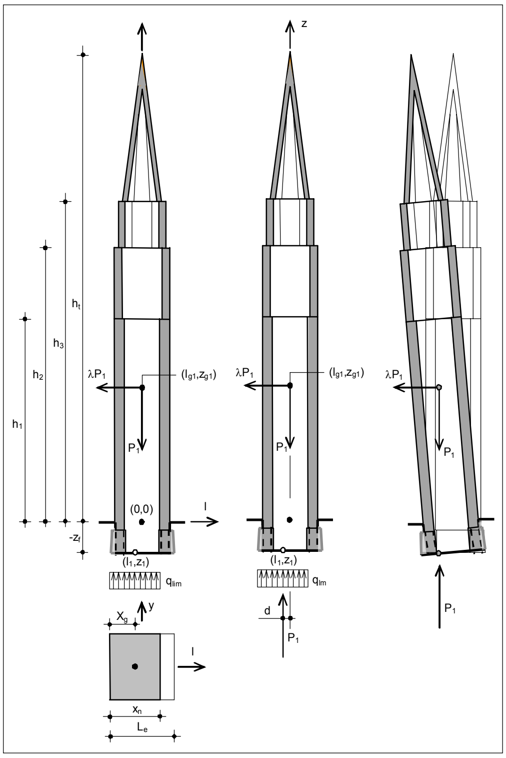

Fig. 13 Geometrical model for limit analysis: overturning mechanism at foundation level

Equilibrium between the self-weight of the tower and soil reaction resultant gives the widthof the compressedarea (figure 13):

$$x_n = \frac{W_{tot}}{L_e . q_{lim}} = 9.7\ m \tag{12}$$

W_{tot} = 85546 kN being the tower weight, L_e = 12.4 m the side of the *squared* foundation area and q_{lim} = 0.714 MPa the soil strength.
The fracture line intercepting the edge of the compression part at foundation level was calculated and the corresponding collapse multiplier results *l* = 0.022

4.3. Uncracked tower overturning

In order to evaluate the influence of considering an inclined fracture line, the previous analyses were also performed considering a horizontal line of fracture at both base level (for masonry infinite/finite compressive strength) and foundation level.
These results in terms of λ and those presented in the previous paragraph are compared in figure 14.

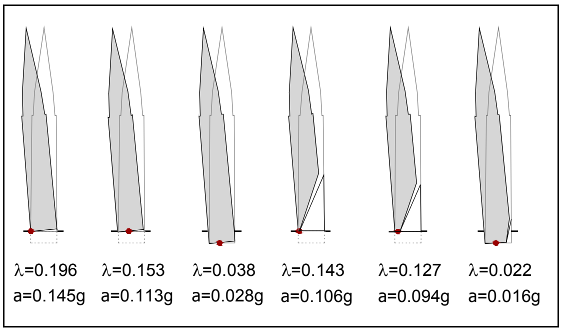

Fig. 14 Collapse multiplier and acceleration values

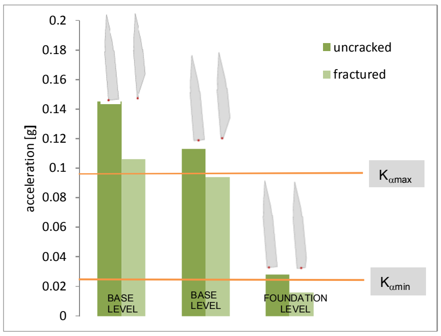

Fig. 15 Comparison between seismic demand and seismic capacity for different configurations of global overturning

4.4. Seismic assesment

According to [12] to a horizontal load multiplier a corresponding spectral acceleration activating the mechanism can be associated: seismic capacity for the uncracked and fractured tower was than compared to the seismic demand, defined in paragraph 3.2 (figure 6).In figure 15 the collapse multipliers calculated and the corresponding accelerations are compared for different mechanisms.
Overturning at foundation level isthe mechanism with the lowest collapse multiplier, due to the small dimensions of foundation area and the slight increase of the global centroid height; in this case almost the whole foundation area is needed to respect the condition on soil bearing capacity, hence the line of fracture separates just a small part of the masonry; nevertheless a relevant influence on multiplier values can be observed.
For the other cases, where the condition on materials strength determines a fracture that propagates higher in the tower, the effect of considering the inclined line of fracture reduces the resistance of the tower to overturning by36% (neglecting masonry compressive strength) and by 20% (considering masonry compressive strength), indicating that for a safe simplified assessment these conditions must be evaluated.

5. CONCLUSIONS

This paper was aimed at presenting a simplified but effective vulnerability assessment for masonry towers. Two major aspects were considered: the influence of soil-structure interaction and the influence of the masonry tower as a no-tension material.
In particular, the influence of soil non-linear behaviour was taken into account, by selecting an upper and a lower value of soil stiffness, representative of small strain and large strain response, appropriate for low and high intensity seismic events.
The obtained results support the conclusion that the structure-soil interaction strongly affects the seismic demand, depending on the soil stiffness, as well as on the system capacity, related to soil strength.

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