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PARAMETERS FITTING ON THE DYNAMIC BEHAVIOURS OF AN ANCIENT PAGODA DAMAGED BY WENCHUAN EARTHQUAKE

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ABSTRACT

The complexity of establishing the dynamic behaviours analysis model of a damaged masonry pagoda structure is determined by the architectural features, existent material behaviours and defects of a damaged pagoda. Taking Deyang Longhu Ancient Pagoda as an example, this paper probes the way to fit the model parameters of damaged pagoda's structure based on FEM and the sensitivity of dynamic behaviours influenced by various structural elements of a pagoda.

Keywords: *Ancient pagoda structure, Dynamic behaviour, Parameters fitting, Sensitivity*

1. INTRODUCTION

The dynamic behaviours analysis of an ancient pagoda structure is crucial to evaluate the reliability of pagoda and to reinforce a pagoda structure. Whether the dynamic behaviours analysis model in the mathematical relationship can be equivalent to the physical system of ancient pagoda structure is one of the most important factors, which can accurately evaluate seismic behaviour of pagoda structure. According to the existent references, many scholars to establish the dynamic behaviours analysis model by various ways [1-3], such as the method of discrete concentrated mass cantilever beam model, the method of continuous distributed mass beam model, the method of plane finite element model, the method of spatial finite element model [4]. Comparing with the results of each method, we conclude that the first three methods are suitable for the regular pagoda structure. And the fourth method can get more ideal analysis results for irregular pagoda. This paper mainly introduces how to compose an analytical model for dynamic behaviours of complex and seismic damage pagoda structure by the method of spatial finite element and how to fit corresponding model parameters.

2. THE ESSENTIAL FEATURES OF A CHINESE ANCIENT MASONRY PAGODA AND THEIR INFLUENCE ON THE DYNAMIC BEHAVIOURS OF PAGODA STRUCTURE

The parameters on the dynamic behaviours of an ancient pagoda are determined by its special architectural features which are different from general building.

2.1. The essential features of a Chinese ancient masonry pagodas

Generally, an ancient masonry pagoda was built with bricks and mortar made from clay, limestone and sticky rice a long time ago. It has been suffered from various disasters and damaged badly. And it has been reinforced and repaired for several times. There are some common features in an ancient pagoda as following: more complex structural type, non-uniform distribution of quality and stiffness along the height, main structure was made by materials with various different physical and mechanical

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properties, the coexistence of the various damages, special geological structure, etc. The dynamic behaviours of an ancient masonry pagoda are determined by the particularity and complexity of an ancient pagoda.

2.2. The structural parameters related to the dynamic behaviours of ancient pagodas

According to the basic theory of the structural dynamics, the dynamic behaviours of an ancient pagoda are determined by the mass, stiffness of the pagodas and their distribution along the height. The mass of a pagoda is easily to be obtained by measuring the size and the density of each component of pagoda. The stiffness of the pagoda is determined by many factors, such as the structural forms, the properties of structural material, structural dimensions. It is difficult to compose an analytical model of an ancient pagoda more accurately under the damaged condition and reinforcement which are difficult to be detected and described quantitatively.

2.3. The sensitivity of the dynamic behaviours responding to the adjustment on structural parameter of a pagoda

The degree of sensitivity on the dynamic behaviours depends on the adjustment to the type of structural parameters whose influence on the dynamic behaviours is significant difference. The dynamic behaviours of pagoda are described by the characteristic value, the eigenvector, the modal flexibility, etc. In order to describe dynamic behaviours of a pagoda's structure, $p_1, p_2, \ldots, p_n$ is individually represented the mass, the stiffness, the structural geometrical parameters and the material properties of a pagoda. The dynamic behaviours are regarded as the derivable function of the structural parameters, denoted by F.
Here,

$$F = F(p_1, p_2, \ldots, p_n) \tag{1}$$

So, the sensitivity of F based on structural parameter: $p_i(i = 1,2 \ldots,n)$ could be denoted as:

$$S_F = \frac{\partial F(p_1, p_2, \cdots, p_n)}{\partial p_i} \tag{2}$$

The greater the absolute value of sensitivity S_F becomes, the more sensitive the structural parameters on modal characteristic equations are. That is to say, the sensitivity S_F will change much as long as p_i changes a little. The positive value indicates that the modal behaviours are in the increasing position .Oppositely, negative value indicates that modal behaviours are in the decreasing position.
In order to compose an analytical model of a pagoda's dynamic behaviours by fitting model parameters and adjusting the structural parameters, it is the basis to establish the evaluating system on the sensitivity that shows the relationship between the structural parameters and the dynamic behaviours of a pagoda.

3. THE WAY TO FIT THE MODEL PARAMETERS OF A SEISMIC DAMAGED PAGODA

The way to fit the model parameters of dynamic behaviours by the spatial finite element method is combined the theory of structural dynamics with experimental technique. Firstly, in order to analyse the sensitivity, the relationship between the dynamic behaviours and the structural parameters of the ancient pagodas with damages is identified. Secondly, obtaining the sensitive parameters and corresponding modified direction. Thirdly, the structural behaviours could be discussed based on the perturbation principle. Finally, a mathematical model to be equivalent to the physical system by gradually approximating the theoretical date to the experimental data was built up.

3.1. Preliminarily estimating structural parameters and establishing corresponding initial model

By surveying and mapping on site, experimenting, and studying the original files, the structural composition of pagodas, the damaged condition and reinforcement, the condition of foundation, the structural geometry size and part of material behaviours, etc could be obtained more exactly. Some of preliminary parameters which are unable to measure could be estimated on the existing references.

The initial model of the dynamic behaviours by spatial finite element method on the basis of the above structural parameters is composed. The preliminary results of the dynamic behaviours which are the foundation of fitting model parameters could be obtained.

3.2. Identifying the relating structural parameters by contrasting calculating date to the experimental data

Contrasting the calculating data on the dynamic behaviour parameters of the initial model to the experimental data and find out the difference, the reason why the difference is made by uncertain factor could be analyzed for the corresponding structural parameters.

3.3. Modifying the structural parameters and fitting the analytical model parameters

Because the mass and its distribution of a pagoda are the almost real in an initial spatial finite element model, the focal points of the modification are the parameters related to the structural stiffness, such as the structural geometric parameter related to various kinds of damage on structural solid, material physical features in different parts. By means of the sensitive response analysis on the dynamic behaviours of a pagoda, the structural parameters which can accurately conveys the dynamic behaviours of the pagoda's structure could be identified [5].

The dynamics of structure-theory shows that the *r* th Eigenvalue λ_y and Eigenvector $\varphi^{(\gamma)}$ of structural parameter function is given by

$$([k]-\lambda_y[M])\varphi^{(\gamma)}=0 \tag{3}$$

Thus, solving the partial derivative of the *i* th parameter in the equation (3-1) leads to the sensitivity of the Eigenvalue as

$$\lambda_{ri}=\varphi^{(\gamma)T}([K]_i-\lambda_y[M]_i)\varphi^{(\gamma)} \tag{4}$$

The sensitivity of the Eigenvector as

$$\varphi_j^{(\gamma)T}=\sum_{\substack{k=1\\k=\gamma}}^{n}\frac{-\varphi_j^{(\gamma)T}([K]_i-\lambda_y[M]_i)\varphi^{(\gamma)}}{\lambda_k-\lambda_\gamma}\varphi^{(k)}-\frac{1}{2}\varphi_j^{(\gamma)T}[M]_j\varphi^{(\gamma)} \tag{5}$$

If the variable is the mass m_{ij} or the stiffness k_{ij}, the *r* th sensitivity of the Eigenvalue and the Eigenvector on the mass m_{ij} or the stiffness k_{ij} could be given by

$$\frac{\partial\lambda_y}{\partial m_{ij}}=-\lambda_\gamma\varphi_{i\gamma}\varphi_{j\gamma}\,,\quad \frac{\partial\lambda_y}{\partial k_{ij}}=\varphi_{i\gamma}\varphi_{j\gamma} \tag{6}$$

$$\frac{\partial\varphi^{(\gamma)}}{\partial m_{ij}}=\sum_{\substack{k=1\\k=\gamma}}^{n}\frac{\lambda_\gamma\varphi_{i\gamma}\varphi_{j\gamma}}{\lambda_k-\lambda_\gamma}\varphi^{(k)}-\frac{1}{2}\varphi_{i\gamma}\varphi_{j\gamma}\varphi^{(\gamma)} \tag{7}$$

$$\frac{\partial\varphi^{(\gamma)}}{\partial k_{ij}}=\sum_{\substack{k=1\\k=\gamma}}^{n}\frac{-\varphi_{i\gamma}\varphi_{j\gamma}}{\lambda_k-\lambda_\gamma}\varphi^{(k)}-\frac{1}{2}\varphi_{i\gamma}\varphi_{j\gamma}\varphi^{(\gamma)} \tag{8}$$

When $\lambda_\gamma=\omega_\lambda^2$, the sensitivities of the natural frequencies on the mass and stiffness of a pagoda are

$$\frac{\partial\omega_\gamma}{\partial m_{ij}}=-\frac{1}{2}\omega_\gamma\varphi_{i\gamma}\varphi_{j\gamma}\,,\quad \frac{\partial\omega_y}{\partial k_{ij}}=-\frac{1}{2\omega_\gamma}\varphi_{i\gamma}\varphi_{j\gamma} \tag{9}$$

4. PARAMETER FITTING OF THE DYNAMIC MODEL OF DEYANG LONGHU ANCIENT PAGODA

4.1. The main features of Longhu Ancient Pagoda

Longhu Ancient Pagoda is in the Jingyang District, Deyang City. Its early architectural feature shows that it is the only existing ancient pagoda of Yuan dynasty in Sichuan province, China. Its precious historical and scientific value shows that it is an important example for probing the evolution of the masonry pagoda [6].

Fig. 1 Longhu Pagoda

Longhu Pagoda is a masonry pagoda with square plan and thirteen layer cornices along the height. It is about 33 meters high. The tower body was gradually shrunk with the height increasing. There are five masonry floors and roof welkin in the pagoda. There is a Buddist shrine in the open space of every floor of the pagoda. There are bucket arches and caisson on the top from the first floor to the fourth floor, and an empty well on the top of the fifth floor which connects to the welkin. The welkin is a dome running through from the eleventh layer to the thirteenth layer. The appearance of Longhu Ancient Pagoda is shown as Fig. 1.

4.2. The basic parameters and initial model

According to the drawings and data provided by the Research Institute of Chinese Cultural Heritage Protection, the structural geometry size of Longhu Pagoda is shown in Table 1. The physical entity model was made by AutoCAD (Fig. 2). Most of damage defects are factually presented in the model. Then the entity model is converted into the general finite element model by the interface tools embedded in ANSYS Workbench (Fig. 3).

Table1 Dimension of Longhu Ancient Pagoda

Position	Floor Height (m)	Length of a side (m)	Space of the entral room (m^2)	external Wall Thickness (m)	Wall thickness of central room(m)	Width of door or window opening (m)	Overhang size of cornice (m)
Welkin	3.827	5.494	–	1.897	–	–	0.616
Fifth floor	2.585	6.4	15.68	1.002	–	0.32	0.598
Forth floor	5.266	7.223	9.35	0.988	2.238	0.535	1.008
Third floor	5.109	7.4	8.04	1.04	2.325	0.590	0.971
Second floor	5.406	7.451	18.74	1.003	2.263	0.526	1.144
First floor	5.557	7.967	10.34	0.942	2.779	1.608	–

Fig. 2 Solid model

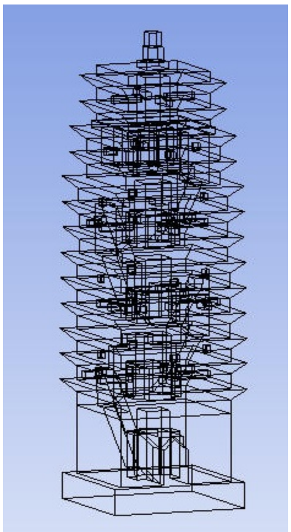

Fig. 3 Perspective

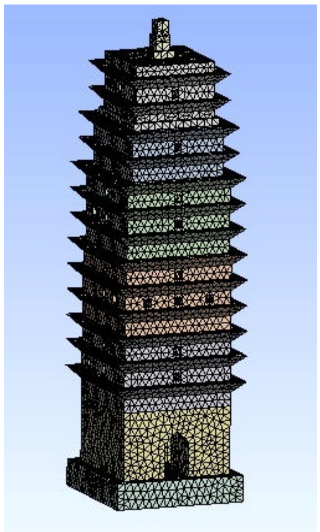

Fig. 4 Numerical Model

During reinforced and repaired after wenchuan earthquake, the strength of the brick block and the bonding material in Longhu Ancient Pagoda is tested on site. The strength grade of the brick is shown in Table 2. The bonding material (mortar) is a kind of mixture with mud and sand etc. The strength grade of mortar of each floor is uneven due to different material in reconstructions. The strength grade of mortar is shown in Table 3. The mass density of masonry is about 2000 kg/m³, and the Poisson's ratio is nearly 0.15. The finite element model is set up (Fig. 4). And the attribute of elements are defined according to the definite structural parameters.

Table 2 Strength grade of the brick in Longhu Ancient Pagoda

The site of test	**First floor**	**Forth floor**	**Fifth floor**
Strength grade	MU10	MU10	MU10

Table 3 Strength grade of the mortar in Longhu Ancient Pagoda

The site of test	**The penetration depth of test (mm)**	**Strength of mortar (MPa)**
First floor	9.8	1.1
Forth floor	10.20	1.0
Fifth floor	13.20	0.6

4.3. The modification of model parameters

Generally, there are two kind of method on evaluating the accuracy of an analytical model [7]. One is half theory -half experience formula method, the other is the method of dynamic test on site. The former one is obtained by special pagoda model, and its errors of result are bigger for the different pagodas. The latter one has been a reliable method for gathering dynamic behaviours on site. So, the result from the latter one would be the best choice for modifying corresponding model parameters.
Considering the influence of the cornice of pagoda, interior details, material properties, the damaged condition and reinforcement, modify the stiffness of model compared with the experimental result on the frequency and vibration. Stiffness is determined by the properties and the geometry size with the spatial finite element method. Therefore, the initial elastic modulus including the influence on the stiffness with damage, reinforcement, etc could be conveniently shown and modified. Here, the elastic modulus is not the real material elastic modulus. It could be named as generalized modulus.

4.4. The comparison results of Longhu Pagoda

Considering the condition of different details, the damage and material properties, the result of calculation could be quickly produced by changing the generalized modulus. After several turns of contrasting and modifying as the generalized modulus of 1~3 floor is 1260 Mpa, the generalized modulus of 4~5 floor is 637 Mpa, the nature frequencies and vibrating mode from the analytical model by the finite element software are nearly the same as the testing values. The frequency of simulation and the dynamic test are shown in Table 4, and the vibrating mode are shown in Figure 5-6.

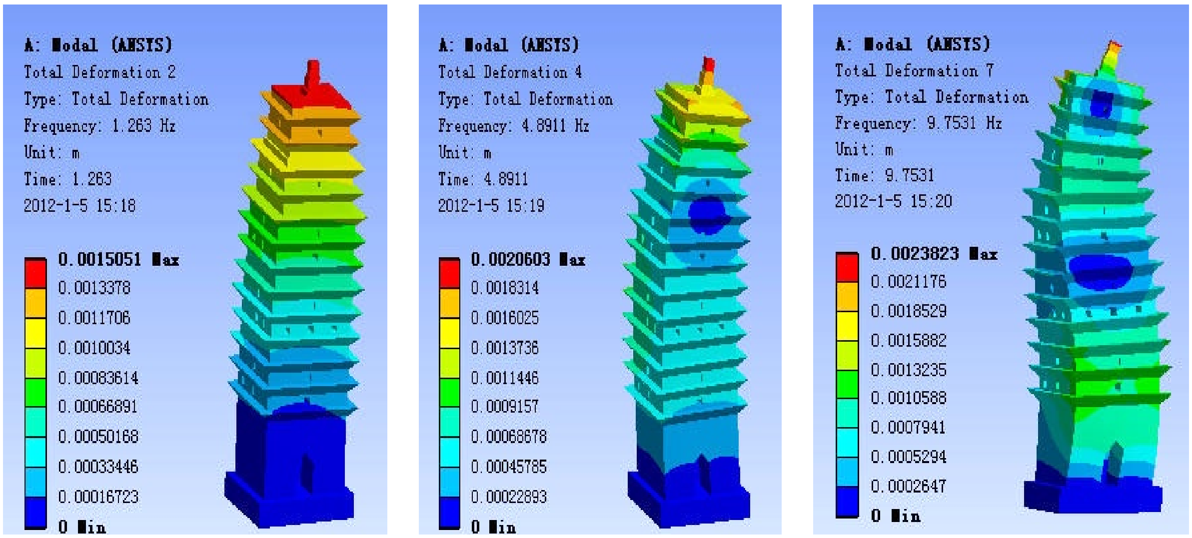

Fig. 5 The first three vibration mode from simulation l

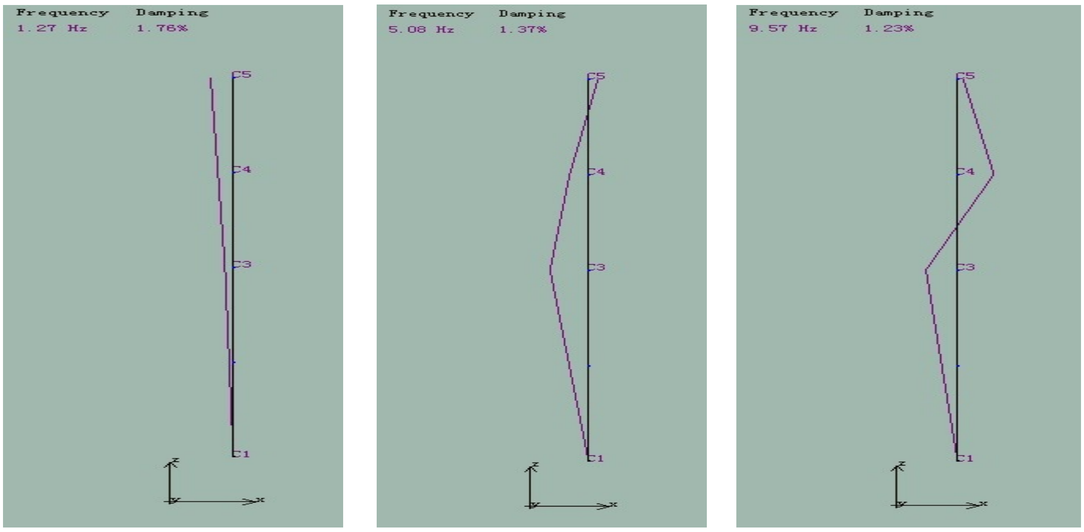

Fig. 6 The first three vibration mode from dynamic test

Table 4 Analysis of comparing of the natural frequency

vibration	Simulation results (Hz)	Experiment results (Hz)	Errors (%)
First	1.263	1.27	0.55
Second	4.891	5.08	3.7
Thrid	9.753	9.57	1.9

5. CONCLUSIONS

It is more efficient to analyse the dynamic behaviours of a pagoda in properly simplified finite element entity model. Modifying the model parameters based on the sensitive analysis of the dynamic behaviour could help to find out the sensitive parameters on dynamic behaviours of a pagoda structure by using the finite element analysis software ANSYS. It would be appropriately that smaller grid element could be finer in the connection parts or damaged parts. In order to reduce the number of the element, the details of a pagoda should be simplified. The larger scale damages and all different kinds of material properties in the analytical model should be included in. However, it is hard that the small scale damage are accurately described and presented in an analytical model. It needs more explorations.

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