Structural Analysis of Historical Constructions – Jerzy Jasieńko (ed)
© 2012 DWE, Wrocław, Poland, ISSN 0860-2395, ISBN 978-83-7125-216-7

STATIC AND SEISMIC ANALYSES OF THE MINARET IN JAM

Clemente P.[1], *Buffarini G.*[2], *Platania L.*[3], *Saitta F.*[4]

ABSTRACT

The study carried out with reference to the Minaret of Jam, which has been declared as the Afghanistan's first World Heritage Site by UNESCO in 2002, is shown in this paper. The static analysis in the present configuration has first been performed and the stability factor has been evaluated. Then the stability of the tower has been studied, in the hypothesis of increasing bending moment at the base section, assuming an elastic-perfect plastic behaviour for the soil. Finally the push-over analysis has been carried out and some consideration about the seismic performance has been discussed.

Keywords: *Leaning tower, Minaret, Stability*

1. INTRODUCTION

Monuments and historical buildings, with their cultural, historical and artistic value, need a particular seismic protection. High rise structures have usually a higher seismic vulnerability, even thought several minarets withstood earthquakes in the past. In addition, it is known that all the masonry towers present a slope, even if not apparent [1, 2]. For these reasons, towers require a detailed study, which includes the historical analysis, the knowledge of the present configuration and the analysis of the all possible collapse mechanism [3].

The Minaret of Jam (Fig. 1) has been the focus of conservation and research for over 45 years and in 2002 UNESCO declared Jam as the Afghanistan's first World Heritage Site [4]. The Minaret is merely the most visible element of the surrounding archaeological site, largely uninvestigated. It is located in a narrow valley, at the confluence of the Hari Rud river with the Jam Rud river.

The first official record of the Minaret dates from 1944, in the journal Anis of the Society for Afghan History. It was rediscovered in 1957 by André Maricq of CNRS, France [5]. A topographical survey was carried out in 1959 by Fischer and co-workers from the University of Cambridge. Finally, in 1962 the Italian architect Andrea Bruno conducted an architectural survey with proposal for restoration and in the following years the reinforcement of the base of the Minaret was done with a stone and timber dam [6]. At that time the Jam rud and the Hary rud were just bordering the structure. Between 1971 and 1975 further surveys were performed to determine the degree of leaning of the Minaret, which results not so important to compromise the stability of the monument [7, 8]. In 1978 UNESCO financed the intervention by means of large stone-filled metal gabions to protect the base of the tower from flooding and erosion and facilitate the archaeological diggings. After the civil war, Najimi also recommended the construction of a gabion [9], which was built in 1999 along the Jam river. In fact, the water was flowing very close to the Minaret producing the under-excavation of the foundation and, as a result, the reduction of bearing capacity.

Between 2001 and 2007 several technical mission of UNESCO experts (among these Bruno, Margottini and Orlando) were done. A consolidation work was completed, consisting in the reconstruction of the missing parts of the octagonal basement (Fig. 2). The work was finalized to the reorganization of portions of masonry seriously deteriorated, using cooked fire-bricks similar

---

[1] PhD, ENEA, Rome, Italy, paolo.clemente@enea.it
[2] Dr. Eng, ENEA, Rome, Italy, giacomo.buffarini@enea.it
[3] Freelance Architect, Rome, lauraplatania@gmail.com
[4] PhD, ENEA, Rome, Italy, fernando.saitta@enea.it

to the original ones, in terms of form, size, rigidity and strength, with the aim of connecting the new elements to the existing masonry in order to achieve the maximum homogeneity of the repaired wall. These intense activities, which restored the base sections of the tower, allowed to improve the global stability.

Unfortunately, a severe flood occurred at the end of April 2007, destroying part of the gabions, very close to the structure. At that time, the first Author was involved in the stability analysis of the tower and in the definition of a list interventions necessary. The first results were presented in a meeting at ICCROM in Rome, in June 2008. Then a complete analysis was carried out and shown in a parallel paper [10]. In this paper the main results are presented. The stability analysis of the tower in the present configuration is first shown, and then in the hypothesis of increasing bending moment at the base section. Finally, the seismic performance is analyzed.

Fig. 1 The minaret of Jam

Fig. 2 Consolidation works at the basement

2. THE MINARET AND SITE CHARACTERISTICS

Jam is located in the Ghur province of western Afghanistan, an inaccessible mountainous region about 260 km East of Herat. The geographic isolation and harshness and the impoverished environment had a profound influence on the historical and cultural developments in Ghur. The Minaret probably marks the site of the ancient city of Firuzkuh, the capital of the Ghurid dynasty that ruled Afghanistan and parts of northern India, from Kashgar to the Persian Gulf, in 12th and 13th centuries. Ghurid fortunes declined in 1202, and finally the Mongol armies completed the destruction in 1222, under Ghengis Khan. The remains of castles and towers of the Ghurid settlement are on the opposite bank of the Hari river and North of the Minaret. Remains of fortifications are East of the Minaret, which was built on the south bank of the Hari river at the intersection of two canyon-like river valleys.

The Minaret probably started to lean after it had been completed. It rises to a height of 60.41 m from the level of the ground and is composed by four tapering cylindrical shafts. The tower structure, constructed with bricks and lime mortar, is composed of two parts: the external one, made of four tapered truncated blocks with ring cross-section, one upon the other, and the internal one, composed by a tapered block with circular cross-section, that ends at the height of 43.50 m. Two spiral staircases are between the external and the internal structures, and also join them.

The double helicoidal staircase runs from the base to the first cylindrical tier, leading to the top; there are windows at regular intervals along the staircase (Fig. 3). Above the staircases there are six square vaulted brick platforms, connected by narrow steps projecting from the curving wall of the minaret.

The first balcony, 34.55 m above the present ground level, was made of bricks and reinforced with projecting wooden beams; however, in its present condition it is difficult to determine its original form. A wood staircase was probably in the ending part, which allowed to reach the top of the Minaret (Fig. 4), where there are six arches, which supported a small cupola, now disappeared.


(a) (b)

Fig. 3 The helicoidal stairs

These platforms are anchored at each corner into the ribs protruding from the external shaft (Fig. 4). The exterior of the shaft of the Minaret is completely covered with geometric decoration in relief laid over the plain structural bricks. The inscription records the date of building. The first cylinder is the most decorated. The text is the entire Sura of Maryam, the 19th chapter of the Koran, which tells the stories of the prophets. There are further Kufic inscriptions between the first and second balconies and above the latter.


(a) (b) (c)

Fig. 4 The platforms (a and b) and the top of the Minaret (c)

A detailed description of the geological characteristics and the experimental analyses carried out is given by Bruno and Margottini [11]. The local geology is strongly affected by mountain environment, with a bedrock composed by massive granite/granodiorite, and covered by few orders of alluvial deposit. This is characterized by few orders of sediments, related to different alluvial stages and is mainly composed by sand and gravel, etheroclastic and etherometric with an abundant presence of sand matrix.

A geophysical campaign, performed in March 2005 and including georadar, seismic refraction profiles and vertical electric soundings [12], gave information about the local stratigraphy and the typology of foundation. It also pointed out the presence of buried archeological remains at about 2.0 m from the surface.

The seismic refraction analysis evidenced that the soil can be schematized in 3 layers:

- the first layer, starting from the topographic surface, has a variable thickness from 2 to 10 m and can be correlated with the weathering layer; it shows a velocity of the seismic waves of about 0.5 km/s,
- the second layer can be correlated with the sandy-gravel formation and shows a seismic wave velocity of about 2.5 km/s,
- the third layer, associated to the granite formation, shows a velocity of about 4.5 km/s.

The bedrock near the Minaret is at a depth of about 20-25 m. These results were confirmed by the vertical electric soundings (VES).

Detailed analyses of the seismic hazard were carried out by several Authors [13, 14, 15, 16]. The earthquakes historical and instrumental catalogues, from 25 A.D., confirm that no large earthquakes have occurred at the site. The nearest epicenter cited in the literature lies 170 km from Jam. However, the catalogues are certainly not complete, especially in remote areas of the country. Actually, Jam lies in proximity of the Herat fault, a prominent right lateral strike-slip lineament running along north Afghanistan for about 1100 km, which for most of its length was not associated with occurrence of earthquakes. Residents of the neighboring village have experienced earthquakes, therefore earthquakes of moderate intensity cannot be totally ruled out, as demonstrated by the recent tectonic uplift in the region. So the region was classified as weak seismicity area.

In the elastic spectrum [16] relative to the return period of 475 years, the amplitude varies from 0.06 g and 0.04 g for periods between 1.0 and 2.0 s. The amplitudes are much higher with reference to higher values of T_R, which are more consistent with the historical importance of the structure.

3. THE PRESENT CONFIGURATION

The Minaret (Fig. 6) is composed of four tapered cylindrical shafts. The diameter varies from 9.74 m at the ground level to 2.19 m at the top. The cross-section has octagonal form for the first 4.0 m, but a circular cross-section has been considered in the analysis, neglecting the external decorations. The actual position of the basement is not known in detail, but from the existing surveys we can suppose that it is at least at 2.60 m under the entrance level. So the total height is 65.5 m, and the height from the level of the ground is about 60.40 m.

Detailed information about the characteristics of the soil are not available, so a wide parametric analysis was carried out, whose results are shown in another paper [10]. Here, in order to have a first glance at the stability assessment of the tower, the results obtained for average values of the strength are presented. In more details, an elastic-perfect plastic behavior with a compression strength $f_t = 1.0$ N/mm^2 was assumed for the soil; masonry has a rigid-plastic behavior with a compression strength $f_m = 4.0$ N/mm^2. Both soil and masonry do not have any tensile strength.

Just after the construction the minaret was not leaning. The axial force at the basement was equal to the total weight $N = W = 33.3$ MN, which corresponded to a uniformly distributed compression stress $\sigma_m = 0.447$ N/mm^2. This is also the average value in the present situation.

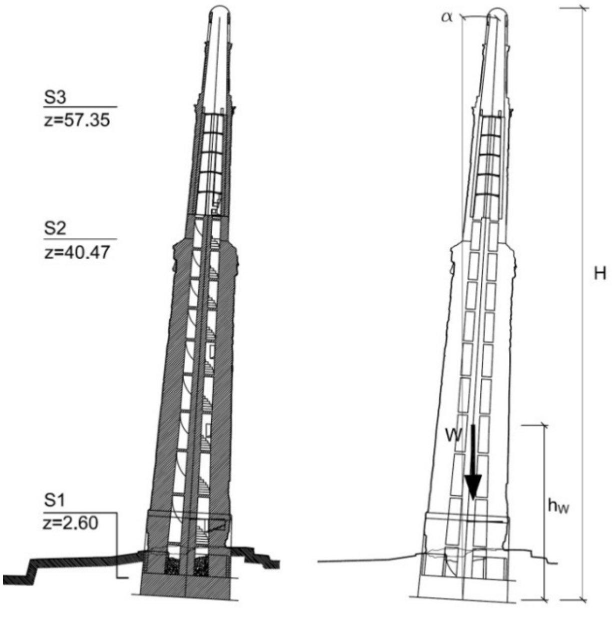

Fig. 5 The minaret of Jam

With the present rotation $\alpha^* = 0.059$ rad the resultant W, applied at the centre of gravity at $h_w = 19.84$ m from the base section, produces there an external moment $M^* = W \cdot h_w \cdot \alpha^* = 39.19$ MNm. The base section at the interface between the Minaret and the soil is fully compressed. The maximum and minimum values of the stress are $\sigma_{max} = 0.824$ N/mm^2 ($< f_t$) and $\sigma_{min} = 0.011$ N/mm^2, respectively. These value are consistent with the assumed soil compression strength.

The rotation α^* is actually composed by an inelastic component α_0, which was the rotation imposed may be by a soil collapse, and the elastic component ($\alpha^* - \alpha_0$), which followed the soil subsiding. In the hypothesis of elastic behaviour, these are related by the relationship:

$$\alpha_0 = \left(1 - \frac{6\left(1-\nu^2\right)}{E \cdot D^3} \cdot W \cdot h_w\right) \cdot \alpha^*$$

For reasonable values of the ratio $E/(1-\nu^2)$, which characterizes the elastic soil, α_0 happens to be very close to α and the stability factor [17] is high enough to ensure the stability of the Minaret.
In order to analyze the structural behaviour of the tower under increasing horizontal loads, for example simulating the seismic actions, it is important to analyze the actual behaviour of the tower cross-sections. Assume the tower to be rigid and the deformability concentrated at the interface section between the structure and the soil.

4. LIMIT BEHAVIOUR OF THE BASE CROSS-SECTION

The limit states of a circular cross-section subject to bending and compression are the following:

- limit state 1 (*LS1*): the compression stress is zero at one edge;
- limit state 2 (*LS2*): the compression stress is equal to f_t at one edge;
- limit state 3 (*LS3*): the effective cross-section is uniformly compressed with stress f_t.

These limit states are represented in figure 10 by the corresponding limit domains, plotted in the non-dimensional shape. For $f_t = 1.0$ N/mm^2, $N_0 = 74.51$ *MN* and $M_0 = 77.00$ MNm.

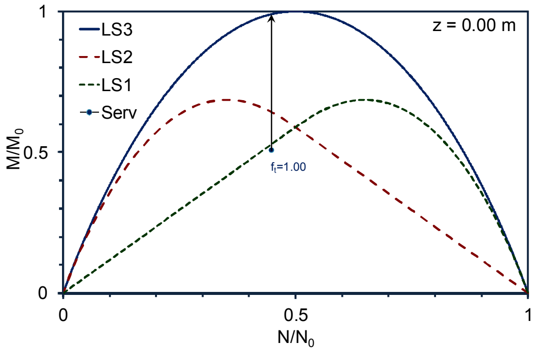

Fig. 6 Limit domain of a circular cross-section and stress points for different values of f_t

It is worth pointing out that in our case study $N < N_0/2$ and *LS1* occurs first if $N < N_0/2$.
The loading path under seismic actions is represented by the vertical line passing through the stress point. Following the stress path, passing through the stress point relative to the case $f_t = 1.0$ N/mm^2, the limit cases shown in Fig. 7 and the corresponding values are listed in Tab. 1.

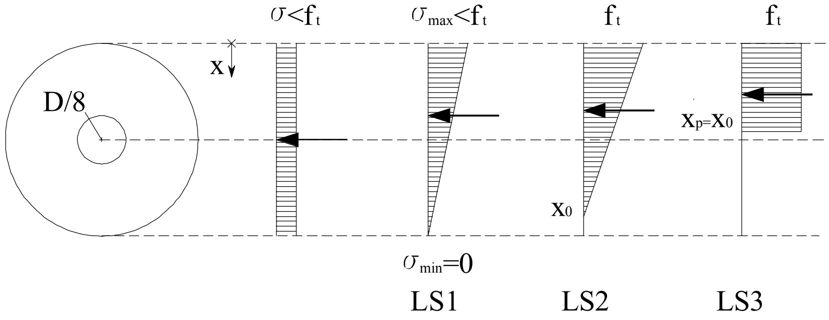

Fig. 7 Stress distributions in the limit states at the base cross-section

Tab. 1 Stress values at the base cross-section in the limit states

	N/N_0	x_0/D	M/M_0	σ_{max} (N/mm^2)	σ_{min} (N/mm^2)
Initial	*0.447*	-	0.000	0.447	0.447
LS1	*0.447*	1.000	0.527	0.894	0.000
LS2	*0.447*	0.900	0.644	1.000	0.000
LS3	*0.447*	0.458	0.989	1.000	0.000

5. BEHAVIOUR UNDER INCERASING HORIZONTAL ACTIONS

For each couple (x_p, x_0), N being fixed and equal to W, the bending moment and the eccentricity can be evaluated. The rotational stiffness is:

$$k_\alpha = \frac{E}{\left(1-\nu^2\right)} \cdot \frac{B^3}{I_\alpha}$$

The factor I_α is a function of the foundation shape and soil model and B is a characteristic geometrical parameter. In the hypothesis of elastic-perfect plastic soil, the portion of the foundation, which contributes to the stiffness, is that in the elastic phase only. Therefore, in the following the foundation shape has been assimilated to a rectangular one and the corresponding values of I_α in the literature, relative to an elastic soil, have been assumed [17].
These considerations allow a first approach to the non linear analysis of the tower, for the assumed rigid model of the structure. For each position of the neutral axis, the stress distribution and the bending moment at the base can be evaluated and so the corresponding stiffness. The rotation can be evaluated by using a step by step procedure. In Fig. 8 the relation between M and α is plotted for $E/(1-\nu^2) = 100$ N/mm^2.
It is worth noting that the curve is linear up to M^*, so the stability factor at the present configuration is that before evaluated. In fact, the base cross-section is fully effective with $\sigma_{\max} < f_t$ in the present configuration, so $I_\alpha = 6$ and k_α ($= k_{\alpha 0}$) is constant.
If the increment of the bending moment M is related to a horizontal load acting in the plane yz, which is the vertical plane containing the leaning axis of the tower, whose resultant F is at h_F from the base section, from the equilibrium condition we deduce the relation between F/W and the rotation α, in which the eccentricity e is a function of α:

$$\frac{F}{W} = \frac{1}{h_F}\left[e - h_w \cdot \alpha\right]$$

In figure 15 the characteristic curve of F/W versus the generalized displacement d [18] are plotted for $E/(1-\nu^2) = 100$ N/mm^2 in the hypothesis of horizontal load due to a seismic acceleration. The limit values are very low if compared with the spectral amplitudes at the site [16].

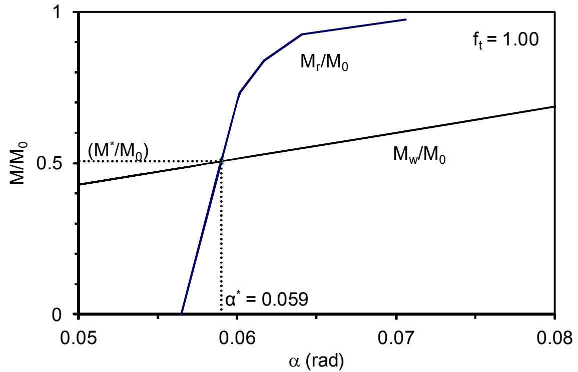

Fig. 8 M/M_0 versus α for $E/(1-\upsilon^2) = 100$ and $f_t = 1.0$ N/mm^2

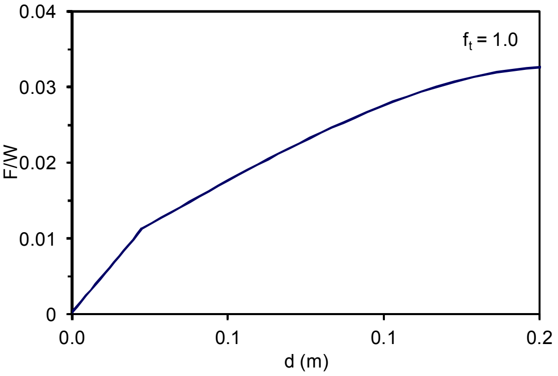

Fig. 9 F/W versus displacement d at h_F for $E/(1-\upsilon 2) = 100$ and $f_t = 1.0$ N/mm^2

The hypothesis of $f_t = 1.00$ N/mm^2 seems to be the most suitable. Anyway, a deeper investigation is advisable for both the soil characteristics and the actual deep of the foundation basement.
A complete investigation cannot neglect the analysis of the other sections of the tower. The results obtained demonstrated the safety of the other sections in comparison with the base section [10].

6. FEM AND SEISMIC ANALYSIS

A finite element model, which reproduces the geometrical characteristics of the real structure (Fig. 10), has been set up, in order to better verify what previously done and to carry out a push-over analysis accounting for the flexibility of the tower. At the lower part, up to 4.0 m above the ground level, the actual cross-section has octagonal form, but a circular cross-section has been considered in the model, neglecting the external decorations. The main structure, i.e., the external wall and the inner shaft, have been modeled by means of solid elements. The regular mesh has been locally modified in order to accounting for the effective openings and thickness. The two helicoidal stairs have been modeled by means of frame elements that represent the steps, spaced of 45°. The stiffening elements and the vaults have been modeled by means of shell elements, having axial and flexural stiffness; the stairs and the openings at the upper levels have not been considered. Masonry has a Young's modulus $E = 3200$ N/mm^2 and a weight per unit volume $\gamma = 16$ kN/m^3.

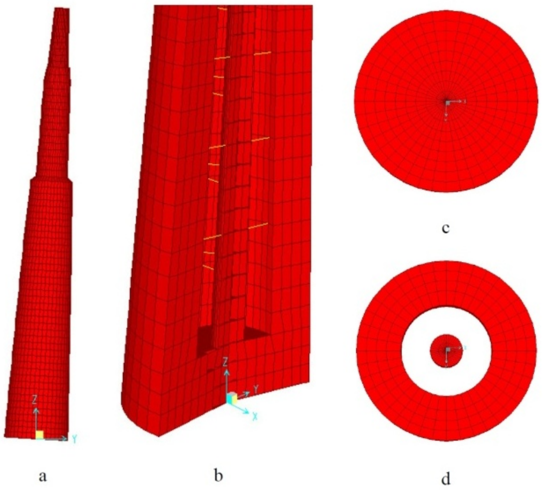

Fig. 10 FE model: a) vertical view, b) detail of the lower portion, c) interface with the soil, d) cross-section at 2.60 m

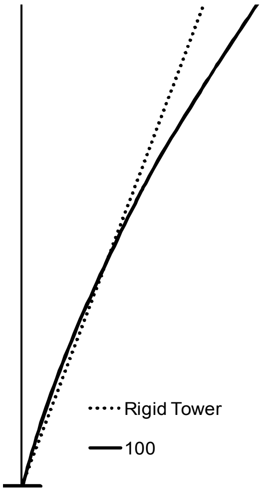

Fig. 11 Modal shapes for the rigid and elastic models

The static analysis has first been carried out. Link elements, which have no tensile strength and an elastic modulus that reproduces the soil stiffness, have been introduced, in the vertical direction, at the interface between structure and soil. Besides, only the central joint of the base cross section has a horizontal restraint. Obviously the static analysis has given results very similar to those of the analysis shown in the previous paragraphs.

The modal analysis produced the resonance frequencies and the modal shapes of the structure. These are plotted in Fig. 11 for the two cases of rigid tower and elastic tower. In the second case the soil is characterized by $E/(1-\nu^2) = 100$ N/mm^2. The natural period of vibration is strongly influenced by the soil stiffness, varying from 1.0 s for $E/(1-\nu^2)=\infty$, which is relative to the model fixed at the base, to 1.6 *s* for $E/(1-\nu^2) = 100$ N/mm^2. It is worth noting that when the period increases, the spectral amplitude S_e decreases with $1/T$ law.

Then the push-over analysis was performed by considering a load distribution acting in the *yz* plane, which is the vertical plane containing the leaning axis of the tower. The load distribution, which should be consistent with the first modal shape, was very influenced by soil stiffness. The analysis, carried out keeping the vertical load constant and increasing the horizontal acceleration with a step of 0.01*g* and accounting for *P*-delta effect, gave results very similar to the previous ones.

7. CONCLUSIONS

With the hypotheses assumed about soil and masonry characteristics, the Minaret is stable under dead loads. A high value of the soil strength has been assumed, which could be justified by the actual depth of the foundation, which could be much higher than the hypothesized one. Anyway, the stability check is very sensitive to the soil properties, which should be investigated in detail, as well as the foundation structures and their depth. As a matter of fact, if the soil strength is lower than the assumed value, a wide portion of the interface section between the Minaret and the soil could be yielded. This imply that the structure could be closer to the collapse point than it appears.

The seismic check confirmed the high vulnerability of the tower to earthquakes. In fact the maximum acceleration value are much lower than the spectral amplitudes at the site for usual return periods but also the seismic vulnerability is very sensitive to the soil characteristics.

Obviously, the hazard due to soil erosion related to the two rivers requires an effective preservation and a continuous control of the protection systems, which should be deep enough. This kind of intervention is certainly more urgent with reference to the others.

The retrofit should be based on a deeper investigation especially on the foundation size and depth but also on the mechanical properties of the masonry. If necessary, the realization of small diameter inclined piles should be a suitable solution for the soil-structure interaction, especially against the effects of horizontal actions. This intervention would allow both to enlarge the base section and to better restrain the tower to the soil. Besides the soil characteristics could be improved by means of injections. The retrofit of the structure could be made by means of the insertion of vertical steel and/or shape memory alloys elements. In any case the intervention on the structure must preserve the historical and artistic value of the Minaret.

ACKNOWEDGEMENTS

The authors are very grateful to UNESCO World Heritage Centre, Paris (France) and the UNESCO Office in Kabul for encouraging and supporting the present investigation. The Authors are particularly grateful to Prof. Andrea Bruno and Prof. Claudio Margottini, Experts and Advisors of UNESCO, and thank very much Prof. Anna Sinopoli for her very helpful contribution to this study.

REFERENCES

[1] Heyman J. (1992). “Leaning towers”. Meccanica, Springer, Vol. 27, No. 1.
[2] Macchi G. (1998). “Seismic Risk and Dynamic Identification in Towers”, Proc. Monument ’98, Workshop on Seismic Performance of Monuments, Lisbon, Portugal, November 12-14.
[3] Burland J. B., Jamiolkowski M., Viggiani C. (2003). “The stabilization of the leaning tower of Pisa”. Soil and foundations, Vol.43, No 5.
[4] Bruno A. (2003). “The Minaret of Jam, Afghanistan”, World Heritage, 29, UNESCO Publishing and Ediciones San Marcos.
[5] Maricq A., Wiet G. (1959). "Le Minaret de Djam: la Décou-verte de la Capitale des Sultans Ghorides (XIIe-XIIIe siè-cles)". Paris: Délégation archéologique française en Afghanistan.
[6] Bruno A. (1963). “Notes of Discovery of Hebrew Inscriptions in the vicinity of the Minaret of Jam”. East and West, New series, Vol.14, No. 3-4, ISMEO, Roma.
[7] Bruno A. (1979). "The Minaret of Jam. A UNESCO project to restore an historic Afghan monument". UNESCO Courier 20 Oct: 32-34.
[8] Bruno, A. (1981). "Background and Justification of the Project and its immediate Objectives". In Restoration of Monuments in Herat. UNDP/AFG/75/022. Unesco.
[9] Najimi, W.N. (1995). "Jam", An Assessment Mission Report, July-August 1995
[10] Clemente P., Buffarini G., Platania L., Saitta F. (2012). Stability analysis of the leaning Minaret in Jam. To be published.
[11] Bruno A., Margottini C. (2011). "The Jam Minaret in Afghanistan: Emergency Works and Long Term Stabilization". Proc. 14th Asian Regional Conference on Soil Mechanics and Geotechnical Engineering, Hong Kong May 23-27.
[12] Orlando L. (2005). Geophysical survey in the minaret of Jam, Afghanistan". UNESCO Internal Report.
[13] Ambraseys N., Bilham R. (2003). The tectonic setting of Baniyam and seismicity in and near Afghanistan for the past 12 centuries. UNESCO.
[14] Ambraseys N., Bilham R. (2003). “Earthquakes in Afghanistan”. Seismological Research Letters, Vol.74, No 2.
[15] Menon, A., Lai G.C. (2004). Seismic hazard assessment of Jam in Afghanistan. 13th World Conference on Earthquake Engineering Vancouver, B.C., Canada, August 1-6, 2004. Paper No. 2314.
[16] Menon, A., Lai C.G., Macchi G. (2004). "Seismic Hazard Assessment of the Historical Site of Jam in Afghanistan and Stability Analysis of the Minaret." Journal of Earthquake Engineering 8: 251-294, Sp. Iss.1.
[17] Viggiani C. (1999), Fondazioni, Hevelius Edizioni, Benevento (in Italian).
[18] Raithel A., Clemente P. (1989). “La verifica delle strutture snelle: il grado di stabilità”. Proc., XII Convegno C.T.A. (Capri, October 22-25), C.T.A., Milan, 983-992 (in Italian).