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MACROELEMENT IDENTIFICATION OF MASONRY CHURCHES BY MEANS OF THEIR DYNAMIC PROPERTIES

Peña Fernando[1], *Meza Miguel*[2], *Chávez Marcos*[3]

ABSTRACT

A novel approach for the identification of macroelements of old masonry churches by means of their dynamic properties is presented. A macroelement can be defined as the part of a church that can be evaluated independently of the rest of the structure. The macroelements are commonly proposed by mean of the damage of the structure. In general, the decomposition of the structure into macroelements is based on empirical observations of the damage of the church after an earthquake. Therefore, the independent behaviour of the macroelement is due to the separation of the macroelement from the rest of the structure after damage. In other hand, when numerical models are used to assess the seismic behaviour of old masonry structures, the dynamic response of the macroelements should be equal as the dynamic response of this part of the entire structure. In this way, a novel approach for the identification of macroelements considering the dynamical properties of the entire structure is proposed. The approach shows that the macroelements can be defined even when they are attached to the rest of the structure and their definition only depend on the dynamic properties of the undamaged church.

Keywords: *Macroelement, Churches, Masonry, Dynamic Properties*

1. INTRODUCTION

The seismic assessment of old masonry churches is not an easy task, mainly due to the fact that the churches are large structures with complex geometries and built with materials with highly nonlinear behavior. Therefore, several simplified methods have been proposed for assessing the vulnerability of churches, which are based on the collection of information on past earthquake damages. One of them is based on dividing the whole structure into elementary parts or substructures, commonly named macroelements. This method is also based on the assumption that possible extended damage may be seen as the combination of damage patterns on single substructures [1-3]. In this context, a three-dimensional complex structural system is considered as an ensemble of a series of simpler components [4, 5].

The use of macroelements to assess the seismic behavior of old masonry churches has demonstrated to be useful. It reduces the time of preparation and analysis of numerical models and interpretation of results. Moreover, macroelements allows the easy identification of damage and partial collapse of parts of the structure. The application of local structural evaluation models considering the seismic response of the macroelements is strongly suggested, rather than conventional evaluation of the overall structural behavior [6].

This technique has been widely used by many authors. For example, Casarin and Modena [7] have used the concept of macroelement for the definition of the structural safety of a historical masonry structure. A complex building, the Santa Maria Assunta (Our lady of the Assumption) Cathedral in Reggio Emilia, Italy, was studied in order to evaluate its structural behavior by using different

[1] Associate Professor, Instituto de Ingeniería, Universidad Nacional Autónoma de México, fpem@pumas.iingen.unam.mx
[2] Ph.D. Student, Instituto de Ingeniería, Universidad Nacional Autónoma de México, jmeza.mendez@gmail.com
[3] Posdoctoral Researcher, Instituto de Ingeniería, Universidad Nacional Autónoma de México, MChavezC@iingen.unam.mx

investigation and analysis methodologies applied to numerical macroelements. Recently, Lucibello et al. [8] analyzed four churches strongly damaged during the L'Aquila earthquake. In this work, each building was analyzed in the linear elastic range with a 3D finite element model, in order to determine the static and dynamic properties, the distribution of internal forces, the points of stress concentration and the strength demand on each macroelement. Subsequently the complex 3D structure was decomposed in its constituting macroelements, and each macro-element was analyzed in the non linear range, up to collapse, in order to determine its horizontal strength capacity.
In general, the decomposition of the structure into macroelements is based on empirical observations of the damage of the church after an earthquake [1, 2]. Therefore, the independent behavior of the macroelement is due to the separation of the macroelement from the rest of the structure after damage. In other hand, when numerical models are used to assess the seismic behavior of churches, the dynamic response of the macroelements should be equal as the dynamic response of this part of the entire structure.
In this way, a novel approach for the identification of macroelements considering the dynamical properties of the entire structure is proposed. Thus, the approach shows that the macroelements can be defined even when they are attached to the rest of the structure and their definition only depend on the dynamic properties of the undamaged church.

2. DEFINITION OF THE MACROELEMENTS

The study made by Doglioni et al. [1] can be considered the first systematic study about the macroelements in old masonry churches. They observed that churches generally have geometric configurations that define sections with different stiffness, which once damaged by an earthquake has an independent dynamic behavior. These sections are not necessarily related to the architectural or structural configuration. They are related to the damage caused by the earthquake and have been called macroelements. The typical macroelements of Italian churches identified by Doglioni et al. are [1]: facade, towers, apse, nave, side wall, triumphal arch and side chapel (Fig. 1).
Until now, the definition of the macroelements is based on the criteria of the engineer. This criteria is mainly based on past experience of the seismic damage of the churches. As the macroelements have been defined after the damages caused by an earthquake, in this work they will be named ***damage macroelements***. This means that it is necessary to know the typical damages patterns and/or collapse mechanism of the churches.

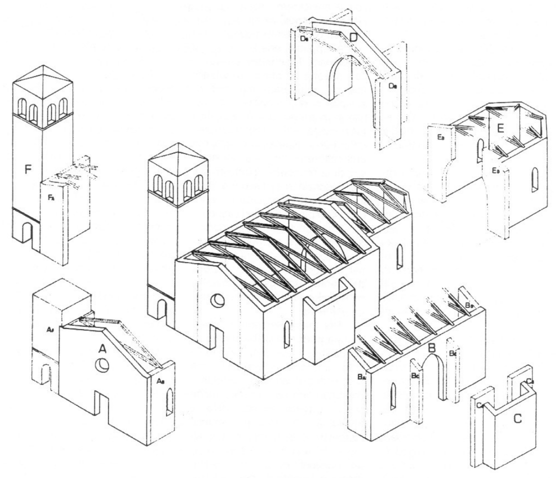


Fig. 1 Typical macroelements defined by Doglioni et al. [1]

The concept of *damage macroelement* is the fundamental basis to create a numerical model. The numerical model contains the *damage macroelement*, but it is necessary considering a bigger part of the structure so that it may be possible to develop the configuration of the damage during execution of the non-linear analysis. In this work, this numerical model will be called ***numerical macroelement***.
The main problem with the numerical macroelements is the application of boundary conditions. The structural elements of the churches are usually continuous and may present interactions on the contact areas, which should be considered in the model. Some authors consider the poor continuity between parts of the structure to define the numerical macroelement. Due to this, the use of macroelements is

considered as an easier way to describe the seismic behavior of a church and also the most correct one [2]. However, the macroelement to be defined so, probably it is under or overestimating their strength or failure mechanism because they do not consider the continuity conditions. In addition, Petrini et al. [3] suggest that unintentional or undetermined restraints can change significantly the ultimate strength of the model.
Therefore, there are not methods or criteria well based to define the dimensions of the numerical macroelements with enough reliability to determine the response of the analyzed parts of churches. Thus, the aim of this paper is to propose a methodology to define the correct dimensions of the numerical macroelements in order to have reliable results that can be compared with the damaged macroelements.

3. MACROELEMENT IDENTIFICATION

The proposed criteria for the macroelement identification is based on the Modal Mass of the structure. Because, as the modal mass is the mass distribution in each degree of freedom and whose magnitude is the normalized to the total mass contained in particular mode of vibration, this is used to estimate the amount of mass contributed by each numerical macroelement. Considering the partial modal mass of each macroelement, the total sum should be equal to the modal mass of the entire model. Computer codes generally normalized the modal mass in order to obtain a unitary modal mass. The modal mass can be calculated by means of [9]:

$$M_n = \{\phi_n\}^T [M] \{\phi_n\} \tag{1}$$

where: M_n = modal mass of the *n* mode, *[M]* = mass matrix, $\{\varphi_n\}$ = amplitude vector of mode *n*.

If a macroelement contributes with a greater amount of modal mass for a particular mode, it means that the modal behavior is due to that particular macroelement. Therefore, the macroelement has an independent behavior of the rest of the structure. If more than one macroelement provide greatest amount of modal mass by one particular mode, it means that these macroelements cannot be analyzed separately, so that there would be a larger macroelement.
On the other hand, modal correlation (Ec. 2) indicates the relationship between two modal shapes. The correlation uses the generalized coordinates of the mode of a macroelement (φ_A) and of the full church. A high correlation exists when is between 0.80-1.00. Low correlation when is between 0 - 0.40 [9]. Zero (“0”) indicates that the modes are orthogonally and one (“1”) that the modes are equals.

$$Corr(\phi_A \phi_B) = \frac{(\phi_A^T \phi_B)^2}{(\phi_A^T \phi_A)(\phi_B^T \phi_B)} \tag{2}$$

With the use of modal correlation it is possible to identify if the modes that are presented in the church are also presented in the macroelements.

Thus, the proposed methodology to obtain the macroelements by means of their dynamical properties follows the next steps:

1. Macroelements are proposed.
2. The dynamical properties of the church are obtained.
3. The contribution to the modal mass of each proposed macroelement is obtained. The modal mass is obtained considering the entire model of the structure.
4. Identification of the main modes for each macroelement is performed.
5. The dynamical properties of each macroelement model are obtained.
6. The shape modes of each macroelement is compared with the shape modes of the full model by means of their modal correlation.
7. If the period and modal shape of the macroelements are similar with the modes of the whole structure, then the proposed macroelement is valid. Otherwise, it is necessery to redefine a new macroelement.

The application of this methodology is explained in the next section by using a typical colonial Mexican church model.

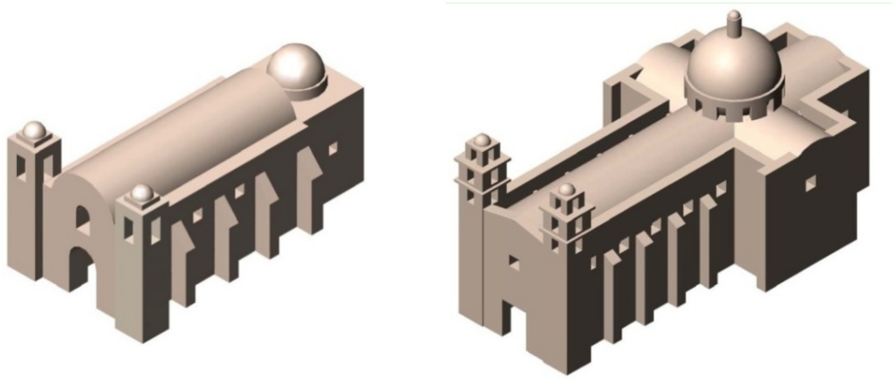
4. EXAMPLE OF APPLICATION

4.1. Typical colonial Mexican churches

Thousands of churches were built in Mexico from XVI to XVIII century, and persist to date in rather good conditions; they vary in size and architectural sophistication, but follow some basic typologies. It is often found that these buildings have been modified throughout the years due to a variety of factors, such as architectural fashions and structural problems. Also, the evolution of their architectural features has been the experience of damage suffered from earthquake activity. Churches can be distinguished by the type of vault that covers the nave. There are basically four types of vault: continuous barrel vaults, barrel vaults with arches, barrel vaults with lunettes, and ribbed cross-vaulting. The first type is the most representative. Structural elements of historical buildings were made of a masonry constituted by stones of different size and shape, agglutinated by lime-sand mortar. In earliest buildings this kind of stone masonry was used in all structural elements; by the end of the sixteenth century, vaults, domes, and arches were most commonly built with bricks [10].

The simplest among them are rather small parochial churches which are found in every “barrio” of Mexican towns and villages. It is in the area of the Pacific Coast, and more specifically in the state of Oaxaca, where the recurrent destruction of the early construction produced an evolution towards low rise, heavily buttressed buildings with scarce external ornamentation (Fig. 2a). In other regions the lower concern for seismic failures favoured taller and more slender constructions, as for example in the state of Puebla (Fig. 2b).

The proposed criteria for the macroelement identification was applied to a typical Mexican church of Oaxaca's town [10]. The Oaxaca’s church has only one rectangular nave, with a simple façade and one or two small bell towers. Dimensions in plan are 15 × 28 m^2, and the heights of the vaults and bell towers are 11 and 14 m, respectively (Fig. 2a). The belfry has only one body. The roof has barrel vaults and a hemispheric dome on the apse. Trapezoidal buttresses are placed along the longitudinal walls of the nave and in the apse. In the first bay there is an intermediate floor for the chorus.



a) Oaxaca church b) Puebla church

Fig. 2 Typical Mexican churches [10]

4.2. Numerical model

Modal analysis was performed through a finite element model to identified periods and the most important mode shapes according to the amount of modal mass. The first 100 modes should be considered because they are enough to reach a mass participation of 90%. Material properties required for the analysis are shown in Table 1. The “X” axis of the church coincides with the longitudinal axis of the model, the “Y” axis with the transverse axis, and the “Z” axis with the vertical axis.

Table 1 Material properties used for the analysis

Properties	Stone masonry	Brick masonry
Mass density [kg/m^3]	1830	1580
Young's Modulus [MPa]	441	1177
Poisson's ratio	0.3	0.3
Compressive strength [MPa]	1.28	3.92

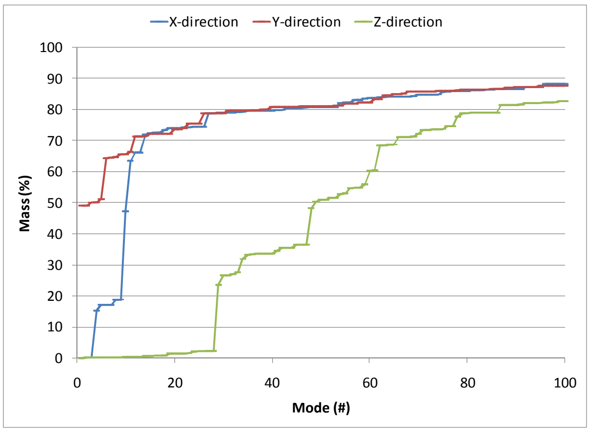


Fig. 3 Modal mass in each direction

Figure 3 shows the more important modes of the model. Modes with higher participating mass are: 1, 10 and 29 in the Y, X and Z directions, respectively. The 20 first modes in the Y-direction have the highest amount of mass, near to 73.6% of total mass. This is equal as in the X-direction, with 73.9%. In the X-direction, the 20 first modes only contribute with 1.5% of the mass. In the X-direction, the mode that provides greater mass is the mode 10, for the Y-direction is the mode 1, and for the Z-direction is the mode 29. However, although in the X-direction mode 10 has a higher content of mass, this contains only 28% of the total mass of the model. In the Y-direction, mode 1 contains nearly 50% of the total mass, therefore, this mode can be considered as a mode to be more participative in the response of the model. Because in the Z-direction the mass contribution of the first mode is about 65%, it will be necessary to consider a greater number of modes to get a seismic response near to the real.

4.3. Proposed macroelements

The first step for the identification of the macroelements is proposing them. In this particular case, the church was divided in four macroelements (Figs. 4, 5): facade, chorus, nave and apse. Then, the mass contribution of each macroelement is obtained by using Eq. 1. It must bear in mind that the calculation of modal mass is made with the complete model.

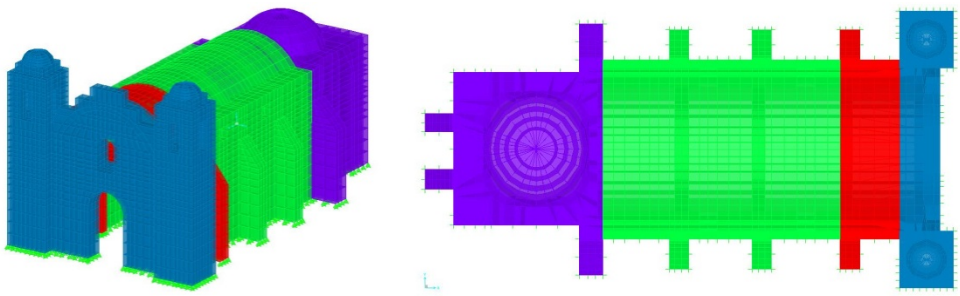
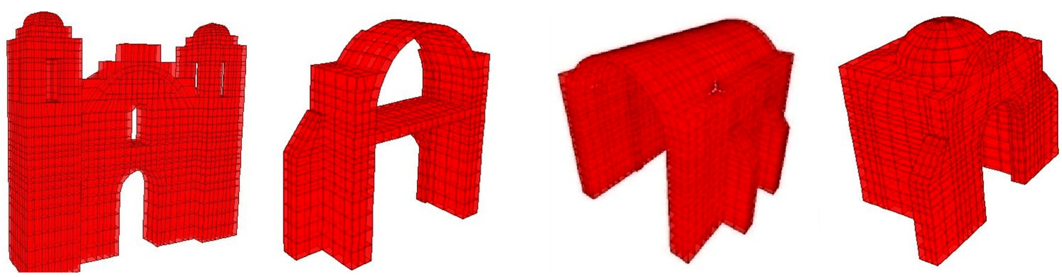


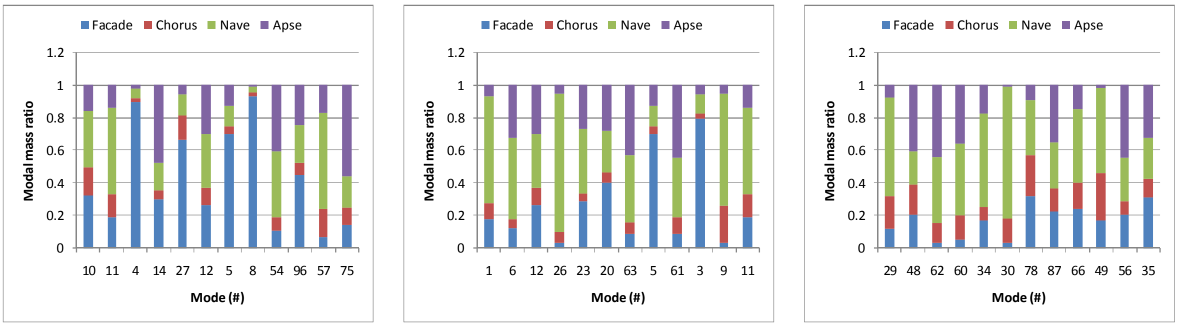
Fig. 4 Proposed division for the macroelements: façade (blue), chorus (red), nave (green) y apse (purple)



(a) (b) (c) (d)

Fig. 5 Proposed macroelements of the Oaxaca's church: Facade, b)Chorus, c) Nave and d) Apse

Figure 6 shows the modal mass for each macroelement in each direction. This figure shows that there are macroelements that provide the greatest quantity of modal mass. For example, in the X-direction, the modal mass of modes 4, 5 and 8, is due only by the vibration of the facade. Modes 11 and 57 can be considered as the nave section. Continuing the comparison in the other directions with other modes, it can be shown that the modal mass is a good indicator to find the relationship between the macroelements and mode shapes.



a) X-Direction b) Y-Direction c) Z-Direction

Fig. 6 Modal mass for each macroelement

There are modes that correspond to the global motion of the church, because the modal mass is divided into all macroelements. For example, mode 10 is due to the vibration of the macroelements facade, chorus, nave and apse. This means that it is possible to identify particular modes for each macroelement and modes of the global vibration of the church.

4.4. Identification of the macroelements

The next step consists into divide the church in macroelements. Thus, the dynamical properties of the numerical models of the proposed macroelements are calculated. In this step, the modal correlation between the macroelements and the church are calculated. The modal correlation will help to identify if the proposed macroelements are valid or they should change.

Table 2 shows the modes of the facade macroelement that have the higher correlation with the most important modes of the church. It is clear that the number of the mode for the macroelement, in parenthesis, is different than the number of mode of the church. In this case, a good correlation among modes is found. The periods between macroelement and church as well as were compared. The first mode for the facade is similar as the fourth mode of the church. However, the period is three times greater in the facade than in the macroelement. This is due to the out of plane vibration of the facade, because it is not more attached to the nave.

Table 2 Modal correlation between façade macroelement and church

Mode Direction	**Period [s] (# mode)**		**Correlation**
	Facade	Church	
X	0.3297 (1)	0.1149 (4)	0.7137
X	0.0903 (5)	0.0877 (8)	0.9699
Y	0.1269 (3)	0.1554 (1)	0.9024
Z	0.0451 (12)	0.0463 (29)	0.8607

Table 3 Modal correlation between macroelements and church

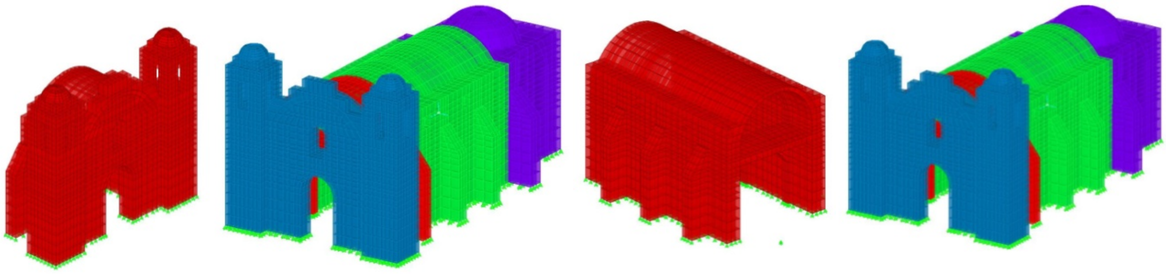
Mode Direction	**Period [s] (# mode)**		**Correlation**
	Chorus	**Church**	
X	0.1923 (2)	0.1126 (5)	0.7671
Y	0.0994 (5)	0.0720 (12)	0.7239
	Nave	**Church**	
Y	0.2721 (1)	0.1554 (1)	0.9717
	Apse	**Church**	
Torsion	0.1094 (2)	0.1126 (5)	0.9756
Z	0.0401 (13)	0.0380 (48)	0.7296

Similar comparison was made for the other three macroelements (Table 3). The apse has a very good correlation and the periods are similar between the macroelement and the church. Although the chorus and nave have good correlation, the periods are not similar in these macroelements.

When the macroelement is well defined as the apse, good correlations and similar periods among modes should be founded. Therefore, it is possible to conclude that the nave and chorus macroelements are not well defined. The facade macroelement is “almost” well defined because several modes are similar in periods and correlations are good. However, the first mode is more flexible than the similar mode in the church.

4.5. Macroelement correction

If the macroelement has not the same dynamical properties of the church, then the macroelement should be modified. In this case, the chorus is the macroelement which has the worst modal correlation. This means that it is not really a macroelement and this part of the structure should be part of the other macroelement or macroelements. As the facade and the nave have a good modal correlation, but they do not have the same modal periods with the church, then the chorus should be a part of these macroelements (Fig. 7).



a) Macroelement Facade   b) Macroelement Nave

Fig. 7 Macroelement correction

Therefore, the new facade macroelement includes the chorus, the main facade wall and the towers, while the nave macroelement includes only the arch and buttresses of the chorus. With these new macroelements the periods and modal correlation are recalculated. As Table 4 shows, a very good modal correlation is obtained and the macroelements have similar periods as the church. This means that these are the correct definition of the macroelements.

Table 4 Modal correlation between redefined macroelements and church

Mode Direction	Period (s)		Correlation
	Facade	**Church**	
X	0.1684 (1)	0.1149 (4)	0.7550
X	0.0886 (5)	0.0877 (8)	0.9651
X	0.0517 (12)	0.0480 (27)	0.8238
Y	0.1311 (3)	0.1554 (1)	0.6864
Y	0.0558 (11)	0.0553 (20)	0.8742
Y	0.1311 (3)	0.1229 (3)	0.8492
Y	0.0778 (6)	0.0803 (9)	0.9327
Z	0.0469 (13)	0.0463 (29)	0.7023
Z	0.0332 (25)	0.0332 (60)	0.8248
	Nave	**Church**	
X	0.1897 (1)	0.1149 (1)	0.9277
Z	0.0323 (47)	0.0323 (62)	0.7380
	Apse	**Church**	
Torsion	0.1094 (2)	0.1126 (5)	0.9756
Z	0.0401 (13)	0.0380 (48)	0.7296

5. CONCLUSIONS

It was shown that using the dynamical properties of the church (periods and modal shapes) is possible to identify the macroelements. Thus, the macroelement definition is not based on the possible damage of the structure, rather on the properties of the structure (mass and stiffness).

With the proposed methodology, they were identified macroelement modes and global modes. Macroelement modes are related to the motion of a particular part of the church and the global modes

with the full motion of the church. It means that macroelement modes will be important for the dynamical behavior of each macroelement and global modes for the behavior of the whole church.
It was concluded that the division of the macroelements should be in the stiffer part of the structure between macroelements. For example, the facade macroelement considers the façade wall, towers and chorus, which includes the buttress. The nave macroelement should include the triumphal arch and the arch of the chorus and the apse macroelement should consider the triumphal arch. This means that the macroelements would have common parts of the structure in the boundary.

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