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MACRO AND MICRO-BLOCK MODELLING FOR LIMIT ANALYSIS OF MASONRY STRUCTURES WITH NON-ASSOCIATIVE FRICTIONAL JOINTS

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ABSTRACT

This paper presents a comparison between the macro and micro-block modelling for the assessment of the lateral capacity of masonry walls loaded in-plane. The macro-modelling is based on the assumption that the failure involves a single crack which separates the structure in two macro-blocks and all the possible relative motions among micro-blocks are concentrated along the crack. By using a limit-state analysis approach and assuming that the interaction at surfaces is governed by Coulomb friction, two limiting conditions for the ultimate load factor are kinematically computed with simple formulations in closed form, by use of minimization routines. The macro-block model and the solution procedure are validated through three illustrative examples of shear walls existing in literature and comparisons against a micro-block model from a recently proposed strategy, herein revised, show a good agreement of the results in terms of both the ultimate load multiplier and the crack slope. The analysis of the influence of the overload on the ultimate load factor is also presented both to validate the procedure and compare the two models.

Keywords: *Limit-state analysis, Non associative frictional joints, Macro and micro-modelling*

1. INTRODUCTION

Many works on limit-state analysis of masonry block structures can be found in literature, starting from Kooharian [1] and Heyman [2], but most of them are related to the Heyman's assumption of failure modes only governed by overturning mechanisms or to the assumption of sliding in presence of associative friction [3-6]. Actually, i.e. for lateral loaded masonry walls with failure modes modelled as macro-block mechanisms, pure rotation of a whole portion with respect to the other is consistent with the discrete model assumptions only when the crack line has the same inclination as the staggering between the units, usually defined by their shape ratio. Some authors [7] have adopted this limit crack slope in the macro-block analysis of shear walls so that no resisting force due to friction may be exploited, because of the uplift during overturning and others [8] have proposed a similar simplification through a homogenisation procedure. This means that the inclination of the crack is assigned as a given parameter instead of being considered as a variable of the problem.
Nevertheless, when non-standard frictional resistances are considered in the macro-block analysis the computed multipliers generally overestimate the true ones, as shown by a recent comparison between experimental and analytical results for in-plane and out-of-plane mechanisms [9]. To account for this, the same authors have proposed a partial efficiency factor as a function of the length of the walls and the height of the failing portion, modifying the analytical formulation and predicting results closer to the experimental ones. Another way to avoid the overestimation of the capacity has been proposed by [10] through the introduction of an equivalent friction coefficient, representing the average condition of the joints when the contact surface is not ideally plane.

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The novel strategy of analysis presented here is based on a more accurate macro-block model which accounts both for the crack slope as a variable of the problem and for the treatment of the overestimation other than the introduction of corrective coefficients. This procedure is herein only applied to in-plane loaded walls, namely shear walls.
Firstly, a likely reason for this overestimation is suggested. Then, using a macro-block model and limit state analysis, the upper and lower thresholds for the possible ultimate load factor for shear walls are evaluated by considering how friction forces can develop along the crack lines. Lastly, a brief revisiting of a recently proposed micro-modelling strategy [11, 12] is also presented in order to validate the results of the macro-modelling procedure. The effect of the overload on the safety factor is analysed by using both methods, while a full parametric analysis with the influence of wall aspect, unit aspect, size ratio of units, compressive stress, overload and bond types will be presented elsewhere.

2. THE SIMPLIFIED PROCEDURE FOR THE MACRO-BLOCK MODEL

2.1. The model and the procedure

The Coulomb friction model of dry rigid block masonry with non-standard behaviour (non-associative frictional sliding) is herein adopted and a global macro-block analysis is used instead of a detailed discrete element analysis. According to the concept of macro-modelling, it is herein assumed that the general failure involves a number of cracks, which separate the structures into a few macro blocks, and all the possible relative motions among micro blocks are concentrated along those cracks.
As already described in recent works [13-16], the transfer from the micro to macro scale of the model of rigid blocks implies a loss of information about the real active sliding interfaces along the generic crack line and the frictional resistance assumed for the macro-block model is generally associated with sliding mechanism only, i.e. its maximum possible value. Actually, for a general and more realistic combination of the failure modes of sliding and overturning, the frictional resistance activating along the crack line is smaller than that generally computed, and is quite unlikely to be estimated accurately.
Therefore, the overestimation of the analytical load factors with respect to the experimental ones highlighted above is due to the erroneous assumption of fully friction along the cracks, while, on the contrary, if zero friction is considered for the same crack slopes the result would be underestimated. This last observation has lead to propose a novel strategy of analysis which is aimed at researching upper and lower bounds of the real ultimate load factors. It implies the following steps:

1. evaluation of the maximum frictional resistance of the walls, in function of the height of the crack lines;
2. definition of the inclination and the height of the crack lines corresponding to the minimum kinematic multiplier, based on the hypothesis of the complete activation of frictional resistances along the cracks; such a multiplier represents the upper bound of the range;
3. evaluation of the kinematic load factor corresponding to the same crack pattern (say the same height and inclination of the crack lines), but based on the hypothesis of nil activation of frictional resistances along the same cracks; such a multiplier represents the lower bound of the range.

The classical kinematic approach of the limit-state analysis is then used to define the bounds of the range.

2.1.1. Assessment of the maximum in-plane friction forces

Considering the single micro-block (unit) and the shear wall in Fig. 1, the unit's shape factor is defined as the angle $\alpha_b = \arctan(s/h)$ where s is the length of superposition between two units of length l belonging to two adjacent courses, h is the height of the brick and s/h is the shape ratio of the unit, having assumed $s = l/2$. It is also assumed that: α_c is the average inclination of a generic crack; all units have same dimensions and are laid with the same staggering; $\alpha_p = \arctan(L/H)$ is the shape factor of the wall of width L and height H.
The total friction force can be calculated for the maximum inclination of the crack α_b, since the number of surfaces crossed by the crack line does not change for any $\alpha_c \leq \alpha_b$ and it is always equal to the total number of courses in the portion considered [17]. Mechanisms with $\alpha_c > \alpha_b$ are excluded from the analysis, as they are unlikely to occur under the current assumptions and the wall is assumed to be long enough to be always $\alpha_b \leq \alpha_p$. Different cases have been analysed elsewhere [13].

Therefore, the maximum total friction force acting on the plane of the wall, based on the hypothesis of complete activation of frictional resistances along the crack, is given by the product of the weight of trapezoid OABC in Fig. 2 (plus the overload) times the friction coefficient *f*:

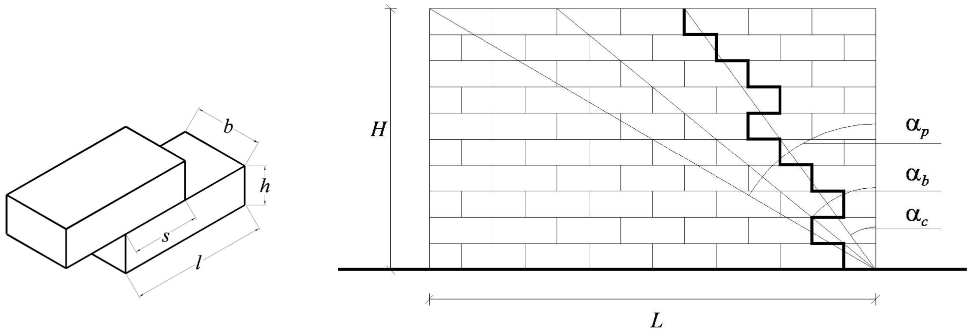


Fig. 1 Unit dimensions and inclinations for the unit and wall shape ratios and for the variable angle of crack

$$F = \left[pH \tan\alpha_b + \frac{(H \tan\alpha_b + s)H}{2} \gamma \right] bf \tag{1}$$

where: p is a uniformly distributed load for unit of surface, γ is the specific weight of the material and the geometrical parameters are simply derived from Figs. 1 and 2.

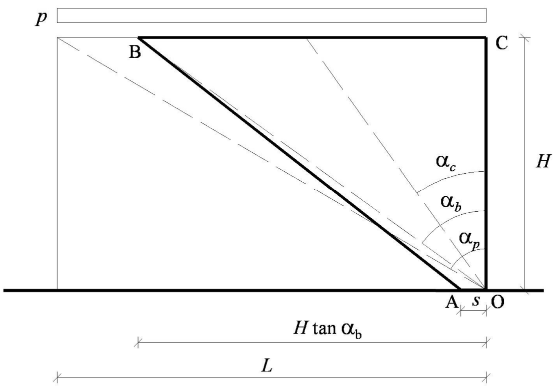


Fig. 2 Masonry wall with $\alpha_b \leq \alpha_p$. The solid line shows the equivalent area of wall considered for the calculation of the friction force

2.1.2. In-plane mechanism and the bounds of the ultimate load factor

The macro-block identified by the angle of crack α_c is that shown in Fig. 3. The tangent of angle β in the same figure is chosen as the variable parameter and relates to α_c through the following:

$$\tan\alpha_c = \frac{H \tan\beta + s}{H} \tag{2}$$

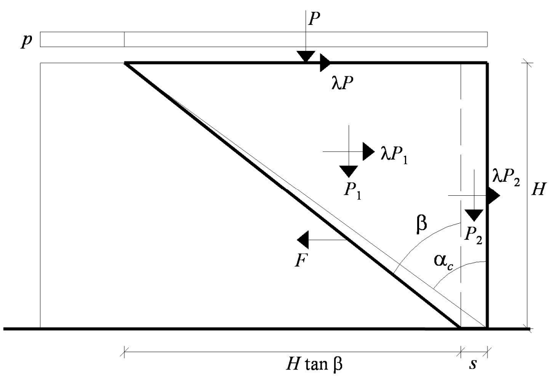


Fig. 3 Macro block identified by the angle of crack $\alpha_c \leq \alpha_p$

Hence having defined the following parameters of weight:

$$P = pb(H\tan\beta + s); \qquad P_1 = \frac{H^2 \tan\beta}{2} b\gamma; \qquad P_2 = sHb\gamma \tag{3}$$

and having simplified out the virtual parameter of distortion ϕ, for incipient rotation around point O of the right hand part of the wall, the virtual work equation yields:

$$\lambda PH + \lambda P_1 \frac{2}{3}H + \lambda P_2 \frac{H}{2} = P\frac{H\tan\beta + s}{2} + P_1\left(\frac{H\tan\beta}{3} + s\right) + P_2\frac{s}{2} + F\frac{H}{3} \tag{4}$$

where: λ is the load factor of the horizontal external forces applied, F is the friction force defined in Eq. (1). Solving Eq. (4) for λ it becomes:

$$\lambda = \frac{P\frac{H\tan\beta + s}{2} + P_1\left(\frac{H\tan\beta}{3} + s\right) + P_2\frac{s}{2} + F\frac{H}{3}}{PH + P_1\frac{2}{3}H + P_2\frac{H}{2}} \tag{5}$$

which can be differentiated with respect to tanβ. This operation yields an equation of second order in tanβ which has solutions:

$$\tan\beta = \frac{-3t(1+2r) + \sqrt{3t^2(1+4r) + 8\nu(1+3r)}}{2(1+3r)} \tag{6}$$

limited by the condition $\tan\beta \geq 0$ and where the adimensionalised parameters are:

$$r = \frac{p}{H\gamma}; \qquad t = \frac{s}{H}; \qquad \nu = \frac{F}{bH^2\gamma} \tag{7}$$

The upper threshold of the real ultimate load factor is obtained from Eq. (5) with its corresponding value of tanβ given by Eq. (6). On the other hand, the lower threshold for the same crack inclination, i.e. the same values of tanβ, is simply obtained from Eq. (5) setting the friction coefficient equal to zero.

3. VALIDATION EXAMPLES

In order to validate the procedure, the ultimate load multiplier, the crack slope and the height of the mechanism must be verified a posteriori by comparing these results with those of the micro-modelling analysis and experimental ones.

Within the framework of micro-modelling, numerical procedures for the limit load analysis of systems formed from rigid-blocks in frictional contact were firstly proposed in [18] and [19], where it is shown that extending the formulation to non-associated flow, results in a nonlinear mathematical problem of a significantly larger size, in comparison with the lower size linear problem resulting from the classical theory. Furthermore, the nonlinear problem arising seems hard to solve and recently the problem has been posed as a mixed complementarity problem (MCP) and a mathematical programming with equilibrium constraints (MPEC) formulation has been proposed [20, 21]. Unfortunately relatively specialised non-linear programming solution methods must be employed and it also seems that solving the MPEC formulation in the way proposed may for practically large problems be prohibitively computationally expensive.

More recently, a new computational limit analysis procedure has been presented [11, 12]. It involves solving a series of LP problems with successively modified failure surfaces and associative friction (rather than working directly with the full Mixed Complementarity Problem (MCP)). The proposed

method appears to be capable of identifying reasonable estimates of the load factor for a wide range of problems and to be particularly suited to comparatively large problems.
This method is herein revised by using Matlab implementation in order to validate the numerical results from the presented macro-block modelling, with reference only to walls subject to in-plane horizontal loading. Further developments and extension to 3D problems are being presented in another work.

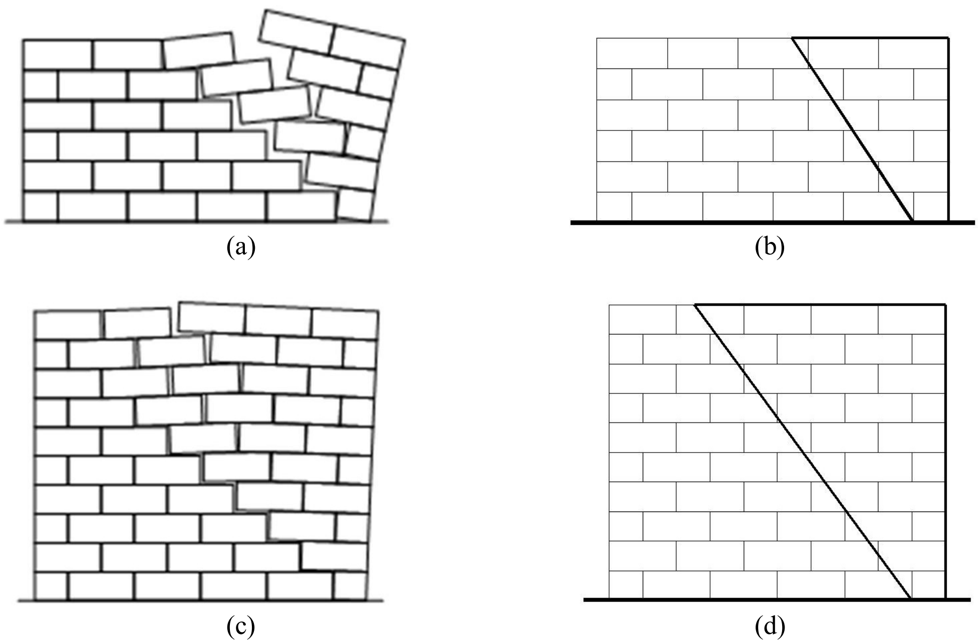
3.1. Freestanding walls subjected to in-plane horizontal loading

Three benchmark wall problems are herein re-run using the proposed procedure. These were previously analysed in [11], [20] and [21]. Each example problem comprises a freestanding wall supported on a base and subject to in-plane horizontal forces applied to the centroid of each block (to represent earthquake-type loading). For these examples the mechanism failure involves the total height of the wall.
Thus, for the first two examples sketched in Fig. 4, each full block has a weight of 1 unit, face area of 4 × 1.75 units, and is subject to a unit horizontal live load (for half blocks these quantities are reduced by half); the friction coefficient is taken as 0.65. Results from both micro and macro-block models are shown in Table 1 and on Fig. 4.

Table 1 Computational results for three benchmarks

Examples no. [12]	Micro-block model			Macro-block model		
	λ_1	λ_2	tanβ	λ_{upp}	λ_{low}	tanβ
1	0.6398	0.6394	0.7619	0.9455	0.4871	0.6597
2	0.5626	0.5601	0.8	0.9094	0.46	0.7379
3	0.5389	0.5384	0.9612	1.07199	0.5363	1.0251

In Table 1, λ_1 is related to the micro-block procedure from [12] and λ_2 to the same procedure herein revised (the slight differences lie inside the numerical tolerances and approximations). Besides, the inclination of the crack line for the micro-block model represented by tanβ is herein calculated as the median of the triangular area of the wall within all the relative motions among blocks are concentrated (Fig. 5). So, being $d = (c+e)/2$ it will be $\tan\beta = d/H_1$ whatever the area involved and provided that $\alpha_c \le \alpha_b \le \alpha_p$ as assumed above. This approximation has been proved to match the results from the macro-block modelling better than the possible alternative bisector of the inner angle of the triangle defined.



(a) (b) (c) (d)

Fig. 4 Two benchmark walls subjected to in-plane horizontal loading. (a)-(c) Examples 1 and 2; (b)-(d) corresponding failure modes using the macro-modelling procedure

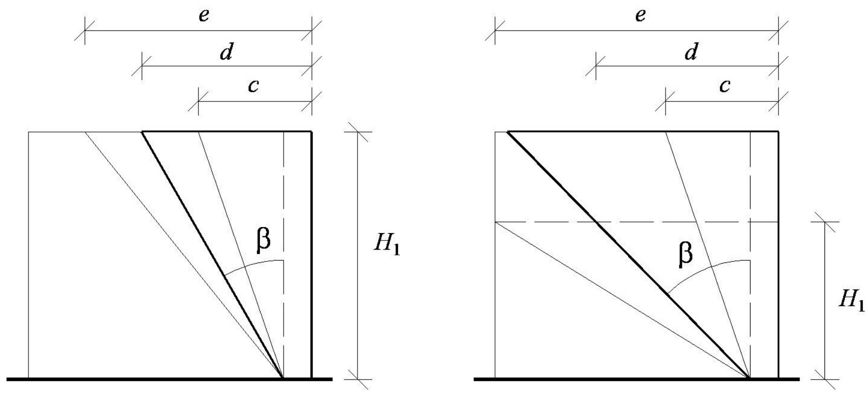


Fig. 5 Micro-block modelling. Inclination of the crack line as the median of the moving area of the wall

A much larger wall problem is represented by the third example (see Fig. 13 in [12]). The wall chosen for study is subjected to the same horizontal live loading as in the previous examples, but the length:height ratio of the constituent blocks is 3:1 and the friction coefficient is taken as 0.75. The results are shown in Table 1.
For all these three examples it is proved that the range for the load multiplier obtained from the macro-modelling always includes that from the micro-modelling, as sketched in Fig. 6, and a good agreement is reached for the inclination of the crack line. Also it is worth noting that the closer to λ_{low}, say, λ_2 is, the more occurrence for the overturning mechanism can be predicted (see example 3 in Fig. 6).

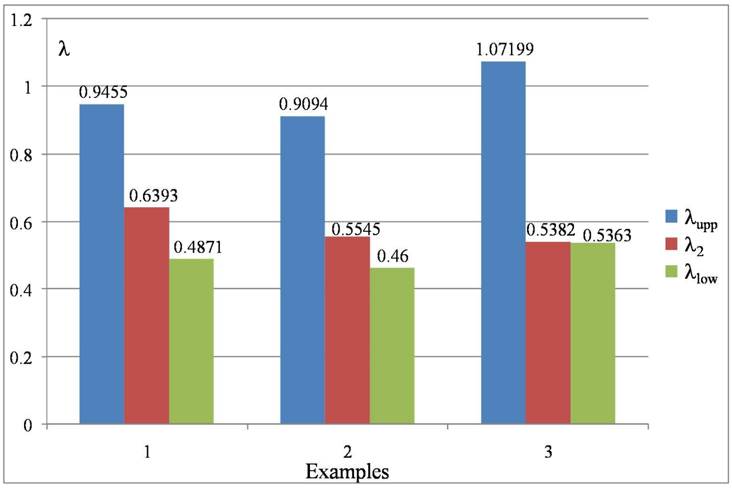


Fig. 6 The ranges of the ultimate load factors for the three illustrative examples

3.1.1. *The influence of the overburden load*

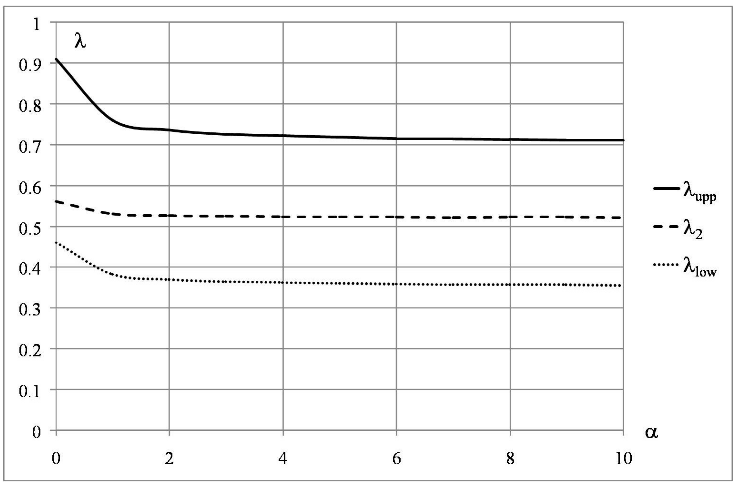


Fig. 7 Overload effect for both macro and micro-block models. Ultimate load factor vs. overload ratio graph

Fig. 7 shows the effect of the overload on the ultimate load factor, assuming that parameter $\alpha = p/(\gamma H)$ measures the amount of overload relative to the wall self weight. The three computed load factors decreases as the overload increases and the effect is less evident for large overload values, in such a way that for overloads larger than three or four times the wall self weight, the load factors remain almost constant. This trend can be explained by the fact that as the overload increases, also $\tan\beta$ tends to be constant and the area of masonry involved into the mechanism is almost the same; this means that the effect of the wall self weight becomes negligible and only the overload position determines the ultimate load factor value.
Lastly, Fig. 7 represents a further validation of the macro-block model and the solution procedure as it shows that the bounds computed by macro-block model always contain the load factor by micro-block model whatever overload amount.

4. CONCLUSIONS

A macro-block model and a simplified procedure are presented in this paper to assess reliable ranges of existence of the collapse load factor and the identification of the crack pattern in dry stone masonry walls with regular units and staggering. These are herein only applied to in-plane loaded walls, namely shear walls.
It is assumed that the general failure involves a number of cracks which separate the structures into a few rigid macro-blocks and that all the possible relative motions among micro-blocks are concentrated along the cracks. By using a limit-state analysis approach and assuming that the interaction at surfaces is governed by Coulomb friction, upper and lower bounds for the ultimate load factor are kinematically computed in closed form, characterised the former by the assumption of full development of the friction force on every contact surface along the crack and the latter by the total absence of friction, respectively. The crack slope is assumed as a variable of the problem and a likely reason for the overestimation of existing analytical results with respect to experimental ones is also presented.
The macro-block model and the solution procedure are validated through three illustrative examples of shear walls existing in literature and comparisons against a micro-block model from a recently proposed strategy, herein revised, show a good agreement of the results in terms of both the ultimate load multiplier and the crack slope. The adopted strategy involves solving a series of LP problems with successively modified failure surfaces and associative friction and appears to be particularly suited to comparatively large problems.
Lastly the analysis of the influence of the overload on the ultimate load factor not only confirms that the bounds computed by macro-block model always contain the load factor obtained by micro-block model, but also shows that for overloads larger than three or four times the wall self weight, the load factor remains almost constant. This means that the lateral capacity of bearing masonry walls is bounded from below, provided that the compressive strength is assumed as infinitive. Further developments and a full parametric analysis with the influence of all the other geometrical and mechanical parameters will be presented in another work.

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