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SIMPLE HOMOGENIZED PLATE MODEL FOR OUT-OF-PLANE LOADED MASONRY TAKING INTO ACCOUNT MATERIAL AND GEOMETRICAL NON LINEARITY

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The buckling behavior of slender unreinforced masonry (URM) walls subjected to axial compression and out-of-plane lateral loads is investigated through a simplified "homogenized" procedure. After a preliminary analysis performed on a unit cell meshed by means of elastic FEs and non-linear interfaces, macroscopic moment-curvature diagrams so obtained are implemented at a structural level, discretizing masonry by means of rigid triangular elements and non-linear interfaces. The non-linear incremental response of the structure is accounted for a specific quadratic programming routine where second order effects are suitably considered adding a further term, quadratic in the nodal displacement, within the total energy of the discretized system.
As validation of the approach proposed, the buckling behavior of some existing experimental pre-compressed four-point bending tests is reproduced; furthermore square panels in two-way bending, exhibiting the classical Rondelet's mechanisms, are also studied. The results obtained are compared with those provided by commercial FE programs and with specifically derived analytical results.

Keywords: *Masonry, Simplified Non-linear Homogenized Approach, Second Order Effects*

1. INTRODUCTION

Masonry structures subjected to seismic events, generally, collapse for the formation of local out-of-plane mechanisms. Following classic works by Giuffrè [1], recent Italian norms [2] prescribe that such mechanisms are considered within a rigid block kinematic analysis, where also geometric non linearity effects can be considered. As a matter of fact, it is well known that especially slender unreinforced masonry (URM) structures and tall reinforced masonry walls are susceptible to instability failure when subjected contemporarily to vertical pre-compression and out-of-plane lateral (i.e., transverse) loads [3], [4]. Current design codes in Europe and United States address stability failure in various ways, e.g. by limiting the allowable compression forces in proportion to member slenderness and the amount of axial load eccentricity. However, such provisions do not address the effects of out-of-plane bending on the behavior of masonry members and the capacity of the cross-section. From a practical point of view, bending from out-of-plane lateral loads affects the stability of a URM compression member because reduces the effective cross-section arising from tensile cracking and P–Δ effects. The effective cross-sectional depth is reduced as flexural tension stresses crack the masonry, which further augments the out-of-plane lateral deflection, thus increasing second-order (P–Δ) moments. The additional bending generates more tension, further reducing the cross-section, leading to possible instability (i.e. buckling).
Despite the huge amount of experimentation available in the literature, e.g. [4], dealing with this important issue, however, at present, second order P–Δ behavior has been studied only for very simple mechanisms, such as the overturning failure of laterally unconstrained façades or simply supported walls in simple horizontal bending.

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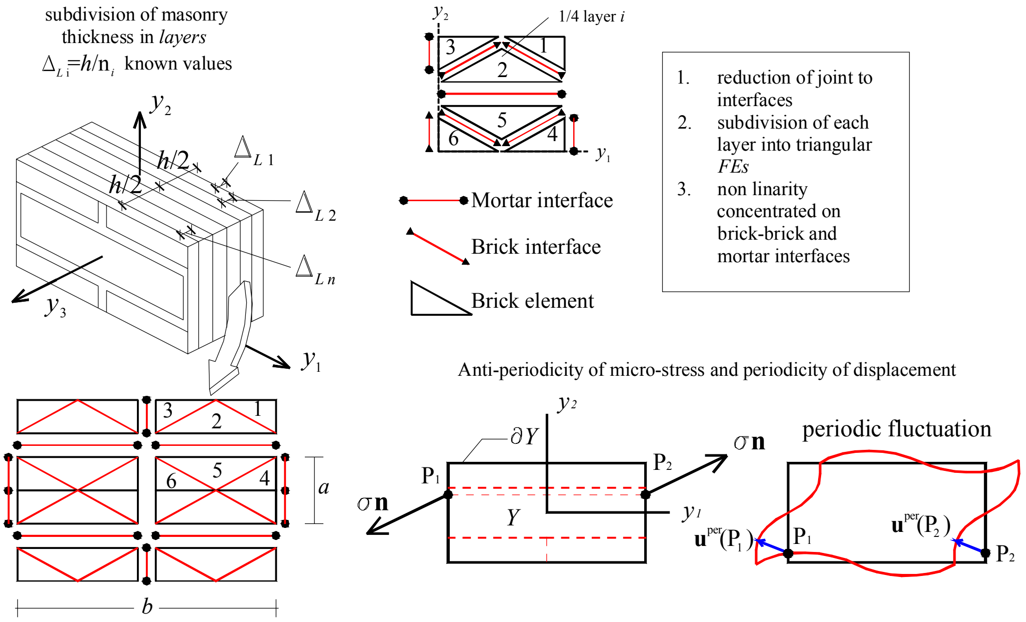


Fig. 1 The micro-mechanical model proposed

In this paper, with the aim to deal with general out-of-plane mechanisms and to consider the texture effect only at the meso-scale, the results of a 3D model accounting for second order effects in an simple but suitable way is presented. In the model, the heterogeneous assemblage of blocks is substituted with a macroscopically equivalent homogeneous non-linear material by means of a simplified FE homogenization. At the meso-scale, a running bond representative element of volume (REV) constituted by a central block, zero thickness mortar joints and four ¼ of blocks is discretized by means of triangular elements and interfaces. Non linearity is concentrated exclusively on joints reduced to interface, exhibiting a frictional behavior with limited tensile and compressive strength with softening. The macroscopic homogenous masonry behavior is then evaluated on the REV solving the incremental boundary value problem on the REV through standard FEs.

The non-linear behavior so obtained is then implemented at a structural level in a non-commercial FE non-linear code, relying on an assemblage of rigid infinitely resistant triangles and non-linear interfaces obtained by the homogenization procedure, exhibiting deterioration of the mechanical properties. Both geometrical non linearity and material properties deterioration are taken into account within a well documented non-linear programming approach. Second order effects are accounted suitably considering the second order contribution of each rigid element within the total internal energy of the structure at each iteration (standard geometric matrix). The model is tested at a structural level on a one-way bended slender masonry wall out-of-plane loaded and subjected to head pre-compression, already tested experimentally and numerically in [4] and on a wall in two-way bending.

2. THE NUMERICAL MODEL PROPOSED

A macro-scale approach is utilized in the paper to study the non-linear behavior of masonry walls in presence of second order effects. In particular, a masonry wall is discretized by means of triangular elements with linear interpolation of the out-of-plane displacement field inside the element and interfaces between adjoining triangles where all deformation occurs.

The non-linear curvature-bending moment relationship to use at a structural level is derived from a recently presented [5][6] simplified homogenization model, which is briefly recalled in the following sub-section. Finally, to solve the non-linear structural problem, a sequential quadratic programming procedure is adopted.

2.1. Curvature-Bending moment non-linear relationship

The representative element of volume *Y* (RVE or elementary cell) depicted in Fig. 1 is considered. *Y* contains all the information necessary for describing completely the macroscopic behavior of an entire wall. If a running bond pattern is considered, as shown in Fig. 1, it can be easily checked that the elementary cell is rectangular.

The basic idea of the homogenisation procedure consists in introducing averaged quantities representing the macroscopic stress and strain tensors (respectively $\mathbf{E}$ and $\mathbf{\Sigma}$), as follows:

$$\mathbf{E} = <\boldsymbol{\varepsilon}> = \frac{1}{A}\int_{Y} \boldsymbol{\varepsilon}(\mathbf{u})dY$$

$$\mathbf{\Sigma} = <\boldsymbol{\sigma}> = \frac{1}{A}\int_{Y} \boldsymbol{\sigma}dY \qquad (2)$$

where A stands for the area of the elementary cell, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$ stand for the local quantities (stresses and strains respectively) and <*> is the averaging operator.
Periodicity conditions are imposed on the stress field $\boldsymbol{\sigma}$ and the displacement field **u**, given by:

$$\begin{cases} \mathbf{u} = \mathbf{E}\mathbf{y} + \mathbf{u}^{\text{per}} & \mathbf{u}^{\text{per}} \text{ on } \partial Y \\ \boldsymbol{\sigma}\mathbf{n} & \text{anti-periodic on } \partial Y \end{cases} \qquad (2)$$

where $\mathbf{u}$ is the total displacement field and $\mathbf{u}^{\text{per}}$ stands for a periodic displacement field.

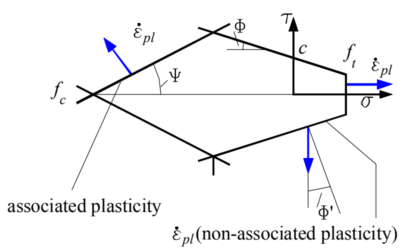
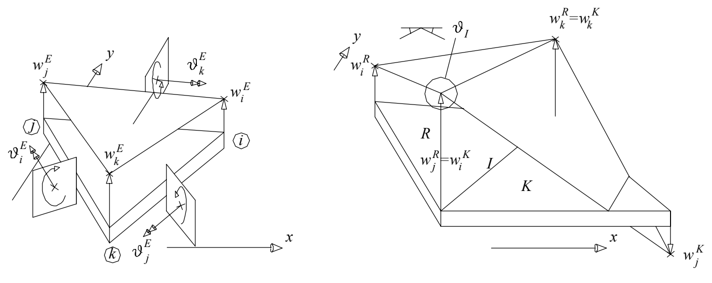


Fig. 2 Modified Mohr-Coulomb criterion for the mortar joint/mortar-brick interface



a) b

Fig. 3 Triangular plate element used for the FE non linear analyses (a), rotation along an interface between adjacent triangles (b), discretization of the 2D domain (c)

Here, it is worth noting that, in equation (2), the periodicity of the displacement fluctuation $\mathbf{u}^{\text{per}}$ forces corresponding boundary segments to exhibit the same shape in the deformed configuration. In the model proposed, joints are reduced to interfaces with zero thickness and blocks are discretized by means of a coarse mesh constituted by three noded plane-stress elastic elements, as schematically sketched in Fig. 1. The choice of meshing 1/4 of the brick through at least 3 triangular elements is due to the need of reproducing the presence of shear stress in the bed joint (element 2 in) in horizontal stretching. All the non linearity in the RVE is concentrated exclusively on interfaces between adjoining elements both on brick and joint. Brick-brick interfaces allow, at least in principle, the reproduction of blocks failure.
The elastic domain of the interfaces is bounded by a Mohr-Coulomb surface with cut-off in tension that includes that includes tension, shear and compression failure with softening, as schematically represented in Fig. 2 and discussed in detail in [5]. The reader is referred there for further details on the solution of the cell problem.
When dealing with out-of-plane internal actions (bending moments and torsion), the same discretization of the elementary cell utilized for in-plane loads is considered, provided that its thickness h is subdivided into n_l layers, as indicated in Fig. 1. For out-of-plane loads, the homogenization problem is solved applying unitary macroscopic curvature tensors, on the cell internal boundary ∂Y. Details on how to handle the problem may be found again in [6], where the reader is referred for further details.

2.2. QP model at structural level

The homogenization model proposed allows obtaining bending moment-curvature diagrams for the homogenous fictitious macroscopic material at different orientations of the bed joint with respect to bending plane. Diagrams are collected in a database and implemented at a structural level.

It has been shown several decades ago that the elastic plastic response of a structure subjected to given proportionally increased loads under the following hypotheses (see e.g. [7]):

1. The plasticity condition is piecewise-linearized with *r* linearly elastic-plastic (or work hardening in the case of ductile materials, which is not the case of masonry) interacting planes in the space of superimposed stress and strain components;
2. Unloading of yielded stress-points does not occur;
3. The continuum is discretized into constant strain and stress finite elements;

is given by the following set of equations and inequalities:

$$\begin{cases} \boldsymbol{\varepsilon}^{plE} = \mathbf{N}^E \boldsymbol{\lambda}^E \\ \boldsymbol{\Phi}^E = \left(\mathbf{N}^E\right)^T \boldsymbol{\sigma} - \mathbf{H}^E \boldsymbol{\lambda}^E \\ \boldsymbol{\Phi}^E \leq 0 \quad \boldsymbol{\lambda}^E \geq \mathbf{0} \\ \boldsymbol{\lambda}^E \boldsymbol{\Phi}^E = 0 \end{cases} \tag{1}$$

where $\boldsymbol{\varepsilon}^{plE}$ is the plastic strain vector of the element E, $\mathbf{N}^E$ is the shape functions matrix of the used finite element, $\boldsymbol{\lambda}^E$ is the plastic multiplier vector, $\mathbf{H}^E$ is the hardening matrix, which in this case is diagonal and with very small non null values, $\boldsymbol{\Phi}^E$ is a vector collecting the r linearization planes of the failure surface, $\boldsymbol{\sigma}$ is the vector of stress parameters which define point by point the stress (or internal actions) acting on the finite element.

It can be shown [7] that, without taking into account second order effects, the solution of problem (1) can be achieved by means of the following equivalent quadratic programming problem:

$$\begin{cases} \max\left\{-\frac{1}{2}\left(\boldsymbol{\lambda}^E\right)^T \mathbf{H}^E \boldsymbol{\lambda}^E + \left(\boldsymbol{\lambda}^E\right)^T \left(\mathbf{N}^E\right)^T \mathbf{D}^E \boldsymbol{\varepsilon}^E \right. \\ \qquad \textit{subject to}: \boldsymbol{\lambda}^E \geq \mathbf{0} \end{cases} \tag{2}$$

where $\mathbf{D}^E$ is the elastic stiffness matrix, $\boldsymbol{\varepsilon}^E$ is the elastic part of the strain vector and all the other symbols have been already introduced.

The finite element model utilized next for the non-linear analysis of masonry panes out-of-plane loaded is based on the simple triangular element proposed by [8]. Such approach seems convenient to solve problem (2)), also in presence of geometric non linearity, since elastic-plastic deformation is concentrated only at the interfaces between adjoining elements and is due exclusively to bending moment. The displacement field is assumed linear inside each element and nodal displacements are taken as optimization variables. Denoting with $\mathbf{w}_E = \left[w_i^E \quad w_j^E \quad w_k^E\right]^T$ element E nodal displacement and with $\boldsymbol{\theta}_E = \left[\vartheta_i^E \quad \vartheta_j^E \quad \vartheta_k^E\right]^T$ side normal rotations, $\boldsymbol{\theta}_E$ and $\mathbf{w}_E$ are linked by the compatibility equation (Fig. 3a and 3b) $\boldsymbol{\theta}_E = \mathbf{B}_E \mathbf{w}_E$, where $\mathbf{B}_E$ is a 3×3 matrix depending only on the geometry of the element under consideration.

In presence of geometric non-linearity, problem (2) may be re-written, within the aforementioned discretization (plate elements with elastic-plastic interfaces) as follows:

$$\begin{cases} \min\left\{\frac{1}{2}\left[\left(\boldsymbol{\lambda}^+ - \boldsymbol{\lambda}^-\right)^T \mathbf{K}_{ep}\left(\boldsymbol{\lambda}^+ - \boldsymbol{\lambda}^-\right) + \boldsymbol{\theta}_{in}^T \mathbf{K}_{el} \boldsymbol{\theta}_{in} - \mathbf{w}^T \mathbf{K}_G \mathbf{w}\right] - \mathbf{F}^T \mathbf{w} \right. \\ \qquad \textit{subject to}: \boldsymbol{\lambda}^+ \geq \mathbf{0}\ \boldsymbol{\lambda}^- \geq \mathbf{0} \end{cases} \tag{3}$$

Assuming that the structural model has n_{in} interfaces and n_n nodes, symbols in equation (3) have the following meaning:

1. $\mathbf{K}_{el}$ is a $n_{in} \times n_{in}$ assembled diagonal matrix collecting elastic stiffness of each interface. It is worth remembering that elastic stiffness values are evaluated through the homogenization model discussed in the previous section. Since masonry is anisotropic both in the elastic and inelastic range, they depend on the interface orientation angle.

2. $\boldsymbol{\lambda}^+$ and $\boldsymbol{\lambda}^-$ are two n_{in} vectors of plastic multipliers, collecting plastic multipliers of each interface for positive (+) and negative (-) bending moment.
3. $\mathbf{K}_{ep}$ is a $n_{in} \times n_{in}$ assembled diagonal matrix of hardening moduli of the interfaces.
4. $\mathbf{K}_G$ is the $n_{in} \times n_{in}$ assembled geometric stiffness matrix.
5. $\boldsymbol{\theta}_{el}$ is a n_{in} vector collecting the elastic part of interfaces rotation, being $\boldsymbol{\theta} = \boldsymbol{\theta}_{el} + \boldsymbol{\theta}_{ep}$, with $\boldsymbol{\theta}_{ep}$ the plastic part of the rotation equal to $\boldsymbol{\lambda}^+ - \boldsymbol{\lambda}^-$.
6. $\mathbf{F}$ is a n_n vector of external loads lumped on nodes and $\mathbf{w}$ is the nodal displacement vector.

Typically, the independent variable vector is represented by nodal displacements $\mathbf{w}$ and plastic multiplier vectors $\boldsymbol{\lambda}^+$ and $\boldsymbol{\lambda}^-$. Problem (3) is solved at increasing values of the external load vector $\mathbf{F}$ and, at each external load value, the initial trust independent variable vector is the solution at the previous step.

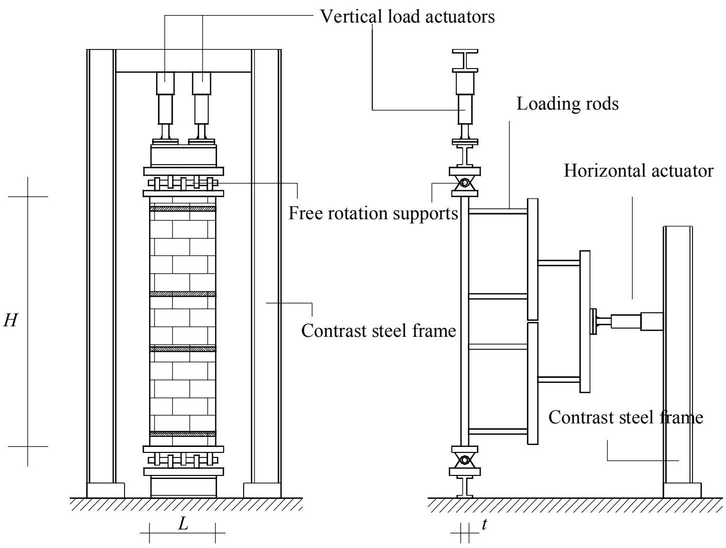


Fig. 4 Load frame utilized to perform stability tests of URM walls in simple vertical bending

3. CASE STUDIES

In order to assess the reliability of the numerical results provided by the present numerical homogenized approach, a set of numerical analyses are performed, relying on a wall in simple vertical bending recently tested in [4] under heavy vertical pre-compression and a two-way bending masonry panel experimentally tested and numerically analyzed without accounting for geometric non-linearity in [10]. The simulations are repeated using the commercial code DIANA [9], under both non-linear material and geometry hypothesis, modeling masonry by means of an orthotropic homogeneous elasto-plastic layered material exhibiting softening. To have an additional insight into the predictive character of the results obtained with the 2D simple numerical approach proposed, an additional, very simple 1 DOF model is also proposed. Details of such model are reported in [6] and the reader is referred there for further details.

3.1. Wall in simple vertical bending

In order to test the reliability of the approach proposed to reproduce the potential buckling instability of slender URM walls in simple bending, a series of non-linear analyses accounting for the material and geometric nonlinearity are conducted on real scale walls originally experimentally tested by [4].

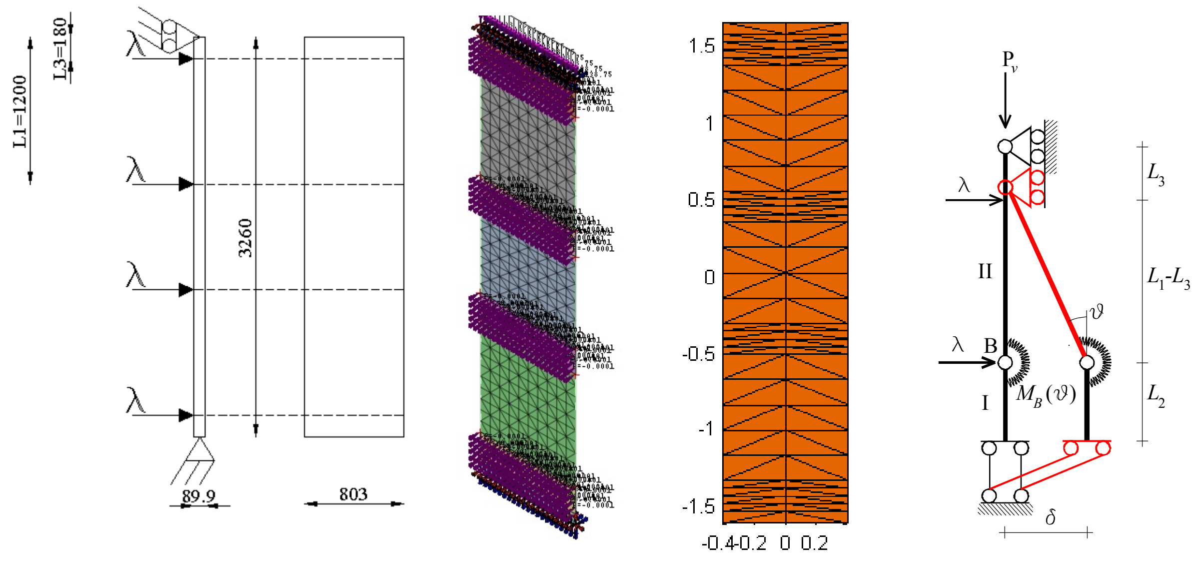
The experimental program conducted by [4] included eight URM wall specimens: four of them were constructed of cored clay brick, while the remaining four were fabricated using concrete block.

In this paper, only the first set of wallets are considered for the sake of conciseness. Standard modular clay bricks of dimensions 89.9 × 57.2 × 193.7 mm laid in running bond were used, with type S Portland cement-lime mortar. A full list of material properties, adopted within the models in agreement with [4] data, is summarized in.

Table 1 Walls in simple vertical bending. Mechanical properties adopted for mortar joints reduced to interfaces and bricks within the homogenization model.

Quantity	Joint	Brick-brick interface		Description
E	13240$^{(*)}$		[MPa]	Young Modulus
G	E/2		[MPa]	Shear Modulus
c	1.0 f_t	2	[MPa]	Cohesion
f_t	0.372	-	[MPa]	Tensile strength
f_{ce}	$1/3 f_{cp}$	-	[MPa]	Compressive hardening/softening behavior
f_{cp}	32.5	-	[MPa]	
f_{cm}	$1/2 f_{cp}$	-	[MPa]	
f_{cr}	$1/7 f_{cp}$	-	[MPa]	
κ_p / e_h	0.008	-	[-]	
κ_m / e_h	0.006	-	[-]	
Φ	37	45	[°]	Friction angle
Υ	45	-	[°]	Angle of the linearized compressive cap
G_f^I	0.010	10	[N/mm]	Mode I fracture energy
G_f^{II}	0.0050	10	[N/mm]	Mode II fracture energy
$^{(*)}$ the value refers to the homogenized elastic modulus found in post processing				

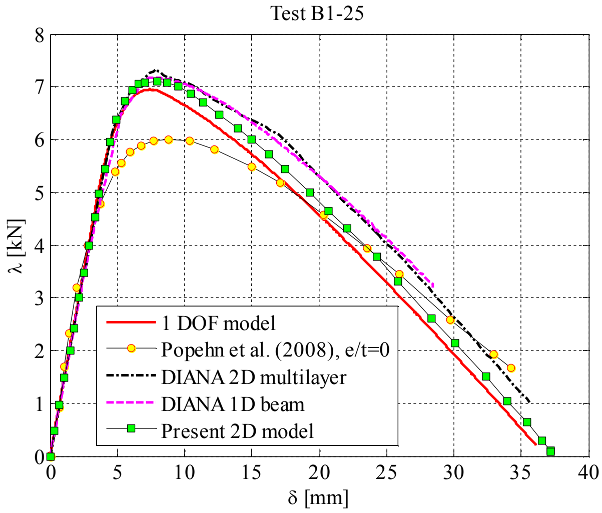
The experimental program was designed around the test setup sketched in Fig. 4. Geometry of the walls is the following: H = 3.26 m (height), L = 80 cm (width), t = 9 cm (thickness). The walls are in four point bending, see Fig. 4, and assumed simply supported at the top and at the base for horizontal out-of-plane loads. The tests began with the application of an assigned axial load on the head of the specimens in force control using two vertical load actuators, see Fig. 4. The axial load was then held constant throughout the tests. Four different walls, labeled as B1-25, B2-50, B3-70 and B4-83.5 were tested, differing one each other only for the vertical pre-compression. In particular B1-25, B2-50, B3-70 and B4-83.5 supported axial loads equal to 111 kN, 222 kN, 311 kN, and 371 kN, respectively. The final number on the labels indicates the initial pre-compression in psi. The specimen response under increasing horizontal loads was measured using internal load cells in the actuators, horizontal load cells on the whiffletree, and displacement transducers at various locations. For the sake of conciseness, in what follows, only wall B1-25 is analyzed.



DIANA model Present 2D model 1 DOF model

a) b) c)

Fig. 5 a) Geometries of the model, b) 2D FE models, c) 1 DOF model



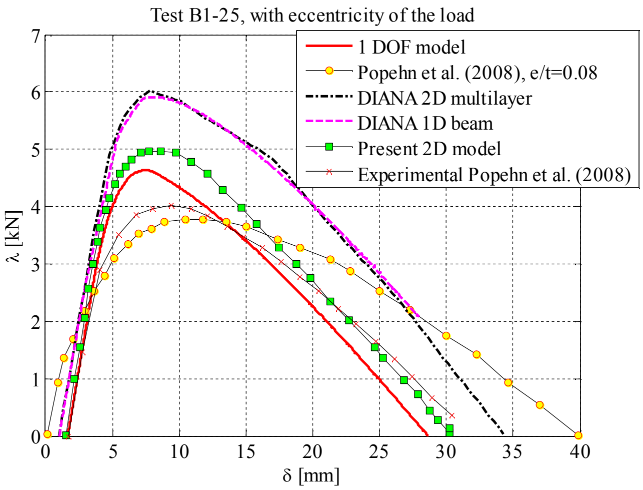


Fig. 6 Force-Displacement curves for model B1-25. Left: without eccentricity. Right: with eccentricity

Two different 2D models are used to study the problem, namely a multilayer model available in DIANA [9] (128 elements) and the present rigid elements with plastic interfaces model (800 triangular elements, see Fig. 5. Within the present approach, a total number of 20 layers is used, which seems a very detailed subdivision, which can be exceptionally handled in this particular case only thanks to the quite reduced time needed to achieve convergence in the non-linear range. A so called "total-strain" model is adopted within DIANA, allowing a definition of masonry mechanical properties through the uniaxial stress-strain relationship in compression and tension. For the analyses hereafter proposed, a multi-linear stress-strain uniaxial behavior is adopted. To properly simulate in-plane vertical compression, a distributed pressure – maintained constant during the simulations is applied on the top of the walls. The incremental non-linear analyses are performed on the walls assuming different hypotheses on the eccentricity of the vertical load P_v, in agreement with [4]. In particular, apart the model without eccentricity, a further numerical simulation is performed, with pre-assigned eccentricity of the vertical load e/t equal to 0.08.

To further assess numerical results, the same walls in simple bending are analyzed by means of both a detailed 1D (Euler-Bernoulli beam elements) FE discretization, each element exhibiting an elasto-plastic behavior and the 1 DOF model of Fig. 5c. In the first case, only the non-linear vertical bending-curvature relationship of the beams is required. Obviously, the same moment-curvature diagrams used in the homogenization model are utilized. Properties of the 1 DOF model are described in detail in [6].

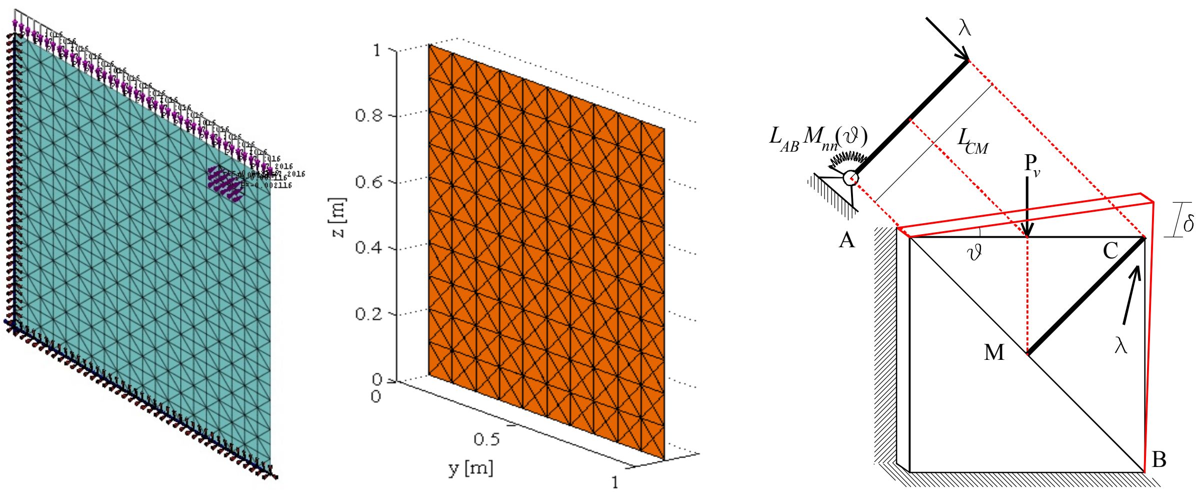
In Fig. 6, horizontal force-displacement curves obtained by means of the homogenized, multilayer 2D, beam 1D and 1 DOF system are compared with numerical and experimental results presented in [4], in absence and presence of eccentricity of the vertical load. Observing the curves obtained by means of all models, it is possible to note that numerical results provided by the three different numerical analyses can be almost always superimposed and in any case a very good agreement among all models is achieved. Conversely, they occasionally deviate by experimental and numerical data provided in [4], in particular those related to the case with eccentricity applied. Such a discrepancy may be related to a different tuning of some numerical parameters that do not find direct correspondence in the model used in [4] and in a different application of the eccentric of the vertical loads, which is here considered through the application of an extra head and foot bending moment.

Table 2 Wall in two-way bending. Mechanical properties adopted for mortar joints reduced to interfaces and bricks within the homogenization model

Quantity	Joint	Brick-brick interface		Description
E	3500$^{(*)}$		[MPa]	Young Modulus
G	E/2		[MPa]	Shear Modulus
c	$1.0 f_t$	2	[MPa]	Cohesion
f_t	0.05	-	[MPa]	Tensile strength
f_{ce}	$1/3 f_{cp}$	-	[MPa]	Compressive hardening/softening behavior
f_{cp}	3.5	-	[MPa]	
f_{cm}	$1/2 f_{cp}$	-	[MPa]	
f_{cr}	$1/7 f_{cp}$	-	[MPa]	
κ_p / e_h	0.015	-	[-]	
κ_m / e_h	0.050	-	[-]	
Φ	37	45	[°]	Friction angle
Ψ	45	-	[°]	Angle of the linearized compressive cap
G_f^I	0.00010	10	[N/mm]	Mode I fracture energy
G_f^{II}	0.00050	10	[N/mm]	Mode II fracture energy
$^{(*)}$ the value refers to the homogenized elastic modulus found in post processing				

3.2. Wall in two-way bending

The ability of the simple numerical model proposed to reproduce second order effects in panels in two-way bending is investigated here, with reference to some experimental campaigns conducted at the University of Calabria (Italy) by Milani et al. [10] on a panel out-of-plane loaded. The wall, hereafter called Series A, with dimensions 100 × 100 cm^2 (length × height) was arranged in running bond texture using 1:3 scale bricks and without mortar joints. In-scale models were used in order to reproduce with the application of reduced forces a 300 × 300 cm^2 masonry wall and trying to represent the real behavior of a wall inside a real building. The wall is supposed pinned at the base and on one vertical edge and subjected to a concentrated out-of-plane load applied near the free top corner, see Fig. 7. Mechanical properties assumed in the simulations are summarized in Table 2.



a) b) c)

Fig. 7 a) DIANA FE, b) present 2D model discretization, c) 1 DOF model

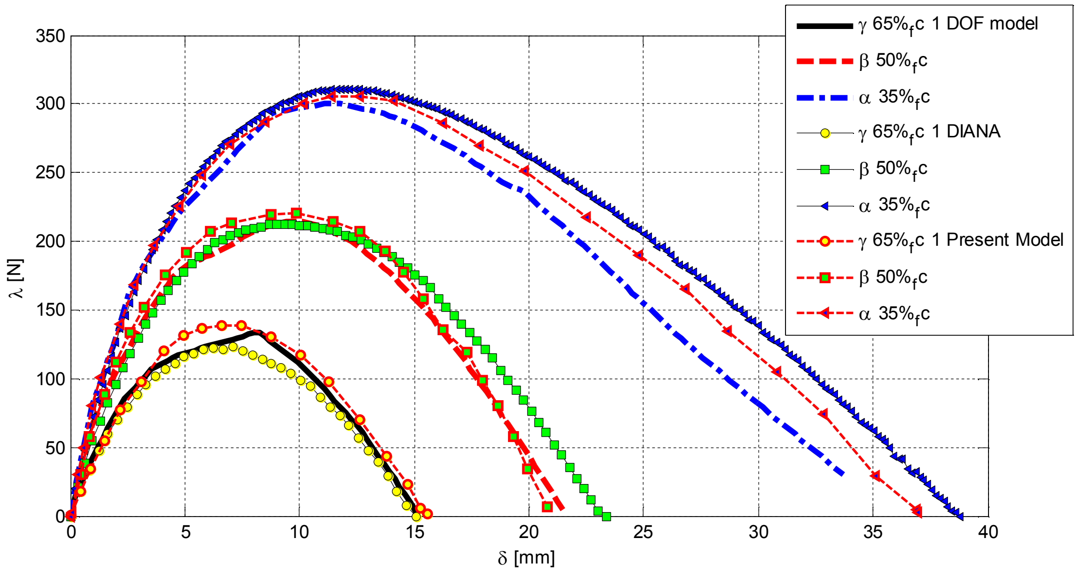


Fig. 8 Series A. Out-of-plane force-displacement curves for different vertical compression loads (α, β, γ with $\alpha = N_V/A = 35\%\ f_{my}$, $\beta = N_V/A = 50\%\ f_{my}$ and $\gamma = N_V/A = 65\%\ f_{my}$)

None of the pre-compression loads applied in [6] may produce perceivable second-order effects on the structure. For this reason, in what follows, pre-compression is suitably increased in order to numerically follow the out-of-plane load-displacement curves in the descending branch. To obtain perceivable second-order effects, Series A is thus supposed subjected to three different pre-stresses, namely $\alpha = N_V/A = 35\%\ f_{my}$, $\beta = N_V/A = 50\%\ f_{my}$ and $\gamma = N_V/A = 65\%\ f_{my}$, where f_{my} is masonry vertical compression strength. Similarly to the previous case, to validate results provided by the 2D numerical model proposed, a FE analysis has been performed using DIANA [9], taking into account both material and geometric non-linearity, with a mesh constituted by 800 triangular elements, see Fig. 7. When dealing with the DIANA mode, layered shell elements degenerated from a three-dimensional formulation are adopted, with a subdivision along the thickness into 10 layers. The adopted yield criterion is based on the plane stress anisotropic yield criterion proposed by Lourenço [11], which includes a Hill type criterion for compression and Rankine type criterion for tension.
Again, to further assess FE results in absence of experimental data available, a 1 DOF simple model, as schematically depicted in Fig. 7c, has been proposed to interpret results. Details of the model may be found again in [6]. In Fig. 8, out-of-plane-force-displacement curves referred to Series A, at the different levels of pre-stress loads (α, β, γ) as previously discussed are reported. Results represented refer to the present simplified homogenized numerical model, the DIANA FE discretization and the 1 DOF simplified approach. Both the good agreement of all numerical results and the effect of geometric non-linearity are worth noting. As expected, also in this case the proposed homogenized model tends to slightly overestimate the actual force-displacement behavior of the structure (upper bound theorem of limit analysis), due to the assumption of rigid elements and the intrinsic dependence of the model from the mesh adopted, dissipation being allowed only at the interfaces between contiguous elements. Actual behavior of the walls is overestimated both near the peak and in the post-peak range.

4. CONCLUSIONS

A simple 2D numerical model for the analysis of masonry structures out-of-plane loaded in presence of relatively high vertical loads and geometric second order effects has been presented. The model is constituted by rigid and infinitely resistant triangular elements connected by deformable nonlinear interfaces. The constitutive nonlinear relationships of the interfaces are derived by a suitable homogenization procedure and take into account both inelastic deformations and the degradation of the mechanical properties. The solution has been obtained by a classic sequential quadratic programming method.
As validation of the approach proposed, the buckling behavior of an existing experimental pre-compressed four-point bending test has been reproduced; furthermore, a square panel in two-way

bending, exhibiting a classical Rondelet's mechanism, has been studied. Results obtained have been compared with those provided by commercial FE programs and with specifically developed single degree of freedom approaches.

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