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STRESS APPROACHES FOR THE ANALYSIS OF MASONRY ARCHES

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ABSTRACT

The paper deals with the static analysis of masonry arches. The equilibrium equations for an arch subjected to a distribution of point-wise forces acting on nodes lying on the line of thrust are written. The equilibrated force distributions for given curves of thrust are deduced. Then, the problem of the determination of the line of thrust for prescribed arch geometry and loading distribution is approached, by formulating a suitable nonlinear constrained minimization problem. The proposed numerical technique is used to derived the profile of the line of thrust for arches subjected to uniform and nonuniform loading distributions. Then, the problem of the elastic arch is formulated and solved making use of the method of consistent deformations, i.e the force method. The effects of the horizontal settlement of the imposts of the arch, which induces a modification of the position of the line of thrust, is investigated.

Keywords: *Masonry Arches, Static Analysis, Stress Formulation*

1. INTRODUCTION

The structural behavior of the arch has been object of many researches and studies during the centuries. The history of the studies on the analysis of masonry arches can be found in several very interesting books and papers. Edoardo Benvenuto in his treatise on the history of Structural Mechanics [1], presented an interesting, deep and detailed history of the scientific developments of the stability of the Arches, Vaults and Domes. Jacques Heyman [2] illustrated the concepts of the Structural Analysis following a historical description, presenting the principal contributions on the researches on masonry arches. The history of masonry constructions, with an interesting interplay between architecture and statics, can be found in the book (in Italian) written by Renato Sparacio [3], where several parts are dedicated to the statics of the arch. Recently, Mario Como wrote a considerable book (in Italian) [4], where the basic theorems of the limit analysis for the so called no-tension material are reviewed and the models for the assessment of stability for different structural masonry elements are illustrated. A critical history of the researches on the statics of masonry arches, paying attention even to the development of the elastic theory can be found in the paper by Sinopoli et al. [5]. Santiago Huerta [6] presented a historic analysis of the researches on the stability arches, from the "geometrical" approach of the old master builders to the "scientific" theory including the elastic and the limit analysis approaches.

The masonry material in existing historic and monumental constructions is characterized by very weak tensile response and by very low values of the compressive working stresses. A simple and effective model for the masonry considers a constitutive law characterized by linear elastic stress-strain relationship in compression with no tensile strength. In the literature, this model is often referred to as the No-Tension Material (NTM). Jacques Heyman in his several memorable works, e.g. [7] and [8], stated the main properties and features of the no-tension model, extending the classical theorems of the plastic design, developed originally for steel frames, to the masonry constructions.

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Several studies were developed regarding the NTM from a mechanical, mathematical and computational point of view, [9]-[13]. Numerical analyses of masonry arches performed in the framework of the displacement or force approaches can be found, for instance, in refs. [14]-[16].
Assuming that in a masonry arch only compressive stresses can be transmitted from blocks, the line of thrust can be introduced as the imaginary line through which the resultant thrust (i.e. the axial force) acts in the masonry arch. The "catenary" is the line of thrust obtained when the masonry arch is subjected only to its own weight. Chosen a coordinate system (O,x,y) as illustrated in figure 1(a), the second order differential equation of the catenary, $y(x)$, and its solution are:

$$y''+\frac{f}{H}\sqrt{1+(y')^2}=0 \quad \Rightarrow \quad y=\frac{H}{f}\left[1-\cosh\left(\frac{f}{H}x\right)\right]+h \tag{1}$$

with f the uniform distributed vertical loading, h the rise of the arch and H the drift force.
The cases of nonuniform distributed vertical loading or of the presence of horizontal distributed loads are more complex, so that the analytical solution of the governing differential equations could not be determined. In such cases, different (simplified, approximated and numerical) approaches can be developed.
In the following, two methodologies based on the stress approach for the analysis of masonry arches are presented. The first approach is derived considering the equilibrium equations for the arch subjected to a distribution of point-wise forces acting on the nodes lying on the line of thrust. A nonlinear constrained minimization problem is formulated for determining the line of thrust for arches subjected to prescribed loading distributions. Then, the elasticity equations for the masonry arch is solved making use of the method of consistent deformations, i.e. the force method.
Numerical applications concerning circular arches subjected to uniform and nonuniform loading distributions are investigated, remarking the possibility of the structural collapse.

2. EQUILIBRIUM OF A MASONRY ARCH SUBJECTED TO POINT-WISE FORCES

The arch is considered subjected to a distribution of point-wise forces acting on the n nodes introduced on the line of thrust. In such a way, the arch is divided in $t=n-1$ segments. The position of the nodes of the line of thrust of the arch are determined adopting a polar coordinate system. Thus, the $k-$th node is defined by the angle θ_k and the radius R_k and, with reference to figure 1(a), its coordinates are:

$$x_k = R_k \sin\theta_k \qquad y_k = R_k \cos\theta_k \tag{2}$$

The components of the force applied on the k-th node ($k=2,..,n-1$), along the horizontal and vertical directions, are denoted by F_k and P_k, respectively (see figure 1(b)). The forces F_k and P_k represent the resultants of the external horizontal and vertical distributed loads, acting in the part of arch whose middle section corresponds to the k-th node.

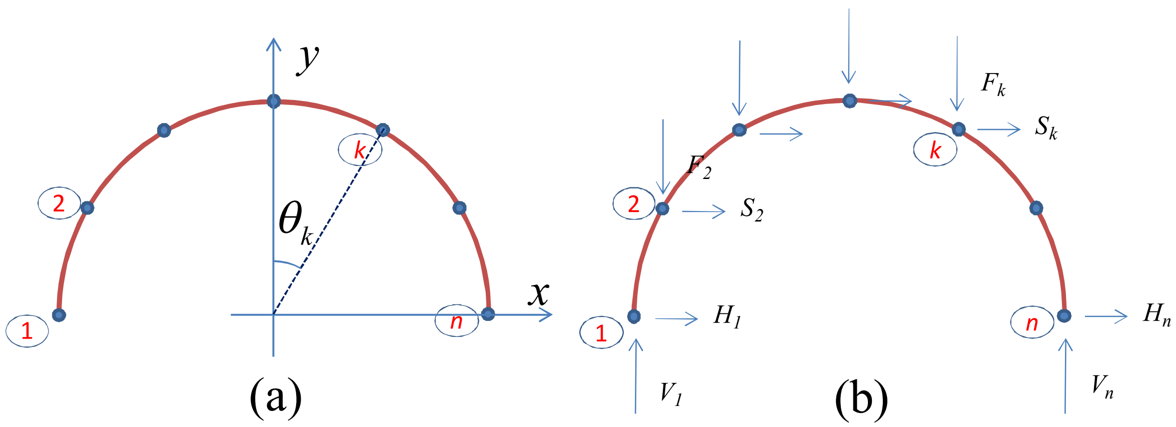

Fig. 1 Coordinate systems(a) and line of thrust of the arch subjected to point wise forces (b)

Note that the first node and the last node are subjected to reactive forces; in particular, the horizontal reactive forces H_1 and H_n are the drift forces acting on the arch.
The equilibrium equations for the first $t = n-1$ nodes of the line of thrust of the arch along the x direction result:

$$\mathbf{A}\,\mathbf{N}+\mathbf{S}=\mathbf{0} \qquad \text{with} \quad \mathbf{A}=\begin{bmatrix} -\alpha_1 & & & & & \\ .. & .. & & & & \\ & \alpha_k & -\alpha_{k+1} & & & \\ & & \alpha_{k+1} & .. & & \\ & & & .. & .. & \\ & & & & \alpha_{t-1} & -\alpha_t \end{bmatrix},\quad \mathbf{N}=\begin{Bmatrix} N_1 \\ .. \\ N_k \\ N_{k+1} \\ .. \\ N_t \end{Bmatrix},\quad \mathbf{S}=\begin{Bmatrix} H_1 \\ .. \\ S_k \\ S_{k+1} \\ .. \\ S_t \end{Bmatrix} \tag{3}$$

Analogously, the equilibrium equations for $n-2$ nodes, i.e. for the nodes from 2 to $t = n-1$, along the y direction result:

$$\mathbf{D}\,\mathbf{N}=\mathbf{F} \qquad \text{with}\ \mathbf{D}=\begin{bmatrix} \beta_1 & -\beta_2 & & & & & \\ & \beta_2 & -\beta_3 & & & & \\ & & .. & .. & & & \\ & & & \beta_k & -\beta_{k+1} & & \\ & & & & \beta_{k+1} & & \\ & & & & .. & .. & \\ & & & & & \beta_{t-1} & -\beta_t \end{bmatrix},\quad \mathbf{F}=\begin{Bmatrix} F_2 \\ F_3 \\ .. \\ F_k \\ F_{k+1} \\ .. \\ F_t \end{Bmatrix} \tag{4}$$

where α_k and β_k are the components of the unit vector along the direction defined by the nodes k and $k+1$:

$$\alpha_k = \frac{x_{k+1}-x_k}{\Delta_k} \qquad \beta_k = \frac{y_{k+1}-y_k}{\Delta_k} \tag{5}$$

with Δ_k the distance between the nodes k and $k+1$.
Considering all the written nodal equilibrium conditions, it is possible to derive $2n-3$ equations, while the unknowns are n, i.e. the $n-1$ axial forces N_1, N_2,..., N_{n-1} and the drift force of the arch H_1 at the node 1; the resulting structural scheme is hypostatic, as there are more equilibrium equations than unknowns.
Nevertheless, the $t \times t$ matrix $\mathbf{A}$ is invertible, so that the vector of the axial forces $\mathbf{N}$ can be computed as:

$$\mathbf{N} = -\mathbf{B}\,\mathbf{S} \qquad \text{with} \quad \mathbf{B}=\mathbf{A}^{-1} \tag{6}$$

Substituting formula (6) into equation (4), it results:

$$-\mathbf{D}\,\mathbf{B}\,\mathbf{S}=\mathbf{F} \tag{7}$$

Hence, assigning the value of the horizontal component of the external forces S_2, S_3, .., S_{n-1} and, moreover, prescribing also the value of the left drift force of the arch, H_1, it is possible to determine the unique set of vertical components of the external force F_2, F_3, .., F_{n-1} which equilibrates the hypostatic structural system.

3. THE LINE OF THRUST

Because of the formula (7), the vertical forces depend on the value of the components of the unit vectors α_k, β_k ($k = 1,2,..,t$), which, recalling expressions (2) and (5), are functions of the radii R_k with $k = 1,2,..,n$ defining the line of thrust. Thus, it can be deduced that the value of the forces is function of the radii R_k ($k = 1,2,..,n$), i.e. $F_i = F_i\left(R_1, R_2,.., R_n\right)$ with $i = 2,3,..,n-1$.

Considering a set of prescribed external vertical forces $\hat{F}_i$ with $i = 2,3,..,n-1$, the following conditions would be enforced:

$$F_i\left(R_1, R_2,.., R_n\right) = \hat{F}_i \qquad i = 2,3,..,n-1 \tag{8}$$

Equations (8) are rewritten in the residual form as:

$$e_i\left(R_1, R_2,.., R_n\right) = \left(F_i\left(R_1, R_2,.., R_n\right) - \hat{F}_i\right)^2 = 0 \qquad i = 2,3,..,n-1 \tag{9}$$

and the total residual quantity is determined as:

$$\Gamma\left(R_1, R_2,.., R_n\right) = \sum_{i=2}^{n-1} e_i\left(R_1, R_2,.., R_n\right) = \sum_{i=2}^{n-1}\left(F_i\left(R_1, R_2,.., R_n\right) - \hat{F}_i\right)^2 \tag{10}$$

As the arch has physical dimensions, admissibility conditions for the line of thrust have to be considered; in fact, the line of thrust cannot be outside of the arch, i.e. it has to be almost contained in the thickness of the arch, defined in each cross-section by the internal and the external radius, R_{int} and R_{ext}, respectively.

Finally, the problem of the determination of the shape of the line of thrust into the arch, which is able to equilibrate a set of external vertical forces $\hat{F}_i$ with $i = 2,3,..,n-1$, can be formulated in the evaluation of the set of radii R_k with $k = 1,2,..,n$, which satisfy the following set of equation:

$$\Gamma\left(R_1, R_2,.., R_n\right) = 0 \quad | R_{int} \le R_k \le R_{ext} \qquad k = 1,2,...,n \tag{11}$$

Problem (11) could have no solution and, because of the strong nonlinearity of the equation (11), when a solution exists it is not simple to determine. Remarking that $\Gamma \ge 0$, an approximated solution for the problem (11) can be evaluated determining the set of radii R_k with $k = 1,2,..,n$, which minimizes the expression of the residual (10), under the constraint that the typical radius R_k ($k = 1,2,..,n$) define a point which lies inside of the physical arch, i.e. the following problem is recovered:

$$\min_{R_k}\left\{\Gamma \,|\, R_{int} \le R_k \le R_{ext} \;\; k = 1,2,..,n\right\} \tag{12}$$

The problem (12) corresponds to a nonlinear optimization, which is also not trivial to solve. It can be remarked that, contrarily to the problem (11), problem (12) always admits solutions, as the residual $\Gamma\left(R_1, R_2,.., R_n\right)$ is not enforced to be zero and, as consequence, equations (8) could be not exactly verified. The minimization problem (12) could not admit a unique solution, but it gives a relation between all the radii R_k with $k = 1,2,..,n$ which can be written in the form:

$$g\left(R_1, R_2,..., R_n\right) = 0 \tag{13}$$

In order to define a unique position for the line of thrust inside the thickness of the arch, further requirements is introduced.

It can be remarked that a solution of the problem (12), or equivalently (13), is determined assigning a value for the drift force H_1. A way to recover a unique position of the line of thrust was proposed by Moseley, who introduced his (well-known) principle, which states that the line of thrust solution of the problem is determined in correspondence of the minimal pressure distribution. It is assumed that the minimal pressure distribution is attained when the drift force reaches the minimum value. On the other hand, the choice to minimize the drift force is also motivated by the circumstance that if the arch is subjected to an abutment settlement, the drift force decreases, until reaching a minimum value. Thus, the following ratio is introduced:

$$\rho = \frac{H_1}{\sum_{i=2}^{n-1} \hat{F}_i} \tag{14}$$

so that the unique line of thrust is determined minimizing the ratio ρ, under the condition that the drift force H_1 corresponds to the solution of the problem (12), i.e. under the constraint (13):

$$\min_{R_k} \left\{ \rho \mid g\left(R_1, R_2, ..., R_n\right) = 0 \right\} \tag{15}$$

Problem (15) can be reformulated as a unique minimization statement:

$$\min_{\lambda, R_k} \left\{ \hat{\Gamma} \mid R_{int} \leq R_k \leq R_{ext} \; k = 1,2,..,n \right\} \tag{16}$$

where

$$\hat{\Gamma}\left(R_1, R_2, .., R_n, \lambda\right) = \sum_{i=2}^{n-1} \left(F_i\left(R_1, R_2, .., R_n\right) - \lambda \hat{F}_i \right)^2 + \frac{\omega}{\lambda} \tag{17}$$

with $\omega = \zeta \rho$, being ζ a multiplier which gives a weight to the minimum condition of the quantity ρ. The formulation (16)-(17) is more suitable than (15) as it allows to fix the value of the drift H_1 and to find the multiplier λ of the forces $\hat{F}_i$ and the radii R_k $(k = 1,2,..,n)$ which correspond to the minimum error of the force distribution and to the ratio ρ.

In the following, the problem (17) is solved using a powerful routine based on the Sequential Quadratic Programming algorithm (SQP).

The ability of the proposed procedure to recover a geometrical position of the line of thrust and the values of the equilibrated forces applied on the structure is investigated considering a round arch. The determination of the line of thrust of minimal pressure for a round masonry arch characterized by a given geometry is performed. The arch has the internal and the external radii R_{int} = 4000 mm, and R_{ext} = 5000 mm respectively. Calculations are performed assuming H_1 = 10000 kN and considering the number of total nodes equal to n = 52.

In figure 2, the line of thrust for the arch subjected to a uniform distribution of vertical forces is plotted. Together with the line of thrust determined using the proposed numerical procedure, in the same figure it is reported also the catenary curve determined from the analytical solution given in formula (1). It can be remarked the perfect agreement of the two determined curves, showing the effectiveness of the simple proposed algorithm in the determination of the line of thrust.

Then, the case of nonuniform distribution of vertical forces acting on the arch is investigated. In particular, a piecewise uniform loading condition is considered for the arch; in fact, the span of the arch is divided in two parts: in the first part, i.e. $x \leq 0$, a uniform distribution of vertical forces is considered, in the second part, i.e. $x > 0$, the vertical forces are obtained multiplying the value of the forces applied for $x \leq 0$ by a factor η, with η varying from 0.75 to 0.18. Note that the solution given in figure 2 corresponds to the case $\eta = 1.00$.

In figure 3, the lines of thrust for the arch subjected to the nonuniform vertical forces distributions are reported. It can be noted that the line of thrust significantly changes for the different loading conditions; in fact, decreasing the ratio η the line of thrust becomes more and more unsymmetrical.

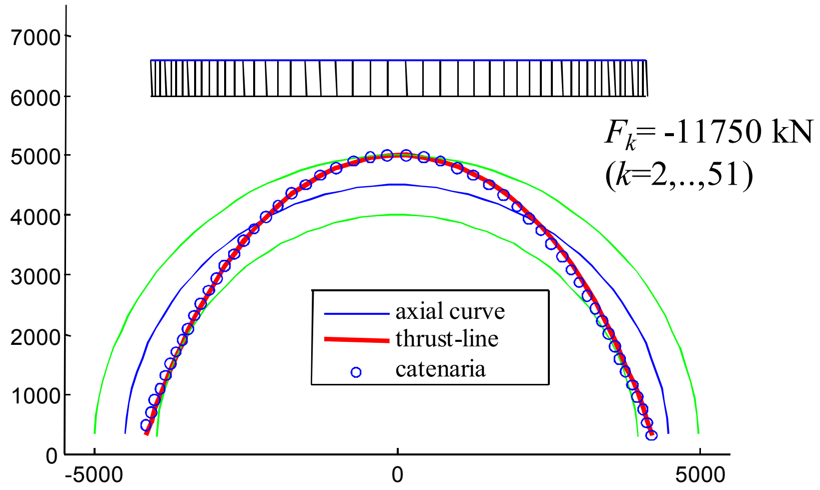

Fig. 2 Round arch subjected to uniform vertical force distribution: line of thrust (continuous line) and catenaria (circles)

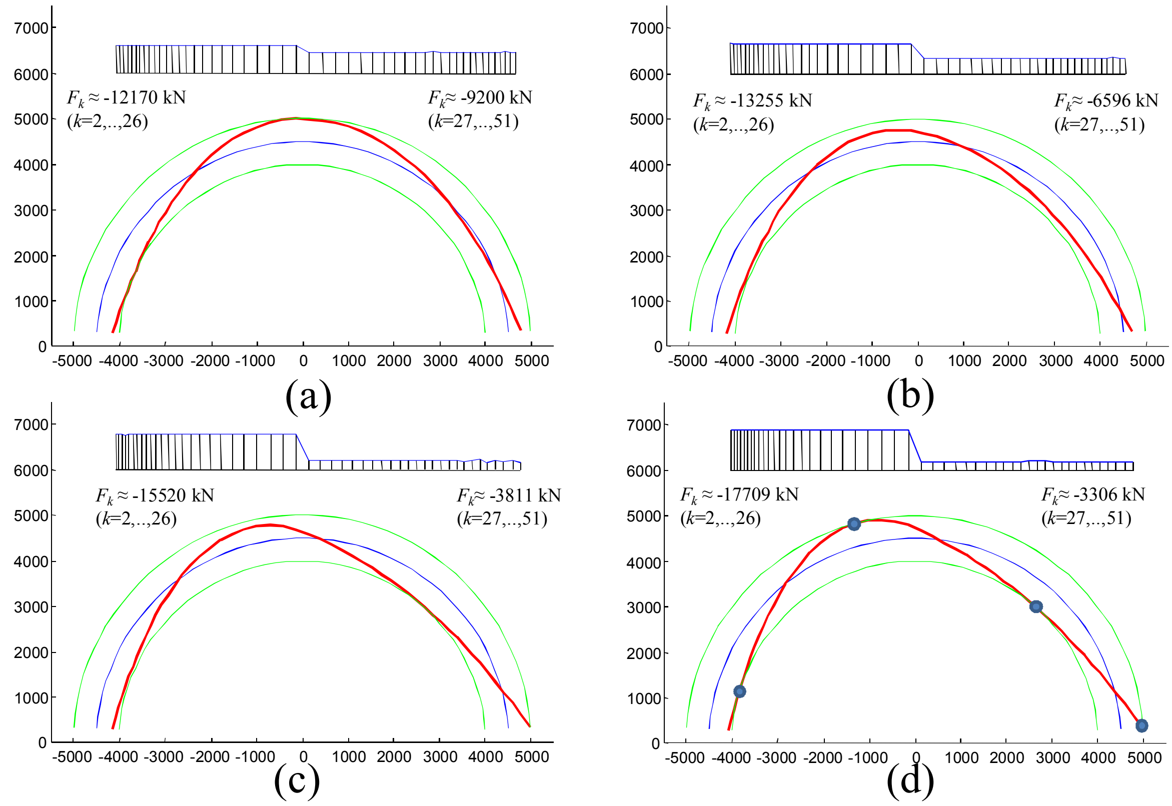

Fig. 3 Round arch subjected nonuniform vertical forces distribution:
(a) $\eta = 0.75$, (b) $\eta = 0.50$, (c) $\eta = 0.25$, (d) $\eta = 0.18$

Values of η lower than 0.18 are not admissible as the line of thrust should be positioned outside of the physical arch. In fact, for $\eta = 0.18$ the curve of the pressures touches the surfaces of the arch in four points, where four hinges take place, two at the intrados and two at the extrados, leading to a hypostatic structural scheme, as illustrated in figure 3. In other words, the loading condition characterized by $\eta = 0.18$ represents a limit equilibrium state for the arch, i.e. a collapse condition. It can be remarked that the proposed procedure and minimization routine is able to find a solution even for values lower that $\eta = 0.18$. Of course, for distributions of the load $\hat{F}$ for which the line of thrust would be outside of the physical arch, the algorithm is able to find a solution whose residual value $\hat{\Gamma}$, corresponding to the error between the prescribed force distribution $\hat{F}$ and the equilibrated forces F, increases, reaching unacceptable values.

4. THE ELASTIC ARCH

The solution of the masonry arch can be derived making use of the elasticity theory, i.e. writing the equilibrium, the constitutive and the compatibility equations.
Regarding the constitutive law, the no-tension model is introduced; it is based on the three well known hypotheses concerning the masonry behavior: the masonry has no tensile strength, the masonry has infinite compressive strength, the sliding failure cannot occurs. Thus, the following stress-strain relationship is considered:

$$\sigma = h(\varepsilon)E\varepsilon \qquad \text{with } h(\varepsilon) = \begin{cases} 1 & \text{if } \varepsilon \leq 0 \\ 0 & \text{if } \varepsilon > 0 \end{cases} \tag{18}$$

with E the Young modulus of the masonry. As consequence of the constitutive equations (18), the typical cross section A of the arch is split in two parts: A_{nt}, i.e. the no tension part of A, where $\varepsilon > 0$ and $\sigma = 0$; A_e, i.e. the elastic part of A, where $\varepsilon \leq 0$ and $\sigma = E\varepsilon$. The parts A_{nt} and A_e are not known *a priori*, as they are defined by the position of the neutral axis, in correspondence of which the strain is equal to zero.
The structural problem is solved adopting the method of consistent deformations, sometimes referred to as the force or flexibility method, as proposed in [16]. In fact, with reference to figure 4, the statically undetermined clamped arch is transformed into a statically determined structure subjected to the unknown reactive forces x_1, x_2 and x_3. The unknown forces are computed, according to Müller-Breslau's principle, enforcing the compatibility conditions $v_B = \delta$, $w_B = 0$ and $\varphi_B = 0$, which states that, at the section B of the arch, the horizontal displacement is equal to the possible settlement δ, the vertical displacement is zero and the rotation is zero.
The compatibility equations can be written in vectorial form as:

$$\mathbf{s}_p + \lambda \mathbf{s}_q + \sum_{i=1}^{3} x_i \mathbf{s}_i = \boldsymbol{\delta} \tag{19}$$

where $\mathbf{s}_p$ and $\mathbf{s}_q$ are vectors of 3 components which assume the physical meaning of the displacements of section B associated to the permanent and to the variable loadings **p** and **q**, respectively, while and $\mathbf{s}_i$ with $i = 1,2,3$ are displacement vectors due to the statically undetermined forces, when they assume the unit value. Moreover, λ represents the multiplier of the variable distributed load **q** and $\boldsymbol{\delta} = \{\delta, 0, 0\}^T$.
It can be remarked that, because of the considered elastic nonlinear constitutive law (18), the vectors $\mathbf{s}_p$, $\mathbf{s}_q$ and $\mathbf{s}_i$ depend on the solution, as they are functions of the partition of the cross section of the masonry into the no tension part and the elastic part. In fact, it is:

$$\mathbf{s}_p = \frac{1}{E}\int_{\theta_i}^{\theta_f} \mathbf{H}\hat{\mathbf{c}}_p \bullet \hat{\mathbf{c}}_j R\,d\theta \qquad \mathbf{s}_q = \frac{1}{E}\int_{\theta_i}^{\theta_f} \mathbf{H}\hat{\mathbf{c}}_q \bullet \hat{\mathbf{c}}_j R\,d\theta \quad \mathbf{s}_i = \frac{1}{E}\int_{\theta_i}^{\theta_f} \mathbf{H}\hat{\mathbf{c}}_i \bullet \hat{\mathbf{c}}_j R\,d\theta \quad \mathbf{H} = \begin{bmatrix} A_e & S_e \\ S_e & I_e \end{bmatrix}^{-1} \tag{20}$$

being A_e, S_e and I_e the area, the statical moment and the moment of inertia of the elastic part of the cross section, respectively. The two components of the vectors $\hat{\mathbf{c}}_p$, $\hat{\mathbf{c}}_q$ and $\hat{\mathbf{c}}_j$ represent the axial force and the bending moment at the typical cross section of the arch, equilibrated with the external loads **p** and **q** and with the unknown reactive force x_j, respectively.
A numerical procedure able to solve the nonlinear problem (19) is developed, able to define the behavior of the structure along the loading path. Furthermore, it is possible to evaluate the limit load for the structure, i.e. the load multiplier λ which induces the collapse of the arch. In order to evaluate the whole nonlinear structural response of the arch and to compute the limit load, an arc-length technique, based on the stress formulation, is developed. Details on the computational procedure can be found in ref. [16].

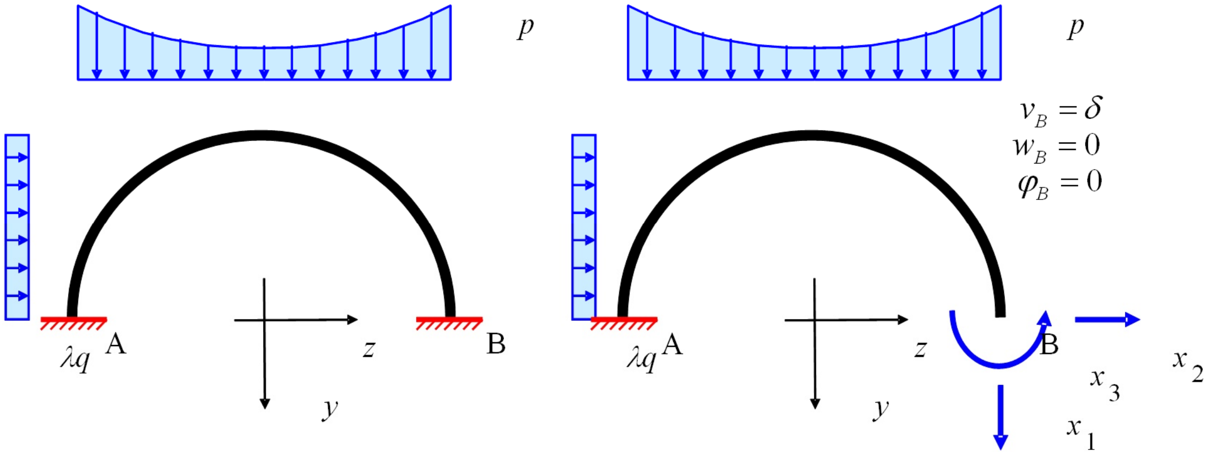

Fig. 4 Clamped arch and statically equivalent arch

The elastic analysis of the arch considered in the previous section is performed, considering different values of the settlement δ. In fact, in figure 5 the position of the line of thrust is plotted for the arch subjected to an uniform distributed load and to the settlement of the right impost, is plotted. It can be remarked that for higher values of δ the position of the line of thrust tends to the one determined in the previous section, where the drift is minimized. In figure 5 the effective part, i.e. the elastic part A_e, of the cross sections of the arch is shown.

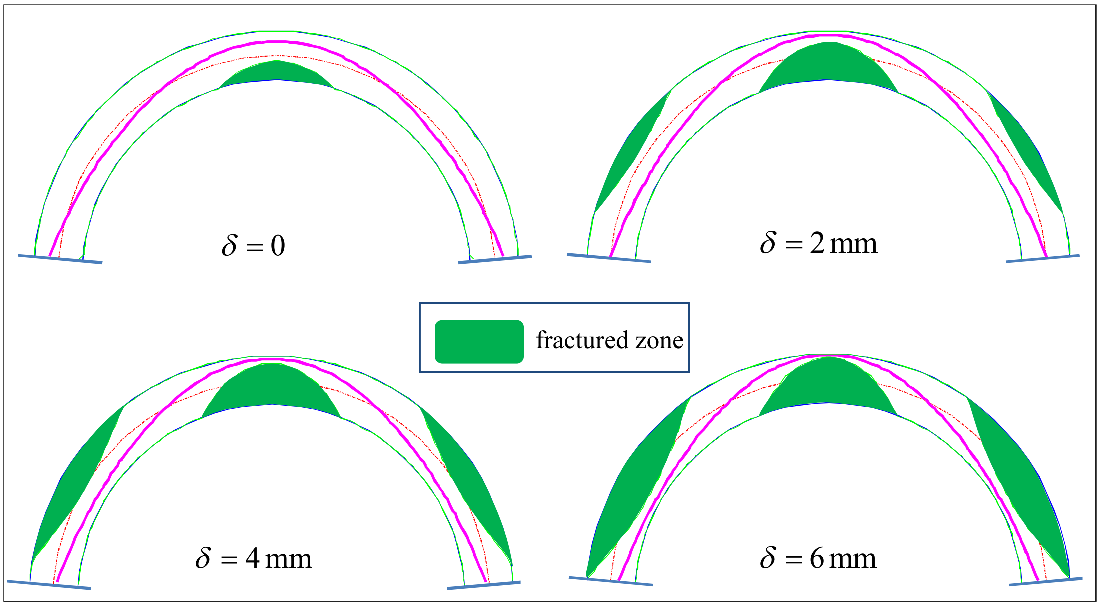

Fig. 5 Arch subjected to a distributed vertical load and to a settlement of the right impost

5. CONCLUSIONS

Although the masonry arch is an ancient structural element and many studies have been developed during the centuries, the research in this field is still active for at least two reasons. From one hand, the research has a historical knowledge intent, as it is aimed to better capture the insights, the ideas and the results of the greatest scientists of the past that have addressed the issue of stability and safety of the arches and vaults. From the other hand, the research is devoted to the use and development of innovative structural analysis techniques for the evaluation of the safety of masonry constructions and for the design of restoration and strengthening interventions.

ACKNOWLEDGEMENTS

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