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EXPERIMENTAL AND NUMERICAL STUDY OF THE BUCKLING FAILURE OF MASONRY WALLS

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ABSTRACT

This paper presents an experimental and numerical research on the buckling failure of masonry walls oriented to assess and improve the provisions of structural masonry codes. Available experimental results on the buckling failure of masonry walls have been reviewed and complemented with new experiments of a large number of 1/4 scale walls. A detailed parametric study on the capacity of walls has been carried out using a well-known numerical tool whose ability to accurately model the wall buckling failure has been verified by comparison with the available experimental results. The study has allowed a wide sample of results corresponding to different values of the slenderness ratio, loading eccentricity, masonry elasticity modulus and masonry tensile strength. The formulation of the European standard EN 1996-1-1:2005 for the verification of walls axially loaded is then compared with these results. Alternative methods are provided for a more accurate estimation of the load bearing capacity of brick masonry walls.

Keywords: *Load bearing brick masonry wall, Buckling failure, Micro-modelling*

1. INTRODUCTION

As is well known, the capacity of load bearing walls subjected to axial loading, or to combined axial and out-of-plane loads, is largely influenced by non-linear geometric effects and instability. Buckling failure is possible, in particular, in thin and/or tall walls showing moderate to high slenderness. Historical and traditional construction may exhibit masonry load bearing walls or piers of significant slenderness subjected to sensible vertical loading, as in the case of urban masonry buildings built during the 19^{th} and 20^{th} Cs. in some European cities. In other cases, as in more ancient or traditional buildings, walls show only moderate slenderness, but instability problems are still possible due to the construction irregularities, producing large load eccentricities, and the larger deformability of the masonries.

Compared to other problems linked to the strength of masonry walls (such as in-plane loading), the buckling failure of walls has deserved only limited research effort. The first studies were carried out early during the first half of 20th c. with the purpose of contributing to the first structural masonry codes. Chapman and Slatford [1], Yokel [2] and Frish-Fay [3, 4] investigated the stability of columns and masonry walls to solve the governing equation for the lateral deflection of the members. Later studies [5-8] obtained different solutions for the critical axial load taking into account the influence of material nonlinearity. The stability of pinned-end masonry walls subjected only to eccentric vertical load has been widely investigated, both analytically [1, 2, 9-12] and experimentally [13-18]. Other authors [19-21] have considered uniformly distributed lateral load. Ganduscio and Romano [7] investigated the case of a cantilever wall subjected to eccentric vertical load combined with concentrated and uniformly distributed horizontal load, while Romano et al. [5] proposed an analytical solution for the case of a cantilever wall subjected to eccentric vertical load and horizontal concentrated load acting on the free end. Stability of a cantilever wall under self-weight and eccentric vertical load acting on free end has been investigated by La Mendola and Papia [22], La Mendola [6]

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and more recently by Mura [8], while the stability of a cantilever wall subjected to self-weight was investigated by Frish-Fay [3]. In this type of problem an analytical solution is difficult to obtain when the non-linearity in the masonry stress-strain relation and the tensile strength are taken into account. In fact, both early and more recent researches [1, 2, 15, 19, 22] have normally assumed a linear behaviour in compression while at the same time have neglected the tensile strength of masonry. A common feature of experimental investigations is found in the large scattering of the obtained results, which appears to increase with the slenderness of the walls. The main reason for such large scattering is found in unavoidable but largely influencing small eccentricities caused by irregularities in the wall geometry and load positioning [23].
Structural masonry codes have generally adopted empirical approaches for the verification of the strength capacity of masonry walls taking into account the buckling failure. In particular, the formulation of Eurocode-6 [24] is based on the capacity reduction factors for slenderness and eccentricities proposed by Kukulski and Lugez [9], originally developed from tests on mortar walls.
The more recent research in the field is mostly oriented to seismic assessment and focuses on the response of walls subjected to axial load in combination with out of plane effects such as transverse loading or lateral movements. However, and due to some evidence showing insufficiently satisfactory agreement between experimental results and the methods adopted by the codes, it is believed that additional research on the behaviour of walls subjected to purely vertical and eccentric axial loading is still necessary.
The research presented in this paper has been aimed to the proposal an improved method for the verification of masonry walls subjected to axial loading. For that purpose, the available experimental results have been reviewed and enlarged with a new experimental programme on 1/4 scale walls. The range of available results has been largely widened through intensive numerical simulation based on a well-known micro-model approach. The ability of this numerical tool to adequate simulate the instability of walls under eccentric vertical loading has been verified by comparison with the available experimental results. Using all the experimental and numerical evidence made available, two alternative methods have been derived for an accurate prediction of the capacity of axially loaded masonry walls. These methods are intended to improve the accuracy of present code formulations (in particular, that of Eurocode-6 [24]).

2. EXPERIMENTS

A first step of the present research has consisted of the review of previous experimental results. For that purpose, the experimental researches by Watstein and Allen [13], Hasan and Hendry [14], Fattal and Cattaneo [15], Kirstchig and Anstötz [16] and, dealing with walls hinged at both ends subjected to concentric or eccentric vertical loading, have been selected. The selected experimental programmes cover a wide range of values of compressive strength and deformation modulus (Table 1).

Table 1 Different parameters of experimental researches

	f_c (N/mm^2)	E/f_c	scale
Watstein and Allen [13]	39.2	540	1:1
Hasan and Hendry [14]	17.3	376	1:3
Fattal and Cattaneo [15]	31.1	620	1:1
Kirstchig and Anstötz [16]	12.2	700	1:1
Present research	14.2	244	1:4

Aiming to enlarge the available experimental evidence on the capacity of vertically-loaded walls, a new experimental programme has been developed on 1/4 scale masonry specimens. The technique used for the construction and testing on the experimental scale walls is directly based on previous experience gathered in the Structural Technology Laboratory of Universitat Politècnica de Catalunya [25, 26]. Solid bricks measuring 72.5 × 35 × 12.5 mm^3 were elaborated on purpose simulating conventional manufacturing conditions. A compressive strength of $f_b = 32.5$ N/mm^2 and an average Young's modulus of $E_b = 4080$ N/mm^2 was determined for the bricks according to standardized procedures. A specific micro-mortar was designed to keep the granulometry in proportion with the scale and to limit the maximum aggregate size to 1 mm to guarantee mortar penetration in the joints.

The micro-mortar compression strength was of f_m = 7.3 N/mm^2. An average masonry compressive strength of f_c = 14.2 N/mm^2 and a Young's modulus E = 3458 N/mm^2 (resulting in E = f_c) were determined by testing five standard masonry prisms. In addition, direct tensile tests yielded an average tensile strength f_t = 0.55 N/mm^2.

The experimental programme included the test up to failure of 36 scale walls with uniformly distributed vertical load (Fig. 1). The walls were built with slenderness of 6.8, 12.6, 18.7 and 25.6 were tested with load eccentricities e = 0, e = t/6 and e = t/3, where t is the wall thickness. Hence, a series of 3 walls was tested for each combination slenderness and eccentricity. In the experiments, both the upper and the lower hinges were located so as to cause the same eccentricity at both ends. The hinged condition was produced by means of neoprene pads placed at the wall ends. A summary of test results is given in Table 2, including the average values ($\bar{x}$) and the coefficient of variation ($C.V.$). As observed in previous experiments, the experimental values show significant scattering with average coefficient of variation of 10%. The scattering is more apparent for the cases with large slenderness and/or large load eccentricity.

The experimental results obtained in the present research are compared with those of other authors in Fig. 2 for the extreme cases e = 0 and e = t/3. In this figure, the term σ/f_c refers the ratio between the average compression stress at failure (σ) and the masonry compressive strength (f_c). The comparison involves results with similar support conditions (top and bottom hinged supports). In the diagrams, the results for each wall tested within each series are plotted individually to allow an appreciation of the scattering. Despite the different scales and materials used for the different campaigns, a similar trend is observed. The differences among the different experimental campaigns are within the limits of the scattering individually obtained in them.

Fig. 1 Wall loading arrangement (left), and typical failure modes (middle and right)

Table 2 Test results (failure average stress in N/mm^2)

Wall Nº	Slenderness ratio h/t	Eccentricity								
		e=0			e=t/6			e=t/3		
		σ	$\bar{x}$	$C.V.$	σ	$\bar{x}$	$C.V.$	σ	$\bar{x}$	$C.V.$
W6-1	6.8	12.3			8.0			3.9		
W6-2	6.8	14.0	12.7	8.6%	8.9	8.5	5.6%	2.9	3.7	19.5%
W6-3	6.8	11.9			8.7			4.3		
W12-1	12.6	10.9			8.0			2.1		
W12-2	12.6	11.6	11.6	6.8%	7.3	7.6	5.0%	2.0	2.2	11.2%
W12-3	12.6	12.5			7.4			2.5		
W18-1	18.7	10.1			4.4			1.5		
W18-2	18.7	9.7	10.4	9.1%	4.8	4.5	5.9%	1.3	1.4	10.5%
W18-3	18.7	11.5			4.3			1.2		
W25-1	25.6	8.9			2.6			1.3		
W25-2	25.6	6.7	7.6	15.1%	3.0	2.8	8.0%	1.1	1.1	13.9%
W25-3	25.6	7.2			2.7			1.0		

3. NUMERICAL SIMULATION

The numerical analyses have been performed using the micro-modelling approach proposed by Lourenço and Rots [27]. A minimum eccentricity of 1 mm has been always applied in cases with zero-eccentricity in order to account for the possible irregularities of the wall geometry or the load positioning. More information on the numerical properties adopted for the numerical simulation can be found in [18, 28]. A comparison between experimental and numerical results, for different experimental campaigns, is presented in Fig. 3. As can be seen, the agreement between the numerical and experimental response is satisfactory. It should be noted that the most of numerical results are contained within the limits of the experimental scattering. The comparison with the present and other author's experimental results shows that the micro-model correctly captures the experimental behaviour in terms of both deformability and strength capacity.

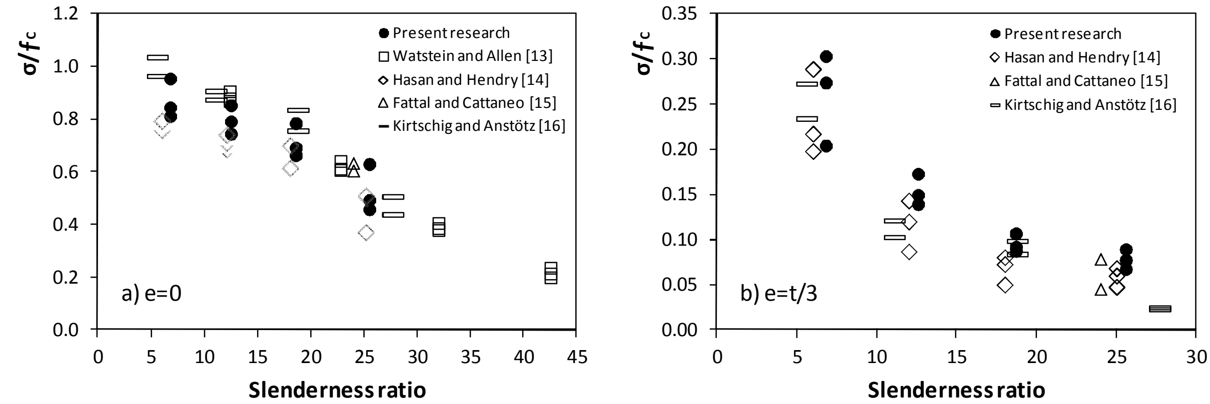

Fig. 2 Compression stress at failure (expressed as $/f_c$) against slenderness ratio for e=0 (left) and e=t/3 (right)

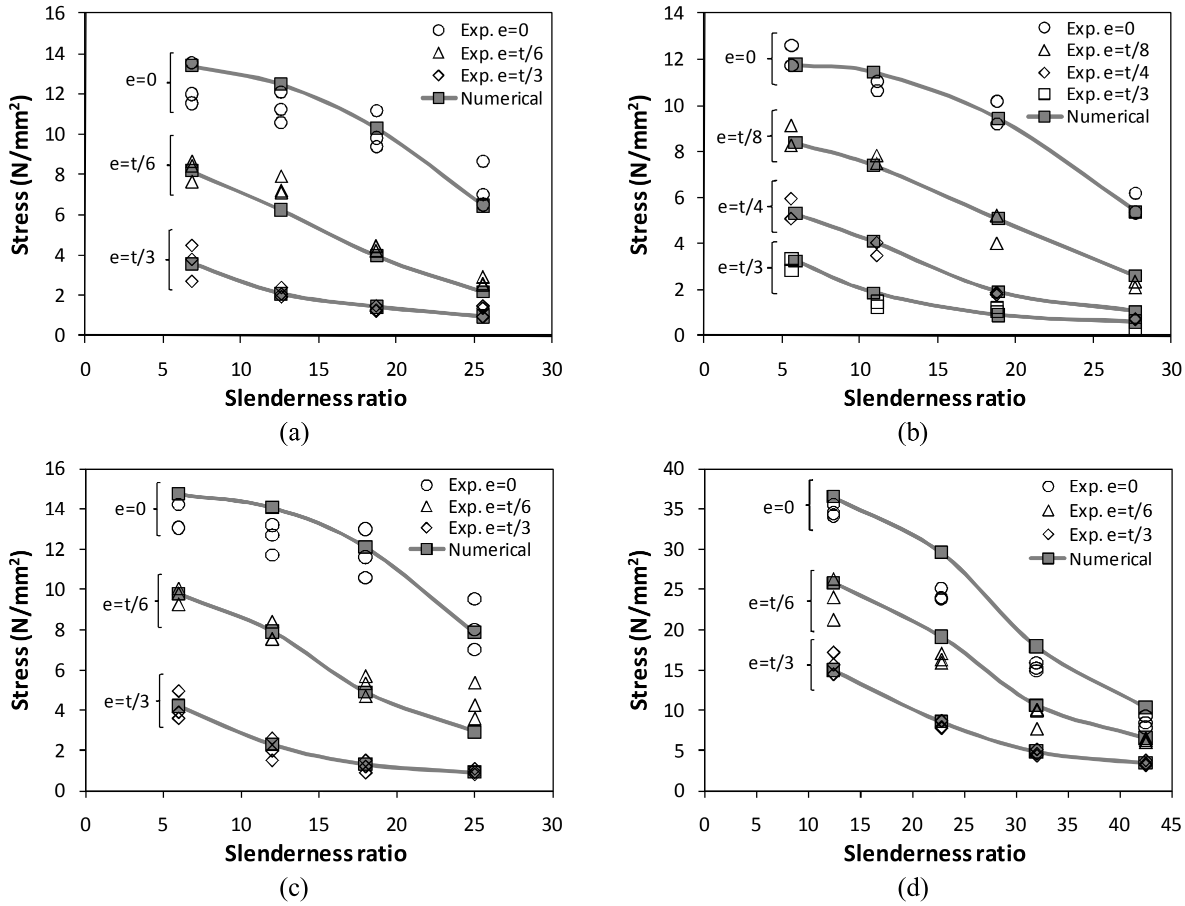

Fig. 3 Comparison between numerical and experimental ultimate capacities for (a) present research, (b) Kirstchig and Anstötz [16], (c) Hasan and Hendry [14], (d) Watstein and Allen [13]

4. COMPARISON WITH EUROCODE-6 [24]

The comparison between selected experimental results and results calculated with Eurocode-6 [24] is presented in Fig. 4. The formulation of Eurocode-6 [24] has been applied taking into consideration the experimentally measured E/f_c ratios. As can be observed, the method proposed by Eurocode-6 [24] tends to conservatively underestimate the strength of the walls. The underestimation increases with the slenderness ratio and eccentricity. In practical calculations, this underestimation may be counterbalanced by the consideration of a larger Young modulus, such as $E = 1000f_c$, as often assumed.

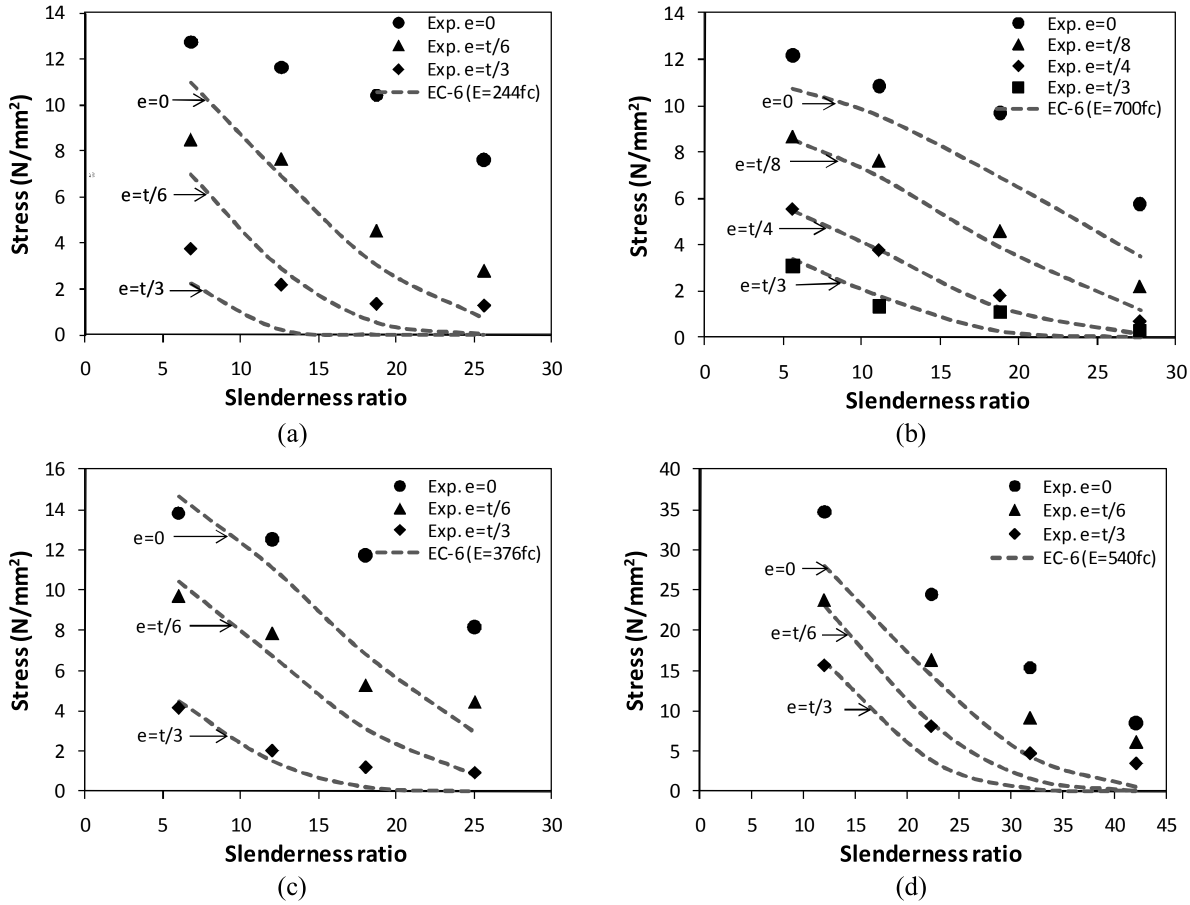

Fig. 4 Comparison between values predicted by Eurocode-6 [24] with experimental results: (a) present research, (b) Kirstchig and Anstötz [16], (c) Hasan and Hendry [14], (d) Watstein and Allen [13]

5. PROPOSED FORMULATIONS

Two different methods are presented for the estimation of reduction factors complying with the experimental and numerical results considered in the present research. For this purpose, two different already proposed equations are adopted and modified to improve their agreement with the numerical simulation results. The first method consists of a slight modification of Eurocode-6 equation [24]. According to Eurocode-6, the reduction factor for slenderness and eccentricity in the middle fifth of the wall height is calculated as:

$$\Phi = \left[\left(1 - 2\frac{e}{t}\right) exp(-0.5u^2)\right] \tag{1}$$

with

$$u = \frac{\overline{\lambda} - \alpha}{\beta - \rho\frac{e}{t}} \tag{2}$$

Using the numerical simulation results, coefficients α, β and ρ are modified and adjusted to 0.0756, 1.075 and 2.26 respectively (the original values used in Eurocode-6 [24] are 0.064, 0.73 and 1.17 respectively). In the equation (2), the non-dimensional parameter $\overline{\lambda}$ is given by,

$$\overline{\lambda} = \frac{h}{t}\sqrt{\frac{f_c}{E}} \qquad (3)$$

Fig. 5 shows a comparison between numerical results and the predictions of equation (1), using the new values adjusted for the parameters α, β and ρ. As can be observed, the use of the new adjusted coefficients improves significantly the agreement with the numerical results obtained for walls with small slenderness and null or small eccentricity.

Fig. 5 also shows a comparison of the reduction factors Φ resulting from Eurocode-6 [24]. The factors Φ are calculated as the ratio between the maximum resisted average stress (taking into account the instability effects) and the masonry compressive strength. As with the experimental results, Eurocode-6 [24] shows some underestimation of the maximum capacity of the wall compared with the numerical simulation. This underestimation increases with the flexibility and slenderness of the wall. In particular, Eurocode-6 [24] predicts very small or null capacity to walls which, according to the numerical results, still have meaningful capacity, even if largely reduced by the non-linear geometric effects.

The second method is based on Lu's equation [29], where the critical load is obtained as the product of an equation for null eccentric by another representing the effect of load eccentricity. Using a similar approach, and after the necessary adjustments, the following equation is proposed for the calculation of the reduction factors,

$$\Phi = \Phi_e\left[\frac{1}{1+1.1\overline{\lambda}^2}\,exp\left(\frac{\overline{\lambda}^2}{1+\overline{\lambda}^{4.5}}\right)\right] \qquad (4)$$

where Φ_e, representing the effect of load eccentricity, is given by,

$$\Phi_e = \left(1-2\frac{e}{t}\right)^{\frac{3.5\overline{\lambda}^2+0.65}{\overline{\lambda}^2+0.65}} \qquad (5)$$

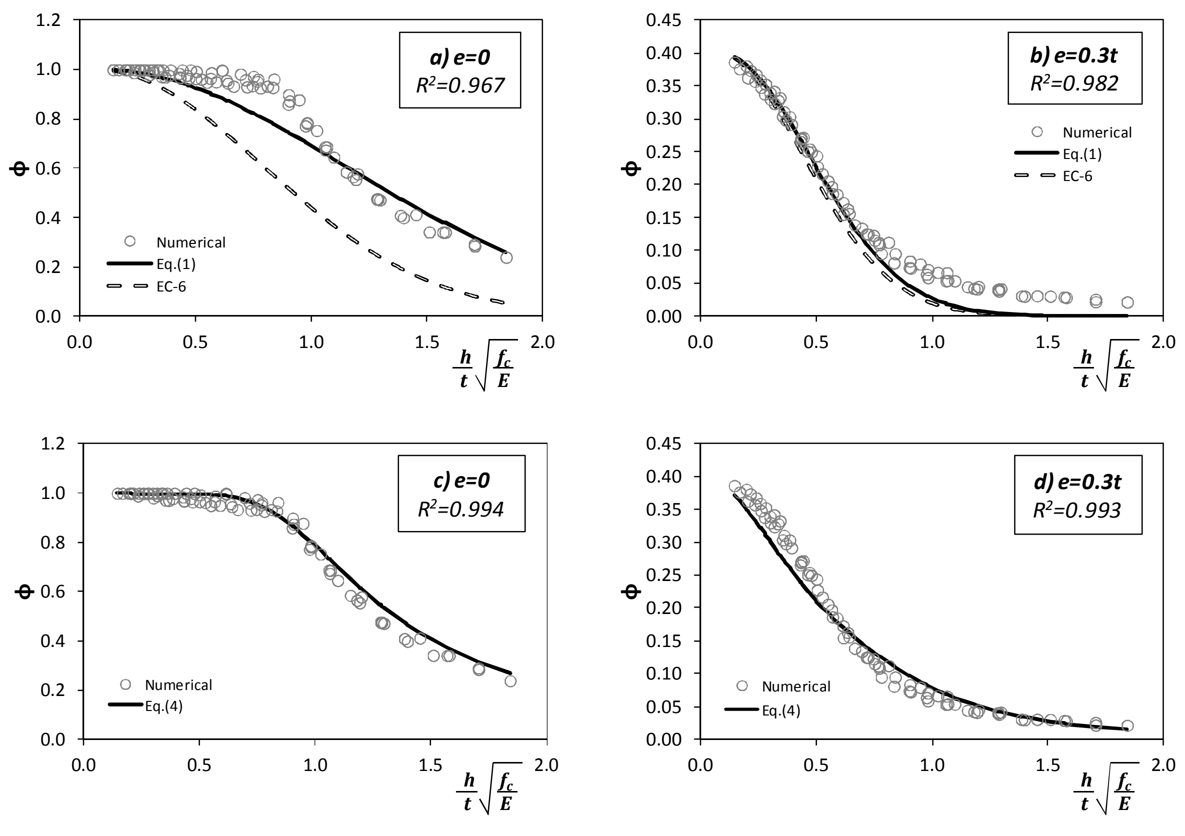

Fig. 5 Comparison of numerical data with predictions of equation (1), with new adjusted coefficients (above), and equation (4) (below)

As can be observed in Fig. 5, equation (4) compares very satisfactorily with the numerical data. The goodness of fit of the resulting curves is high, with an average correlation coefficient of 0.994.
The present research has explored also the use of yet a more accurate method for the calculation of the reduction factors resulting as a best-fit equation among a very large collection of mathematical descriptions. In this third method, the gain in accuracy has a significant cost in terms of algebraic complexity. A modified formulation has been also proposed to including the influence of a non-null tensile strength. More information of these proposals can be found in Sandoval [28], along with a more detailed description of the methods presented and their derivation from the numerical simulation results.

6. CONCLUSIONS

An experimental research has been carried out on the strength capacity of brick masonry walls subjected to concentric and eccentric vertical loading. In spite of the scattering typically obtained in this type of experiments, the results obtained agree satisfactorily with previous experimental evidence obtained by other authors. Based on these experimental results, the applicability of the well-known multi-surface interface model in the simulation of the buckling failure of vertically loaded brick walls has been assessed into detail. Simulations carried out by means of this model have provided satisfactory fits for all the experimental campaigns analysed, with an average error of 11.4%.
The numerical tool has been utilized for a comprehensive parametric analysis involving the variation of the main parameter relevant for the problem (wall slenderness, eccentricity, masonry compression strength, masonry elastic modulus and tensile strength). The comparison of numerical results with the method of Eurocode-6 [24] has shown that the latter tends to conservatively underestimate the strength of the walls. The underestimation increases with the slenderness ratio and eccentricity.
Based on parametric analysis, three alternative methods for a more accurate estimation of the wall load bearing capacities have been derived (two of which are presented in this paper). In particular, a method based on Lu’s approach [29] has allowed very high correlation coefficients (larger than 0.99) when compared with the numerical results.

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