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SIMPLIFIED ASSESSMENT OF THE STRUCTURAL SAFETY OF HISTORICAL CONSTRUCTIONS

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Abstract. *In the studies of conservation above historical constructions the structural analysis is a determining factor to diagnose potential problems of load capacity in their structural elements. However due to the complexity of structural system, it is necessary to use methods of advanced structural analysis, such as: finite elements and rigid elements. Therefore, the objective of this paper is to propose a simplified procedure to carry out a fast preliminary assessment to determine if it's sufficient with this assessment or in its defect warrants a structural analysis more detailed with some advanced method.*

In this research different criteria to evaluate the structural safety of the structural elements (walls, columns and cover systems) that make up the structuring of historical constructions were reviewed. This allowed derive simplified criteria to make a fast preliminary assessment of the structural safety of historical constructions, taking into consideration the following concepts of assessment: vertical load capacity of walls or columns, seismic assessment, allowable tilt in walls or columns, allowable angular deformation in walls, stresses in elements of cover curve and level of damage observed in the building. With these concepts the construction can be qualify in any of the following categories: safe, caution or dangerous. To show the application of the simplified procedure, the structural safety of four parishes were evaluated: Santiago Apóstol, Santa Cruz del Monte, Santa María Nativitas and San Bartolomé Apóstol; these buildings correspond to the colonial period.

The application of the simplified procedure to these parishes allowed to verify that the result is almost similar to the obtained from an advanced analysis using finite elements, therefore the proposed procedure is recommended for a preliminary assessment of this type of buildings; given that if such an assessment finds the building as dangerous, will require an advanced structural analysis for their final assessment.

1 INTRODUCTION

In Mexico, there are several historical buildings, where most of them have been declared heritage of humanity; however have some degree of deterioration that puts at risk its structural safety. The level of damage goes from the appearance of cracks in the masonry until partial collapses of domes and towers; even there have been cases in which is at risk the stability of the whole by what has been required underpinning interior and exterior of buildings. This type of construction has accumulated during its "useful life" problems of progressive instability due to the weathering that degrades the strength of materials, in addition to the damage caused by the action of the recurrent earthquakes and in some cases the effects of land subsidence.

In the studies of conservation a determining factor is the structural analysis for effect to diagnose potential problems of load capacity in their structural elements. Due to the complexity of structural system, it is necessary to use methods of advanced structural analysis, such as: finite elements and rigid elements; both in linear range as non-linear [1]. However, applying these methods involves a deep work of mathematical modeling of the building geometry, which brings with it that may not be able to gain a preliminary evaluation simple and fast on the structural state of construction.

Therefore, the objective of this paper is to propose a simplified procedure to evaluate the structural safety of historical constructions, in such a way that have good approximation with respect to obtained valuations of advanced methods of structural analysis.

2 BASIC STRUCTURAL ELEMENTS OF HISTORICAL CONTRUCTIONS

In historical constructions from Mexico has been seen that usually the foundations were resolved on the basis of spread footing of masonry. However, in some cases the foundation may consist of a grid of foundation beams and a thick masonry mat over a dense array of short timber piles.

The walls of these buildings can reach more than two meters in thickness and heights greater than 20 m. The walls are connected with the systems of cover to receive their loads and transmit them to the foundation (figure 1), and also the walls are accompanied by buttresses. The walls are built of masonry, although in some cases can be of adobe, brick or a composition of several materials [2].

The columns have a cross-section fairly generous came to handle areas higher than 4 m^2 and heights that can reach more than 19 m. Arches and pendentives that receive the download of the barrel vaults and domes are supported on columns (figure 1). The columns were built in one piece or by the union of ashlar stone, the material that was used was masonry or quarry, among others.

The covers vary from one type to another construction, as well as buildings of religious character was peculiar use of great covers of curved surface, mainly constituted by barrel vaults and domes (figure 1); the material of construction may be a heterogeneous masonry consisting of basalt stone, bound together by a mortar, also can be a masonry bulkhead of mud. While in constructions destined to housing were used flat ceilings with wooden beams with tables or with bricks and earth fill.

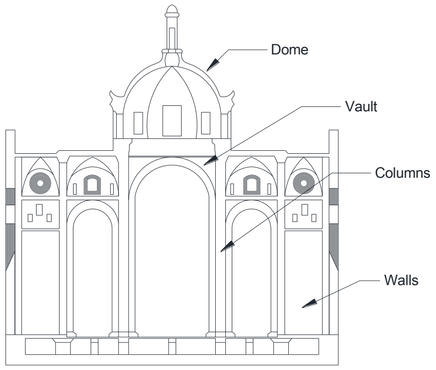


Figure 1: Basic structural elements of historical constructions

3 STRUCTURAL BEHAVIOR

The structural system of these constructions can be described as a gravity system in which the external actions imposed by the action of its self-weight and lateral load is transmitted by compressive stresses to columns, walls and buttresses.

The walls and columns are elements that do not have problems of instability by buckling before the considerable weight who have to endure, furthermore that the compressive stress are kept at moderate levels, which is primarily attributed to the generous dimensions of its cross section.

The lateral loads to which they are subjected these walls must be to seismic forces and to the thrusts that are generated by the covers, both forces can act in the plane of wall or in the direction perpendicular to plane of wall. The presence of crack diagonals are due to lateral loads that act in the plane of wall when the shear stress exceeds the masonry strength and are favored when there are openings for doors and windows [3]. With respect to the lateral loads acting perpendicular to the plane are associated with inertia forces caused by an earthquake [4, 5], therefore can be viewed the following mechanisms of damage: turning the wall (figure 2a), failure by flexure (figure 2b), inclined cracks (figure 2c) and failure in flexion by effect of plate (figure 2d).

The tilt of walls and columns is a transcendent aspect for its stability; this effect is generated primarily by differential settlements and thrusts of covers.

Covers such as vaults and domes are elements that work primarily to compressive stresses. Furthermore, their thicknesses considerable contribute to maintain moderate stress in materials.

The internal forces that are generated in the hemispherical domes, acting in the meridional and parallel of these domes generating stresses known as meridional and annular, respectively (figure 3). A strip of the dome, defined according to the meridians, has a similar behavior to of an arch with the same guideline of dome. The stresses of meridional membrane always are compression and the annulars are of compression or tension as it is taken up or down the neutral plane as shown in figure 3.

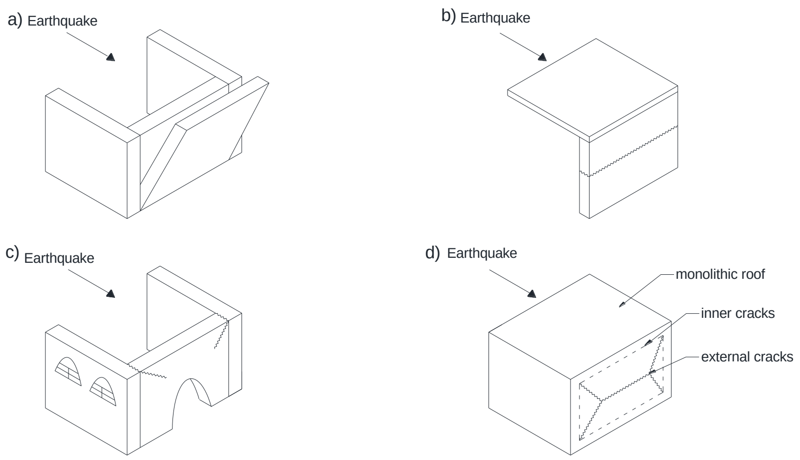


Figure 2: Failure modes in walls due to the earthquake [2]

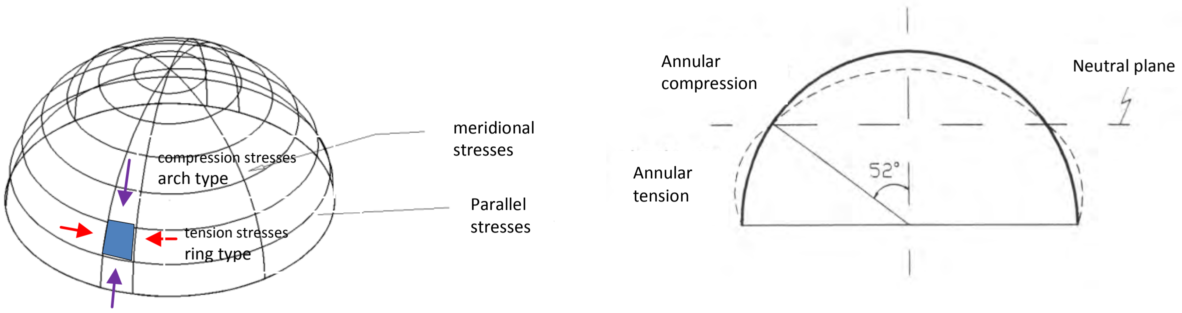


Figure 3: Internal stresses, meridional and parallel, in a hemispherical dome

4 PROPOSAL FOR SIMPLIFIED ASSESSMENT OF THE STRUCTURAL SAFETY

A simplified procedure to carry out a fast preliminary assessment is proposed, in order to determine if it's sufficient with this assessment or in its defect warrants a structural analysis more detailed with some advanced method. The following describes some simple criteria to have an estimate on the conditions of structural safety of their principal structural elements: walls, columns and cover systems.

4.1 Load capacity in walls and columns

To evaluate the loading capacity to compression of walls and columns can be used the equation that recommends the Uniform Building Code (UBC) for masonry structures [6], that is given by:

$$F_a = 0.20 f_m \left[1 - \left(\frac{h}{42t} \right)^3 \right] \quad (1)$$

where, F_a is the allowable compressive strength of element, f_m is the design compressive strength of masonry, h and t are height and thickness of wall, respectively. In the case of columns h and t are height and least lateral dimension of column, respectively. The reason for recommending the equation 1 is due to that the stresses calculated are similar to those that prevail in this type of construction, which has been able to corroborate in studies of the structural safety of historical buildings with the use of finite element models and measurements in the structure [7].

4.2 Seismic assessment

For the seismic assessment of historic buildings the simplified method can be used, which specifies the Federal District Code [8], when taking in consideration that its support elements is based on load-bearing walls. The method consists in comparing the total lateral force imposed by the earthquake (V_a) and capacity of structure that has to resist such seismic force as the sum of all the walls aligned in the direction of analysis (V_R), using the following relationship that gives rise to a safety factor (F_s):

$$F_s = \frac{V_R}{V_a} = \frac{\sum A_m V_m}{cw} \tag{2}$$

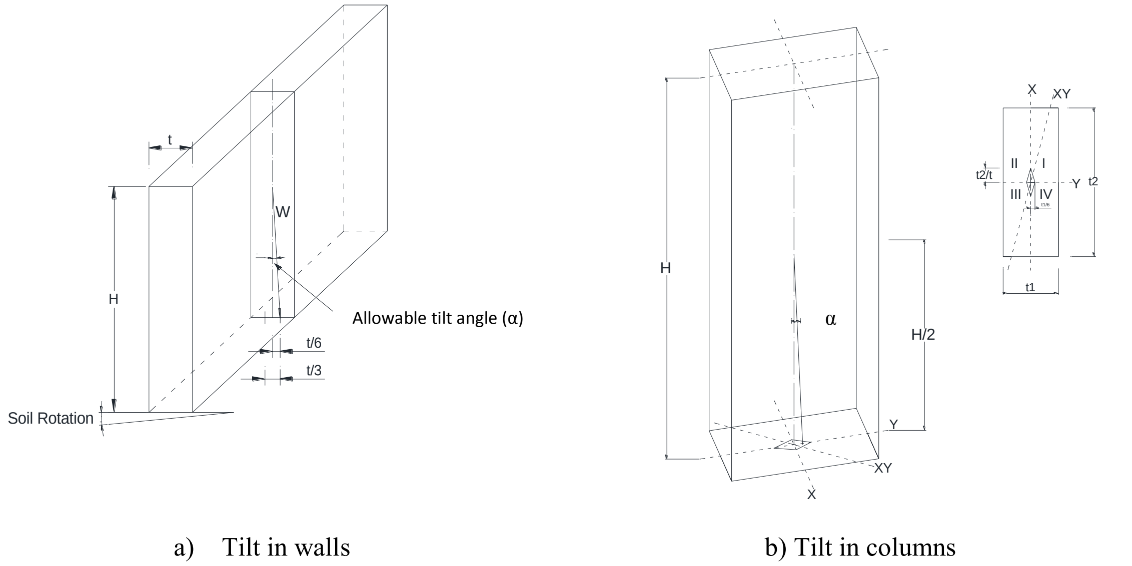
where, A_m is the cross sectional area of wall, w is the total weight of building, V_m shear strength of masonry and c represents the seismic coefficient. To ensure the safety of building, F_S must be greater than the unit.

4.3 Allowable tilt in walls and columns

To analyze the tilt in walls and columns as a result of soil differential settlement the tilt angle of walls and columns can limit to an allowable value, in such a way that they do not suffer overturn and that the stresses do not reach the material resistance. Heyman [9] provides that the allowable tilt angle (α) will be such that the resulting of gravitational force acts just at the limit of middle third of the thickness of wall or the base of column (figure 4), so that the expression to evaluate it is given by:

$$Tan\alpha = \frac{1}{3}\frac{t}{H} \tag{3}$$

where, t and H are the thickness and height of wall or column, respectively.



a) Tilt in walls   b) Tilt in columns

Figure 4: Allowable tilt angle according to Heyman

4.4 Allowable angular deformation in walls

The angular deformation in these walls is caused by the soil differential settlement and it is quantified in terms of drift, relationship of the soil differential settlement and the wall length. This deformation is manifested through the cracks that suffer the walls; therefore the analysis should focus toward an allowable value of the angular deformation that does not generate considerable cracking and therefore guarantee the safety of the wall. Laboratory tests on typical historic buildings from the Mexico City have shown that for a drift of 1/2000 the material develops 50 per cent of its resistance to shear, which could serve as a reference to set this value as allowable drift in this type of constructions [2].

4.5 Evaluation of stresses in elements of cover curve

Based on the membrane equations, can be estimated the stresses in vaults and hemispherical domes. For the case to assess the magnitude of the meridional forces, T, it is necessary to calculate the meridional radio, R, and the angle, ø, which form the radio, in the point of interest, with the upright as shown in figure 5, where, a and f are the base radio and arrow of the vault, respectively, which are calculated from the following equations

$$R = \frac{a^2 + f^2}{2f} \quad (4)$$

$$Sen\phi = \frac{a}{R} \quad (5)$$

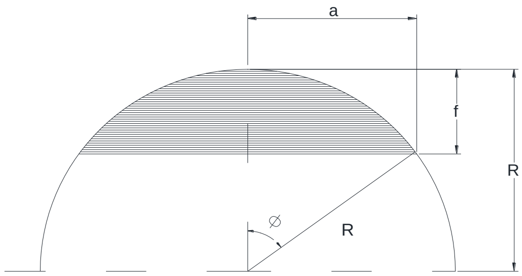


Figure 5: Geometric data to calculate the stresses in a recessed hemispherical vault

Known the selft-weight per unit area of the vault, W, the force per unit meridional paralelo, T, is defined by the following expression:

$$T = \frac{WR}{1 + Cos\phi} \quad (6)$$

If ϕ=0°, $T = \frac{WR}{2}$

If ϕ=90°, $T = WR$

The annular force, H_o, defined by unit of meridian is expressed by:

$$H_o = WR\left(\frac{Cos^2\phi + Cos\phi - 1}{1 + Cos\phi}\right) \quad (7)$$

If ϕ=0°, $H_o = \frac{WR}{2}$ compression in the crown

If ϕ=90°, $H_o = -WR$ tensile in the base

The annular stresses can be of compression and tension; to know where it causes the sign change is necessary to determine the neutral plane. This plane can be obtained by $H_o = 0$ in expression 7, so the angle of this plane is $\phi = 51.49$ '.

In the case of domes, the meridional and annular forces are affected by the presence of the opening that have in their crown to allow lighting in the interior of the temple (figure 6). To take into account the influence of the opening in the structural behavior of dome is proposed the following expressions:

$$T = WR\frac{Cos\phi_o Cos\phi}{Sen^2\phi} \quad (8)$$

$$H_o = WR\left(Cos\phi - \frac{Cos\phi_o - Cos\phi}{Sen^2\phi}\right) \quad (9)$$

The angles involved in these expressions are measured with respect to an axis that passes tangent to the hollow as shown in figure 6b. To define the neutral plane, must be satisfied that the annular forces are null, that is to say, $H_o = 0$; in this case the angle remains in function of the dimensions of the opening in the crown of the dome.

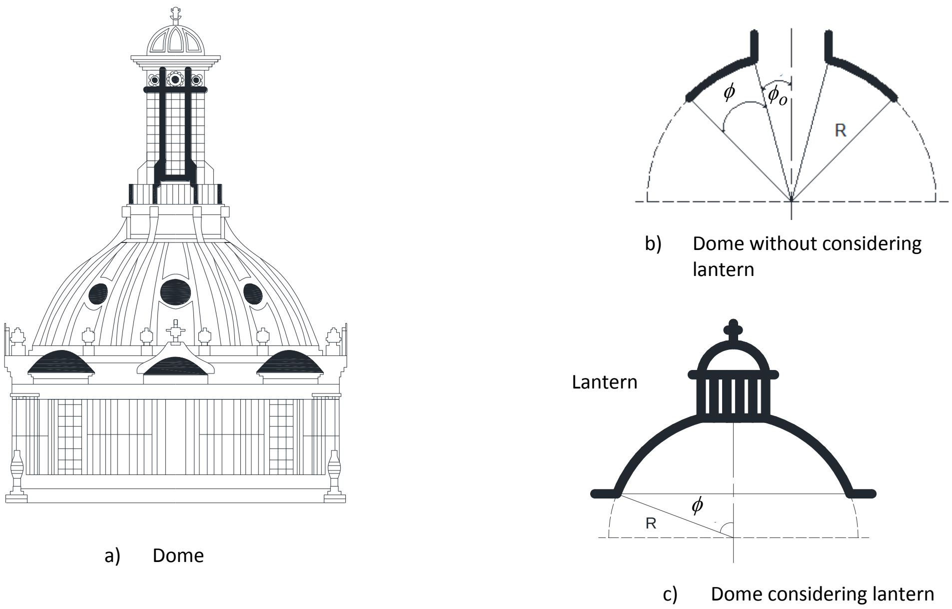


Figure 6: Geometric data to calculate the stresses in domes

4.6 Assessment format

Based on the simplified criteria described above, it is recognize the following basic concepts to make a fast preliminary assessment of the structural safety of historical constructions: vertical load capacity of walls or columns, seismic assessment, allowable tilt in walls or columns, allowable angular deformation in walls, stresses in elements of cover curve and level of damage observed in the building. These concepts give rise to the evaluation format that is shown in table 1.

In table 1 refers to the conditions of security for each of the concepts, where is granted a qualification via the safety index by concept (I_i); $I_i = 1$, if conditions of security are favorable; $I_i = 2$, if conditions of security are in the limit; $I_i = 3$, if conditions of security are unfavourable. The final evaluation of the structural safety is carried out based on the final security index (I_F), whose equation is given by:

$$I_F = \frac{\sum I_i}{6} \tag{10}$$

The interpretation of I_F is described below. If $1.00 \le I_F \le 1.20$, the construction is evaluated as *safe building* and is conceived as a building that presents at least five concepts with favorable conditions of security and one with condition in the limit. If $1.20 < I_F < 2.30$, the construction is considered as *caution building,* because has at least two concepts with unfavorable conditions of security. If $2.30 \le I_F \le 3.00$, the construction is finds as *dangerous building* and has at least four concepts with unfavorable conditions of security.

5 APPLICATION OF THE SIMPLIFIED PROCEDURE

5.1 Cases of study

To show the application of the simplified procedure, the structural safety of four parishes were evaluated: Santiago Apóstol, Santa Cruz del Monte, Santa María Nativitas and San Bartalomé Apóstol (figure 7). These buildings correspond to the colonial period (16th, 17th and 18th centuries) and are located in the municipality of Naucalpan, Estado de México, Mexico.

Figure 7: Analyzed historical constructions

These parishes are comprised of a main nave with cover systems of curved surface, barrel vault and dome, with their respective bell tower at the main facade whose height is approximately 14 m.

In regard to the state that saved these buildings, vertical cracks in some walls with detachment of flattened are observed; in the arches of the barrel vaults and columns can be seen deterioration in their boards.

Table 1: Format for simplified preliminary assessment of the structural safety of Santa Maria Nativitas Parish

Concept	Conditions of security	Safety index by concept (I_i)
Vertical load capacity of walls (F_a) $F_a = 0.2(32.5)\left[1-\left(\frac{740}{42(70)}\right)^3\right]$ $F_a = 6.40\ kg/cm^2$	If $F_a > \sigma_a$, $I_i = 1$ If $F_a = \sigma_a$, $I_i = 2$ If $F_a < \sigma_a$, $I_i = 3$ σ_a acting stress in the wall	$I_i = 1$ In taking as reference the wall with greater stress, acting stress, σa = 2 kg/cm^2, so it is lower to F_a.
Seismic assessment $F_{sx} = \frac{V_{Rx}}{V_a} = \frac{1432.76t}{297.92} = 4.80$ $F_{sy} = \frac{V_{Ry}}{V_a} = \frac{924.44t}{297.92t} = 3.10$	If F_{sx} and $F_{sy} > 1$, $I_i = 1$ If F_{sx} or $F_{sy} = 1$, $I_i = 2$ If F_{sx} or $F_{sy} < 1$, $I_i = 3$ F_{sx} safety factor in x F_{sy} safety factor in y	$I_i = 1$ In both directions the construction offers a large safety factor by earthquake.
Allowable tilt in walls (α) $\alpha = Tan^{-1}\left(\frac{1}{3}\frac{t}{H}\right) = Tan^{-1}\left(\frac{1}{3}\frac{0.70m}{7.40m}\right) = 1.80^o$ $\Delta_{ALLOWABLE} = Hsen\alpha = 0.23\ m$	If $\alpha > \alpha_a$, $I_i = 1$ If $\alpha = \alpha_a$, $I_i = 2$ If $\alpha < \alpha_a$, $I_i = 3$ α_a tilt in the Wall	$I_i = 1$ The walls have inclinations (α_a) lower than 1^o. The wall before soil settlement may experience a tilt less than 23 cm for it to remain stable.
Allowable angular deformation in walls (γ_p) -Allowable settlement $\Delta = \gamma_p\ L = \frac{1}{2000}(600cm) = 0.3\ cm = 3\ mm$ -Start of damage $\Delta = \gamma_{cr}\ L = \frac{1}{1000}(600cm) = 0.6\ cm = 6\ mm$ L Length of wall	If $\gamma_p > \gamma_a$, $I_i = 1$ If $\gamma_p = \gamma_a$, $I_i = 2$ If $\gamma_p < \gamma_a$, $I_i = 3$ γ_a angular deformation in the Wall	$I_i = 1$ The building presents no settlements, so $\gamma_a = 0$. Taking as reference the wall more long (600 cm), it is estimated that if the wall is experiencing settlements exceeding 3 mm begin to suffer some damage.
Stresses in elements of cover curve -Meridional stress (T, compression) $T = WR\frac{Cos\phi_o Cos\phi}{Sen^2\phi} = 1.10\ \frac{kg}{cm^2}$ -Annular stress (H_o, tensile) $H_o = WR\left(Cos\phi - \frac{Cos\phi_o - Cos\phi}{Sen^2\phi}\right) = 0.79\ \frac{kg}{cm^2}$	If T and $H_o < R_m$, $I_i = 1$ If T or $H_o = R_m$, $I_i = 2$ If T or $H_o > R_m$, $I_i = 3$ R_m strength of masonry	$I_i = 1$ The meridional and annular stresses are below the resistance of masonry, on the assumption that the dome is built with masonry of natural rock; compression = 32.50 kg/cm^2; tensile = 6.43 kg/cm^2; according to the reference [2].
Level of damage observed	If Light damage, $I_i = 1$ If Moderate damage, $I_i = 2$ If Serious damage, $I_i = 3$	$I_i = 1$ Light damage are observed in the parish
	Final safety index (I_F)	1
	Final assessment	Safe building

5.2 Simplified preliminary assessment of the structural safety

Table 1 shows an example about the use of the assessment format applied to the parish of Santa Maria Nativitas, which was used for the evaluation of the rest of historical buildings, whose results are summarized in table 2.

Table 2: Summary of the simplified preliminary assessment of the structural safety of the Parishes

Concept	Safety index by concept (I_i)			
	Santiago Apóstol	Santa Cruz del Monte	Santa María Nativitas	San Barto-lomé Após-tol
Vertical load capacity of walls (F_a)	1	1	1	1
Seismic assessment	1	1	1	3
Allowable tilt in walls (α)	1	1	1	1
Allowable angular deformation in walls (γ_p)	2	1	1	1
Stresses in elements of cover curve	1	1	1	1
Level of damage ob-served	1	1	1	1
I_F	1.17	1	1	1.33
Final assessment	Safe building	Safe building	Safe building	Caution building

In the table 2 can be seen that the parishes of Santa Cruz del Monte and Santa Maria Nativitas are considered safe buildings, which is attributed to the acting stresses in walls and covers curves are below the allowable stresses, furthermore the seismic safety is favorable, the tilt angle and deformation of the walls are below the permissible values, coupled with light damage. In the case of the parish of San Bartolomé Apóstol, it was found that the deformation angle of one of the walls is similar to the allowable value, however by having the other concepts in favorable conditions of safety is determined that the building is safe. The parish of San Bartolomé Apóstol is considered as building with caution due to that in the seismic assessment was found that the design earthquake can overcome the resistance that offer the support elements [10]; according to a historical background, during the Acambay earthquake (1912) the structure suffered severe damage, which has relation with the evaluation.

5.3 Comparison of the simplified procedure with advanced methods

In order to compare the simplified procedure with advanced methods, were carried out structural analyzes of parishes based in finite element models (figure 8). As regards the analysis by gravitational load, the calculated stresses are presented in table 3, the stresses reported are lower than the strength of typical masonry of these historical buildings from Estado de México, where its strength to compression and tension are greater than 15 kg/cm^2 and 1 kg/cm^2, respectively, so the revised buildings are safe; this conclusion agrees with the deduced from the simplified assessment.

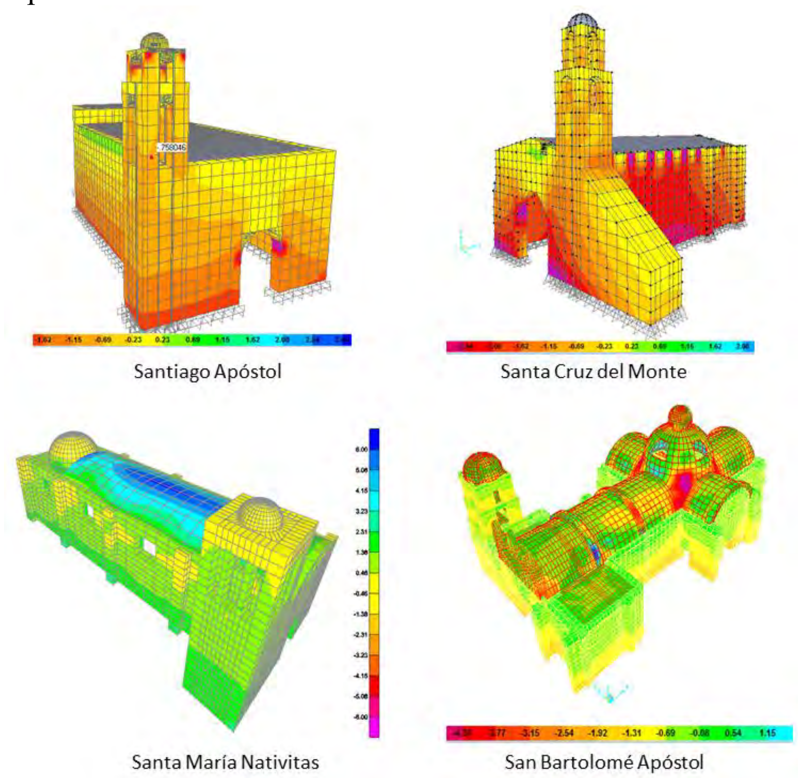


Figure 8: Finite element models of the parishes

For the analysis by earthquake, was carried out a modal spectral analysis and it was found that the values of shear stress demanded do not exceed the strength of masonry, so the buildings are safe; this conclusion is similar to the obtained with the simplified procedure for only three parishes, but not as well, for the case of San Bartolomé Apóstol, where such procedure assessed as building with caution; with the finite element model it was found that the building is safe, because with such modeling is possible to consider the contribution of other structural elements to resist seismic shear, as are: buttresses, pilasters and the structure of the bell tower.

Table 3: Stresses obtained with finite element models

Structural element	Acting stresses by gravitational load (kg/cm^2)			
	Santiago Apóstol	Santa Cruz del Monte	Santa María Nativitas	San Bartolomé Apóstol
Walls	2.72	2.02	2.16	1.32
Barrel vault	-	-	4.73	2.00
Dome				
Meridional strees	0.23	0.70	2.21	5
Annular strees	0.23	0.62	0.46 (tension)	0.29 (tension)

6 CONCLUSIONS

A procedure to carry out an assessment of the structural safety of historic buildings is proposed, it is based on simple criteria to analyze critical concepts of its structural performance:

vertical load capacity of support elements and systems of curved surface, tilt angle and deformation of walls by soil settlement, revision of the seismic safety, furthermore to take into consideration the level of damage observed in the building. With these concepts the building can be qualify in any of the following categories: safe, caution or dangerous.

The application of the simplified procedure to assess the structural safety of four parishes, allowed to verify that the result is almost similar to the obtained from an advanced analysis using finite elements, therefore the proposed procedure is recommended for a preliminary assessment of this type of buildings; given that if such an assessment finds the building as dangerous, will require an advanced structural analysis for their final assessment.

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