SAHC2014 – 9th International Conference on
Structural Analysis of Historical Constructions
F. Peña & M. Chávez (eds.)
Mexico City, Mexico, 14–17 October 2014

TIMOSHENKO COMPOSITE BEAM MODEL OF STEEL BEAM AND BRICK MASONRY FOR FOUNDATION UNDERPINNING OF MASONRY BIULDING

Y. OUYANG[1], **J. J. MA**[1], **N. K. LIU**[2], **X. YANG**[1], Q. FU[3]

[1] Department of Civil Engineering
Shanghai University, 149 Yanchang Road, Shanghai 200072, China
e-mail: oyy_wly@sina.com, 540780789@qq.com, xyang@shu.edu.cn

[2] Shanghai Tongrui Civil Engineering Co. Ltd
Room 802 of Tower B, 1398 Siping Road, Shanghai 200092, China
e-mail: vicliunengke@163.com

[3]Shanghai Conservation Center of Historical Buildings
99 Beijing West Road, Shanghai 200003, China
e-mail: fu_qin2008@sohu.com

Keywords: Brick masonry, Foundation underpinning, Composite beam, Arch effect, Underpinning length.

Abstract. *Shanghai is famous for its diversities of cultures and architectures, and there exist a lot of historical masonry buildings. To preserve them safely, the reinforcement and rehabilitation are need frequently. Compared with the researches and practices on reinforcements of the brick columns and concrete beams, etc, the theoretical study on the composite structure of the steel beam and brick masonry is dropped behind. In this paper, for foundation underpinning of the brick wall of masonry structure, based on the theory of Timoshenko composite beam, regarding the steel clamping beam and brick masonry wall as elastic materials, the governing equation for bending deformation of composite beam composed of steel beam and part of brick masonry wall was established, and the analytical expressions of the deflection and stresses of the steel-masonry composite beam were presented for single stage foundation underpinning. Then, considering the arch effect of the brick wall, the maximum deflection and maximum stresses of the steel-masonry composite beam for different models of the I-type steel clamping beam were obtained, and the maximum length of the foundation underpinning for single stage was given. The results were compared with those of Euler steel-masonry composite beam. It is shown that deflection and stresses of the steel-masonry composite beam decrease with increase of the model number of the I-type steel clamping beam. However, the changes of the loading borne by the steel beam and the tensile force of the steel bolts are trivial, and the influence of the Poisson's ratio on the maximum deflection of the composite beam is little. These results can provide theoretical guidance for specific engineering practices.*

1 INTRODUCTION

At present, a large number of historical buildings constructed with masonry structures in Shanghai, as well as the existing palaces and temples in China are needed urgently to reinforce and rehabilitate. And at the same time, the residential masonry structures constructed earlier in China also turn into the period of the reinforcement and rehabilitation. Therefore, the research on theory and technology of the reinforcement and rehabilitation of the masonry structures has practical significance and has become one of the concerned problems in civil engineering[1]. Compared with the researches and engineering practices on reinforcements of the brick columns and concrete or timber beams[2-5], the theoretical study on the composite structure of the steel beam and brick masonry is dropped behind[6-7]. In the removing wall with supporting beam and foundation underpinning, Tong and Pen[8] fulfilled the wall-removal of an office building by use of the steel-masonry composite beam, and it is revealed that the wall-removal with steel-masonry composite beam has advantages of reducing structure weight and enhancing clear space height of structure, etc. Furthermore, Deng et al[9] studied experimentally the mechanical behavior of the steel-masonry composite beam, and the influences of steel plate thickness and pouring materials on the strains of the cross-section and the bearing capacity of the composite beam were examined. However, to the best of authors' knowledge, there is no research report on the theoretical analysis of the composite beam composed of steel beam and brick masonry wall.

As for the problem of foundation underpinning of an external wall of a historic brick masonry building in Shanghai, China, based on the model of Timoshenko elastic composite beam, the governing equation for bending deformation of composite beam composed of steel beam and brick masonry was established. Considering the arch effect of the brick masonry wall, the maximum deflection and maximum stresses of the steel-masonry composite beam for different model of I-type steel clamping beam were obtained, and the maximum length of the foundation underpinning for single stage was given. The differences of the results between the Timoshenko and Euler composite beam models were examined. All these results can provide theoretical guidance for specific engineering practices.

2 BENDING DEFORMATION OF THE STEEL-MASONRY COMPOSITE BEAM

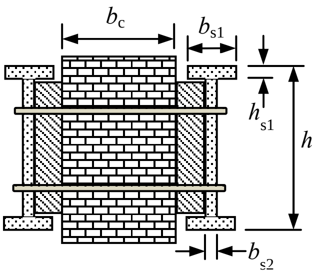

Figure 1: Composite cross-section of brick wall and steel

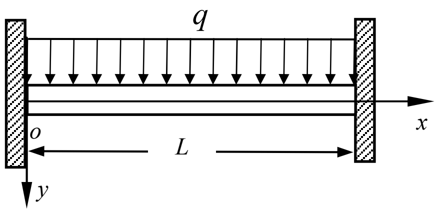

Figure 2: Clamped steel-brick composite beam subjected to a uniform load

A brick masonry wall which is required to underpin its foundation is shown in Figure 1. The brick wall and steel beams are connected by the steel studs which are uniformly distributed along wall length and give rise to tight pressure N per unit length between the steel beam and brick wall. In order to guarantee the tight connection between the steel beams and brick wall, the wood blocks are filled up between the steel beams and brick wall, so that the steel beams, wood block and brick wall deform compatibly. Since the elastic modulus of the wood block is small, the influence of the wood block on the deformation is neglected. The two steel beams and the portion of brick masonry wall between them is equivalent to a Timoshenko composite beam, and the weight of the upper brick masonry wall is equivalent to transversal load q, then, a model of the steel-masonry composite beam with two clamped ends is established and shown in Figure 2.

Denote the length of the steel-masonry composite beam to be L, the thickness and height of the brick wall to be b_{c} and h, respectively, and its cross-section area and inertia moment around the principal centroid axis to be A_{c} and I_{c}, respectively; And denote the width and height of the beam's flange to be $b_{\mathrm{s}1}$ and $h_{\mathrm{s}1}$, respectively, the thickness of its web to be $b_{\mathrm{s}2}$, and the cross-section area and inertia moment of the beam to be A_{s} and I_{s}, respectively (See Figure 1). Let the elastic modulus and shear modulus of the brick wall be E_{c} and G_{c}, respectively, the Poisson's ratio be ν_{c}, and the shear correction coefficient be k_{c}; And let the elastic modulus and shear modulus of the steel beam be E_{s} and G_{c}, respectively, the Poisson's ratio be ν_{s}, and the shear correction coefficient be k_{s}. Suppose that the deformation of the steel-masonry composite beam satisfy the assumption of Timoshenko beam[10], and the deformations of the steel beam and brick masonry be compatible through the steel studs.

Denote $w(x)$ and $\theta(x)$ to be the deflection and the angle between the deformed axial line of the steel-masonry composite beam and coordinate axis ox, then, the governing equations for bending of the steel-masonry composite beam are as follows[10]

$$(EI)_{\mathrm{e}}\frac{\mathrm{d}^3\theta}{\mathrm{d}x^3}=q,\quad (EI)_{\mathrm{e}}\frac{\mathrm{d}^2\theta}{\mathrm{d}x^2}+k\left(GA\right)_{\mathrm{e}}\left(\frac{\mathrm{d}w}{\mathrm{d}x}-\theta\right)=0 \tag{1}$$

And the corresponding boundary conditions are

$$w=0,\quad \theta=0.\quad x=0,L \tag{2}$$

where, $(EI)_{\mathrm{e}}=E_{\mathrm{c}}I_{\mathrm{c}}+2E_{\mathrm{s}}I_{\mathrm{s}}$ and $(GA)_{\mathrm{e}}=G_{\mathrm{c}}A_{\mathrm{c}}+2G_{\mathrm{s}}A_{\mathrm{s}}$ are the equivalent flexural stiffness and equivalent shear stiffness, respectively, of the steel-masonry composite beam, and k is the equivalent shear correction coefficient.

It is easy to obtain that the solution of the boundary value problem (1) and (2) is

$$\begin{cases} w=\dfrac{q}{24\left(EI\right)_{\mathrm{e}}}(x^4-2Lx^3+L^2x^2)+\dfrac{q}{2k\left(GA\right)_{\mathrm{e}}}(Lx-x^2) \\ \theta=\dfrac{q}{12\left(EI\right)_{\mathrm{e}}}(2x^3-3Lx^2+L^2x) \end{cases} \tag{3}$$

With the expression (3), the normal stresses σ_{s} and σ_{c} on the cross-sections of the steel beam and brick wall, respectively, are

$$\begin{cases} \sigma_{\mathrm{s}}=-E_{\mathrm{s}}y\dfrac{\mathrm{d}\theta}{\mathrm{d}x}=-\dfrac{E_{\mathrm{s}}q}{12(EI)_{\mathrm{e}}}y(6x^2-6Lx+L^2) \\ \sigma_{\mathrm{c}}=-E_{\mathrm{c}}y\dfrac{\mathrm{d}\theta}{\mathrm{d}x}=-\dfrac{E_{\mathrm{c}}q}{12(EI)_{\mathrm{e}}}y(6x^2-6Lx+L^2) \end{cases} \tag{4}$$

The shear forces F_{s} and F_{c} on the cross-sections of the steel beam and masonry, respectively, are

$$F_{\mathrm{s}}=\frac{\mathrm{d}M_{\mathrm{s}}}{\mathrm{d}x}=-\frac{qE_{\mathrm{s}}I_{\mathrm{s}}}{2(EI)_{\mathrm{e}}}(2x-L),\quad F_{\mathrm{c}}=\frac{\mathrm{d}M_{\mathrm{c}}}{\mathrm{d}x}=-\frac{qE_{\mathrm{c}}I_{\mathrm{c}}}{2(EI)_{\mathrm{e}}}(2x-L) \tag{5}$$

And the distributions of the shear stresses $\tau_{\rm s}$ and $\tau_{\rm c}$ on the cross-sections of the steel beam and brick wall of the steel-masonry composite beam can be derived with the standard method of material mechanics and can be expressed as

$$\tau_{\rm s} = \frac{F_{\rm s}}{b_{\rm s2} I_{\rm s}} \iint_{A_{\rm s}} y {\rm d}A_{\rm s}, \quad \tau_{\rm c} = \frac{F_{\rm c}}{b_{\rm c} I_{\rm c}} \iint_{A_{\rm c}} y {\rm d}A_{\rm c} \tag{6}$$

It is obvious that the loads $q_{\rm s}$ and $q_{\rm c}$ borne by the steel beam and masonry, respectively, are

$$q_{\rm s} = E_{\rm s} I_{\rm s} \frac{{\rm d}^3\theta}{{\rm d}x^3}, \quad q_{\rm c} = E_{\rm c} I_{\rm c} \frac{{\rm d}^3\theta}{{\rm d}x^3} \tag{7}$$

Since the frictional coefficient at the interface of the wood block and brick wall is much larger than that at the interface of the wood block and steel beam, in order to guarantee the compatible deformation of the steel-masonry composite beam, the tight pressure N per unit length between the steel beam and brick wall should satisfies

$$N \geq \frac{q_{\rm s}}{\mu} = \frac{E_{\rm s} I_{\rm s}}{(EI)_{\rm e}} \frac{q}{\mu} \tag{8}$$

where, μ is the frictional coefficient at the interface of the wood block and steel beam.

The shear strain energy stored in the element dx of the steel-masonry composite beam can be expressed as

$$\frac{1}{2}\left(2F_{\rm s}\gamma {\rm d}x + F_{\rm c}\gamma {\rm d}x\right) = \frac{1}{2} F\gamma {\rm d}x \tag{9}$$

Using the following expressions

$$F = k\left(GA\right)_{\rm e}\gamma = k\left(GA\right)_{\rm e}\left(\frac{{\rm d}w}{{\rm d}x} - \theta\right), \quad F_{\rm s} = k_{\rm s} G_{\rm s} A_{\rm s}\left(\frac{{\rm d}w}{{\rm d}x} - \theta\right), \quad F_{\rm c} = k_{\rm c} G_{\rm c} A_{\rm c}\left(\frac{{\rm d}w}{{\rm d}x} - \theta\right) \tag{10}$$

The equivalent shear correction coefficient k can be expressed as

$$k = \frac{k_{\rm c} G_{\rm c} A_{\rm c} + 2k_{\rm s} G_{\rm s} A_{\rm s}}{G_{\rm c} A_{\rm c} + 2G_{\rm s} A_{\rm s}} \tag{11}$$

Due to the arch effect for deformation of the brick masonry wall and according to the provision 7.3.3 in the Code [11]: “As for the height of the brick masonry wall, when its height is larger than the length L of the steel-masonry composite beam, the height of the brick masonry is taken as length L for computation”. Considering the actual height of the brick wall of the historic masonry building is 15m, the height of the brick wall is taken as the length L of the steel-masonry composite beam in the following analysis. If the unit weight of the brick wall is $\gamma_{\rm c}$, then, the uniform load can be expressed as

$$q = \gamma_{\rm c} b_{\rm c} L \tag{12}$$

3 PARAMETER ANSLYSIS FOR I-TYPE STEEL CLAMPING BEAM

If the I63c steel is used as clamping beam for the foundation underpinning of the brick wall, the material and geometry parameters of the I63c steel beam and the brick wall of the masonry building experimented are given in the Table 1 and Table 2[12]. From these tables, it has

Table 1: Material parameters of steel beam and brick masonry wall

E_s (MPa)	E_c (MPa)	ν_s	ν_c	G_s (MPa)	G_c (MPa)	γ_c (N/m^3)
200000	778	0.3	0.17	76900	332	17000

Table 2: Geometry parameters of cross section of composite beam

b_c (mm)	h (mm)	b_{s1} (mm)	h_{s1} (mm)	b_{s2} (mm)	I_s (mm^4)	I_c (mm^4)	A_s (mm^2)	A_c (mm^2)	$2S_x$ (mm^3)
510	630	180	22	17	2.05×10^{9}	1.06×10^{10}	35958	321300	3.9×10^{6}

$$\begin{cases} q = \gamma_c b_c L = 8.67L \text{ N / mm}, \\ (EI)_e = 2E_s I_s + E_c I_c = 4.18\times10^{14}\,\text{N}\cdot\text{mm}^2, \left(GA\right)_e = G_c A_c + 2G_s A_s = 2.87\times10^{9}\,\text{N}. \end{cases} \tag{13}$$

The shear correction coefficient k_c of the rectangular cross-section beam composed of the brick wall is[13]

$$k_c = \frac{10(1+\nu_c)}{12+11\nu_c} \tag{14}$$

And the shear correction coefficient k_e of the I-type steel beam is[13]

$$k_s = \frac{\alpha}{\beta} \tag{15}$$

where,

$$\begin{cases} \alpha = 10(1+\nu_s)(1+3m)^2, \quad m = \dfrac{2b_{s1}h_{s1}}{hb_{s2}}, \quad n = \dfrac{b_{s1}}{h} \\ \beta = (12+72m+150m^2+90m^3) + \nu_s(11+66m+135m^2+90m^3) + \\ \quad 30n^2(m+m^2) + 5\nu_s n^2(8m+9m^2) \end{cases} \tag{16}$$

Therefore, with the expressions (11), (14) and (15), it can obtain that

$$k_s = 0.563, \quad k_c = 0.844, \quad k = 0.574 \tag{17}$$

From the expression(3), it can obtain that the maximum deflection of the steel-masonry composite beam is

$$w_{\max} = 5.41\times10^{-5} L^5 + 6.58\times10^{-4} L^3 \text{ (mm)} \tag{18}$$

And from the expression (4), it obtains that the maximum normal stresses σ_s and σ_c are

$$\sigma_{s,\max} = 0.11L^3 \text{ (MPa)}, \quad \sigma_{c,\max} = 4.20\times10^{-4} L^3 \text{ (MPa)} \tag{19}$$

Since

$$\left(\iint_{A_s} y\,\mathrm{d}A_s\right)_{\max} = 3.9\times10^{6}\text{ mm}^3, \quad \left(\iint_{A_c} y\,\mathrm{d}A_c\right)_{\max} = 2.53\times10^{7}\text{ mm}^3, \tag{20}$$

it obtains, from expressions (5) and (6), that the maximum shear stresses are

$$\tau_{\mathrm{s,max}} = 0.48L^2\ \mathrm{MPa}, \quad \tau_{\mathrm{c,max}} = 4.00 \times 10^{-4} L^2\ \mathrm{MPa} \tag{21}$$

In order to determine the maximum length $L_{\max}$ of the foundation underpinning of the historic brick masonry wall, denote the allowable normal stresses of the steel beam and brick wall to be $[\sigma_{\mathrm{s}}]$ and $[\sigma_{\mathrm{c}}]$, respectively, and their allowable shear stresses to be $[\tau_{\mathrm{s}}]$ and $[\tau_{\mathrm{c}}]$, respectively, then, from the expressions (19) and (21), it can obtain that

$$L \le L_{\max,1} = \min\left(2.09\sqrt[3]{[\sigma_{\mathrm{s}}]}, 13.35\sqrt[3]{[\sigma_{\mathrm{c}}]}, 1.44\sqrt{[\tau_{\mathrm{s}}]}, 50.00\sqrt{[\tau_{\mathrm{c}}]}\right)\ (\mathrm{m}) \tag{22}$$

If the allowable deflection of the steel-masonry composite beam to be $[w]$, then, from the first of the expression (3), it requires that

$$5.41 \times 10^{-5} L^5 + 6.58 \times 10^{-4} L^3 \le [w] \tag{23}$$

When $L > 11\,\mathrm{m}$, the error caused by the second term of the above expression (23) is less than 9%, which can be neglected in engineering practices, then, in this case, it has

$$L \le L_{\max,2} \equiv 7.14\sqrt[5]{[w]}\ (\mathrm{m}) \tag{24}$$

When $L \le 11\,\mathrm{m}$, the second term of the expression (23) can't be neglected, and it is difficult to obtain the analytical solution of the expression (23). The variations of the maximum deflection (18) with the beam length L are given in the Appendix A. Then, for a prescribed allowable deflection $[w]$, the maximum beam length $L_{\max,2}$ can be determined from the Appendix A when $L \le 11\,\mathrm{m}$. Therefore, considering the constraints of the stiffness and strength, the maximum length $L_{\max}$ of the foundation underpinning for single stage is

$$L_{\max} = \min(L_{\max,1}, \quad L_{\max,2})\ (\mathrm{m}) \tag{25}$$

At the same time, from the expression (7), it gets $q_{\mathrm{s}} = 8.5L\ \mathrm{kN/m}$. Then, with the expression (8), it obtains that the tight pressure N of the composite beam should satisfy

$$N \ge \frac{8.5L}{\mu}\ (\mathrm{kN/m}) \tag{26}$$

Table 3: Results of the steel-brick composite beam for different H-type steel beam

	I 63c	I 50a	I 40a
$w_{\max}$(mm)	$5.41\times10^{-5}L^5+6.58\times10^{-4}L^3$	$1.19\times10^{-4}L^5+1.14\times10^{-3}L^3$	$2.54\times10^{-4}L^5+1.61\times10^{-3}L^3$
$\sigma_{\mathrm{s,max}}$(MPa)	$0.11L^3$	$0.19L^3$	$0.32L^3$
$\sigma_{\mathrm{c,max}}$(MPa)	$4.20\times10^{-4}L^3$	$7.40\times10^{-4}L^3$	$1.26\times10^{-3}L^3$
$\tau_{\mathrm{s,max}}$(MPa)	$0.48L^2$	$0.82L^2$	$1.17L^2$
$\tau_{\mathrm{c,max}}$(MPa)	$4.00\times10^{-4}L^2$	$5.55\times10^{-4}L^2$	$7.6\times10^{-4}L^2$
q_{s}(kN/m)	$8.5L$	$8.5L$	$8.5L$
N/L(kN/m)	$8.5L/\mu$	$8.5L/\mu$	$8.5L/\mu$

Similarly, with the material and geometry parameters of I50a and I40a steel[12], the relation of the maximum deflection, maximum normal and shear stresses of the steel-masonry composite beam with the I50a and I40a steel as clamping beam with the beam length L can be obtained, and the maximum length $L_{\max}$ of the foundation underpinning can be determined. All these results are given in the table 3 and the Appendix A. It can be seen that, for a steel-masonry composite beam with prescribed length, with the model number of the I-type steel clamping beam increase, the maximum deflection, maximum normal stresses and shear stresses of the steel beam and masonry decrease, but the load q_s borne by the steel beam and the tight pressure N borne by the steel studs remain constants.

In addition, the Euler beam model can also be employed to analyze the deformation of the steel-masonry composite beam, which results can be obtained directly from the formula (3) by letting the equivalent flexural stiffness $(GA)_e \to \infty$. Due to the limitation of space, the analysis procedure for the Euler steel-masonry composite beam will be omitted here. For the steel-masonry composite beam with the I63c as clamping beam, the results of the Timoshenko and Euler steel-masonry composite beam are listed in the Table 4. It can be seen that the deflection of the Timoshenko composite beam is larger than that of the Euler composite beam, and it relative error is $12.17/L^2$. However, the maximum normal stresses and shear stresses of the steel beam and masonry as well as the load borne by the steel beam and the tight pressure borne by the steel studs remain unchangeable for both models. Therefore, in design of the foundation underpinning of the brick masonry wall, the simple model of Euler composite beam can be employed for its strength analysis, while the model of Timoshenko composite beam should be employed for its stiffness analysis.

Table 4: Result comparisons for Timoshenko model and Euler model of the I63c steel-masonry composite beam

	Model of Euler composite beam	Model of Timoshenko composite beam
$w_{\max}$(mm)	$5.41\times10^{-5}L^5$	$5.41\times10^{-5}L^5+6.58\times10^{-4}L^3$
$\sigma_{s,\max}$(MPa)	$0.11L^3$	$0.11L^3$
$\sigma_{c,\max}$(MPa)	$4.20\times10^{-4}L^3$	$4.20\times10^{-4}L^3$
$\tau_{s,\max}$(MPa)	$0.48L^2$	$0.48L^2$
$\tau_{c,\max}$(MPa)	$4\times10^{-4}L^2$	$4\times10^{-4}L^2$
q_s(kN/m)	$8.5L$	$8.5L$
N/L(kN/m)	$8.5L/\mu$	$8.5L/\mu$

4 CONCLUSION

The foundation underpinning of the brick masonry wall by using the steel clamping beam was studied theoretically. The two steel beams and the portion of brick wall between them was equivalent to a Timoshenko composite beam, and the governing equation of bending deformation of the steel-masonry composite beam was established. The analytical solution of the steel-masonry composite beam was presented for single stage foundation underpinning. The variations of the maximum deflection and maximum stresses of the steel-masonry composite beam for different models of the I-type steel clamping beam with the length of the single stage foundation underpinning are analyzed, and the maximum length of the foundation underpinning was determined. It is revealed that

a) Under the arch effect of the brick masonry wall, with the model number of the I-type steel clamping beam increase, the maximum deflection, maximum normal stresses and shear stresses of the steel beam and masonry decrease, but the load borne by the steel beam and the tight pressure borne by the steel studs remain unchangeable;
b) With the prescribed beam length, the deflection of the Timoshenko composite beam is larger than that of the Euler composite beam, and the differences between them decrease

with the beam length increase. However, the stresses and tight pressure borne by the steel studs are the same for both models;

c) And, in design of the foundation underpinning, the simple model of Euler composite beam can be employed for strength analysis, while the model of Timoshenko composite beam should be employed for stiffness analysis.

REFERENCES

[1] Zhang Liren, Wei Hai, Detection, *Identification and strengthening of building structures*. Wuhan: Wuhan University of Technology Press, 2012 (in Chinese).

[2] Ouyang Yu, Liu Nengke, Analysis of flexural behavior of masonry column strengthened by wrapped steel outside under axial compression. *Building Structure*, **36**, 27-29, 2006 (in Chinese).

[3] M. Corradi, A. Borri, A. Grazini, Confinement of brick masonry columns with CFRP materials. *Composites Science and Technology*, **67**: 1772-1783, 2007.

[4] H. Alhayek, D. Svecova, Flexural Stiffness and Strength of GFRP-Reinforced Timber Beams. *Journal of Composites for Construction*, **16**, 245-252, 2012.

[5] Lu Xin, Yang Xiao, Song Shaohu. Nonlinear Bending Analysis of Timber Beam Strengthened with Fiber Reinforced Polymer. *Journal of Shanghai University*, **18**, 634-639, 2012 (in Chinese).

[6] S. J. Hardy, Design of steel lintels supporting masonry walls. *Engineering Structures*, **22**, 597-604, 2000.

[7] E. S. Maria, B. L. Vagelis, Dynamic analysis of a masonry wall with reinforced concrete lintels or tie-beams. *Engineering Structures*, **44**: 23-33, 2012.

[8] Tong Min, Pen Shaomin, Application of the steel-masonry structure in a reconstructive engineering of the composite building. *Journal of Wuhan University of Technology*, **29**, 152-156, 2007 (in Chinese).

[9] Jing Denghu, Cao Shuangyin, Shi Lei, Zhou Haitao, Experimental study on the static behavior of steel plate-masonry composite beams and columns. *China Civil Engineering Journal*, **43**: 48-56, 2010.

[10] J. M. Gere, S. P. Timoshenko, *Mechanics of Materials, 2nd Edition*. Van Nostrand Reinhold, New York, 1984.

[11] GB50003-2011, *National Standard of People's Republic of China: Code for design of masonry structures*. Beijing: China Architecture & Building Press, 2011 (in Chinese).

[12] Shen Zuyan, Chen Yangji, Chenyiyi, *Fundamental of Steel Structures*. Beijing: China Architecture & Building Press, 2011 (in Chinese).

[13] Cowper GR. The Shear coefficient in Timoshenko's beam theory. *Journal of Applied Mechanics*, **33**: 335-340, 1966.

APPENDIX A

Responses of the maximum deflection with the beam length of the steel-masonry composite beam

Beam length L(m)	Models of the I-type steel			Beam length L(m)	Models of the I-type steel			Beam length L(m)	Models of the I-type steel		
	I 40a	I 50a	I 63c		I 40a	I 50a	I 63c		I 40a	I 50a	I 63c
	Maximum deflection (mm)				Maximum deflection (mm)				Maximum deflection (mm)		
0.6	3.68×10^{-4}	2.55×10^{-4}	1.46×10^{-4}	4.2	0.45	0.24	0.12	7.8	8.10	3.98	1.87
0.8	9.08×10^{-4}	6.23×10^{-4}	3.55×10^{-4}	4.4	0.56	0.29	0.15	8.0	9.15	4.48	2.11
1.0	1.86×10^{-3}	1.26×10^{-3}	7.12×10^{-4}	4.6	0.68	0.36	0.18	8.2	10.30	5.04	2.37
1.2	3.41×10^{-3}	2.27×10^{-3}	1.27×10^{-3}	4.8	0.83	0.43	0.21	8.4	11.58	5.65	2.65
1.4	5.78×10^{-3}	3.77×10^{-3}	2.10×10^{-3}	5.0	1.00	0.51	0.25	8.6	12.97	6.32	2.96
1.6	9.26×10^{-3}	5.92×10^{-3}	3.26×10^{-3}	5.2	1.19	0.61	0.30	8.8	14.50	7.06	3.30
1.8	1.42×10^{-2}	8.90×10^{-3}	4.86×10^{-3}	5.4	1.42	0.73	0.35	8.9	15.32	7.45	3.48
2.0	2.10×10^{-2}	1.29×10^{-2}	7.00×10^{-3}	5.6	1.68	0.86	0.41	9.0	16.17	7.86	3.67
2.2	3.02×10^{-2}	1.83×10^{-2}	9.79×10^{-3}	5.8	1.98	1.00	0.48	9.2	17.99	8.73	4.08
2.4	4.25×10^{-2}	2.52×10^{-2}	1.34×10^{-2}	6.0	2.32	1.17	0.56	9.4	19.98	9.68	4.52
2.6	5.85×10^{-2}	3.42×10^{-2}	1.80×10^{-2}	6.2	2.71	1.36	0.65	9.6	22.13	10.71	4.99
2.8	7.91×10^{-2}	4.55×10^{-2}	2.38×10^{-2}	6.4	3.15	1.58	0.75	9.8	24.47	11.83	5.51
3.0	0.11	5.97×10^{-2}	3.09×10^{-2}	6.6	3.64	1.82	0.87	10.0	27.01	13.04	6.07
3.2	0.14	7.73×10^{-2}	3.97×10^{-2}	6.8	4.20	2.09	0.99	10.2	29.75	14.35	6.67
3.4	0.18	9.89×10^{-2}	5.04×10^{-2}	7.0	4.82	2.39	1.13	10.4	32.71	15.76	7.32
3.6	0.23	0.13	6.34×10^{-2}	7.2	5.52	2.73	1.29	10.6	35.91	17.28	8.02
3.8	0.29	0.16	7.90×10^{-2}	7.4	6.29	3.10	1.47	10.8	39.35	18.92	8.78
4.0	0.36	0.19	9.75×10^{-2}	7.6	7.15	3.52	1.66	11.0	43.05	20.68	9.59