OPTIMAL FIBER REINFORCEMENT OF MASONRY WALLS LOADED OUT-OF-PLANE

Matteo Bruggi\textsuperscript{1}, Gabriele Milani\textsuperscript{2}, and Alberto Taliercio\textsuperscript{1}

\textsuperscript{1} Department of Civil and Environmental Engineering, Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133, Milan, Italy
e-mail: matteo.bruggi@polimi.it, alberto.taliercio@polimi.it

\textsuperscript{2} Department of Architecture, Built Environment and Construction Engineering, Politecnico di Milano
Piazza Leonardo da Vinci 32, 20133, Milan, Italy
e-mail: gabriele.milani@polimi.it

Keywords: Masonry, Homogenization, Out-of-plane Loads, Limit Analysis, Topology Optimization, Stress Constraints.

Abstract. The problem of the optimal external reinforcement with FRP strips of masonry walls under lateral loads is dealt with, by means of a combined homogenization-topology optimization approach. A homogenization procedure is utilized to deduce both the orthotropic elastic moduli of masonry and the out-of-plane strength domain. The elastic moduli are estimated by minimizing the total complementary energy of any discretized elementary cell. The out-of-plane failure surface is numerically evaluated by means of the lower bound theorem of limit analysis. Joints are reduced to interfaces obeying a Mohr-Coulomb strength criterion with tension cut-off and a cap in compression. For bricks, a classic Mohr-Coulomb criterion is employed. The out-of-plane macroscopic strength domain is deduced by integration of the stresses along the thickness. The optimization problem is written assuming perfect bonding between FRPs and masonry, meaning that a fiber-reinforced layer is modelled as an additional contribution to the out-of-plane stiffness of the underlying brickwork. Constraints on the internal actions in masonry are also imposed at any Gauss point. The proposed approach is applied to the determination of the optimal reinforcement of a rectangular panel. The numerically predicted optimal layout is compared with that given by an energy-based approach, in which the stiffness of the panel is maximized for a given amount of reinforcement.
1 INTRODUCTION

Masonry walls often fail because of out-of-plane actions, typically in earthquake-prone areas. Historical buildings with slender perimeter walls and poor mechanical properties are extremely vulnerable to lateral loads. Accordingly, many engineers are faced with the retrofitting or upgrading of existing masonry walls expected to undergo horizontal actions. Conventional retrofitting techniques include external reinforcements with steel plates, surface concrete coatings and welded meshes. Among the disadvantages of these techniques, the increase in mass added to the existing structure is a critical drawback in seismic areas, because of the increase in earthquake-induced inertia forces.

The use of Fiber Reinforced Polymers (FRPs) is a technique particularly suitable for the strengthening or restoration of masonry buildings, owing to its flexibility, reversibility, and limited increase in structural weight [1]-[8].

So far, the layout of the reinforcing strips has been basically driven by the intuition, owing to the simplicity of the loading conditions in the case of laboratory samples, or by the intent of healing existing cracks in the case of real structures. In presence of complex geometries, constraints and/or loading conditions, more sophisticated approaches, e.g. based on topology optimization, have to be used.

Recently, an innovative methodology based on the combined utilization of homogenization concepts and topology optimization was proposed for in-plane loaded concrete [9] and masonry structures [10], without any a-priori assumption on the optimal reinforcing layout. The same methodology was also applied to masonry structures undergoing lateral loads using an energy-based approach, in which the layout of a given amount of reinforcement that maximizes the structural stiffness is sought [11]. In the present work, the optimal reinforcement of out-of-plane loaded walls is sought including constraints on the maximum allowable moments in the masonry layer.

The paper is organized as follows. In Section 2, a homogenization procedure recently proposed for the derivation of the macroscopic elastic and strength properties of masonry structural elements undergoing out-of-plane actions is briefly recalled. In Section 3 the topology optimization problem that allows the optimal reinforcement of any masonry panel in bending to be identified is formulated. In Section 4 the proposed approach is applied to the optimal reinforcement of a solid panel undergoing different loads. Finally, in Section 5 the main findings of the works are summarized and future perspectives of the research are outlined.

2 MACROSCOPIC PROPERTIES OF MASONRY IN BENDING AND TWISTING

2.1 Discretization of any elementary cell

Let us consider any masonry wall consisting of a regular pattern of units, separated by bed and head mortar joints. Owing to the material periodicity, a single unit cell (Y) can be used as Representative Volume Element (RVE) for the heterogeneous medium. If a running bond (or a header bond) pattern is considered, it is expedient to adopt a unit cell of rectangular shape (see Figure 1a).

Homogenization is a convenient strategy to analyze masonry structures, both in the linear and in the non-linear range, since the mechanical properties of the constituent materials (bricks and mortar) are accounted for only at the cell level, and large scale FE computations at the macro-scale can be performed without the need of meshing joints and bricks separately.

Homogenization has long been used for the analysis of in-plane loaded masonry structures [12]-[17], and has recently been extended to masonry walls subjected to out-of-plane loads (see e.g. [8],[18]-[21]).
According to homogenization theory for heterogeneous bodies in bending [19], averaged quantities representing the macroscopic curvature and moment tensors (denoted by $\chi$ and $M$, respectively) are defined as:

$$\chi = \frac{1}{V} \int_Y \frac{\varepsilon(u)}{z} dY, \quad M = \frac{1}{A_Y} \int_Y z\sigma dY$$

(1)

where $z$ is an axis orthogonal to the wall, $V$ is the volume of the cell, $A$ the area of the cell in the $x$-$y$ plane, and $\sigma, \varepsilon, u$ the local (microscopic) stresses, strains and displacements, respectively. The Kirchhoff-Love’s assumption for thin plates was implicitly adopted, as retrofitting mostly involves slender masonry walls. The local stress and displacement fields must fulfill suitable periodicity conditions:

$$\begin{align*}
\mathbf{u} &= z\mathbf{v} + \mathbf{u}^{\text{per}} \quad \text{in } Y \\
Mn &= \text{anti-periodic on } \partial Y
\end{align*}$$

(2)

where $\mathbf{u}^{\text{per}}$ is the periodic part of the displacement field, $v$ is any point in the RVE in the local reference frame, $\partial Y$ is the boundary of the RVE and $n$ is the unit outward normal vector to $\partial Y$ (see Figure 1a).

![Elementary cell Y and CMT subdivision](image)

**Figure 1:** The micro-mechanical model proposed. (a) Subdivision of any RVE into 24 constant moment triangular elements. (b) Constant moment element: edge bending and twisting moments.

To numerically estimate the macroscopic properties of a masonry wall in bending, joints are reduced to interfaces of vanishing thickness; units are discretized by means of a coarse mesh consisting of constant moment triangles (CMT) [22], [23], see Figure 1b. Three unknown moments (per unit length) per element are introduced, namely the bending moments about the horizontal and vertical axis ($M_{xx}, M_{yy}$), and the twisting moment ($M_{xy}$). 1/4 of the RVE is meshed through 6 triangles, indicated by 1, 2, 3, 7, 8, 9 in Figure 1a. The generalization of the symbols to the whole cell is straightforward. Globally, the stress field within the RVE is defined by 72 unknown moments.
Neglecting body forces, equilibrium within any element is a-priori satisfied, being the moment tensor element-wise constant. Along the 28 interfaces between adjoining triangular elements, two equality constraints involving bending and twisting moments have to be prescribed to ensure the moment vector to be continuous, for a total of 56 conditions.

Anti-periodicity constraints for the moment vector are prescribed at the couples of opposite triangles (i.e. 1-6, 7-12, etc.), leading to additional 16 equalities, 4 of which are shown to be linearly dependent.

The following two subsections briefly illustrate how this numerical model can be used to define the macroscopic elastic and strength properties of out-of-plane loaded masonry walls. Readers are referred to [11] for more details.

2.2 Macroscopic flexural rigidities

The homogenized flexural rigidities can be obtained minimizing the complementary energy of any RVE, which is given by the following quadratic form:

\[
\Pi^* = \frac{1}{2} \sum_{i=1}^{N^r} A_{tr}^{(i)} 12 \left[ (1-V_b^2) \left( M_{xx}^{(i)} - 2\nu M_{xy}^{(i)} - 2 M_{yy}^{(i)} + 2 M_{xx}^{(i)} \right) \right] \]

\[
+ \frac{1}{2} \sum_{i=1}^{N^r} A_{tr}^{(i)} 12 \left[ \frac{M_{xx}^{(i)} \nu}{E_m} + \frac{M_{xx}^{(i)}}{G_m} \right] - t M_{xy} \chi_b, \tag{3}
\]

where \( N^r \) is the number of triangular elements, of area \( A_{tr}^{(i)} \), \( N^t \) is the total number of mortar interfaces, \( A_{tr}^{(i)} \) is the area of the \( i \)-th interface, \( \chi_b \) are prescribed macroscopic curvature components conjugated to the three macroscopic moments \( M_{ij} \). Summation over \( i \) and \( j \) is implied (\( i, j = x \) or \( y \)). \( \Pi^* \) depends on the 72 independent moment components in the elements discretizing the RVE, and on the three macroscopic moments. The constraints recalled in Sec. 2.1 have to be prescribed in the minimization of \( \Pi^* \), in addition to global equilibrium conditions.

Thanks to the very limited number of optimization variables involved, a standard large scale quadratic programming routine is utilized to solve the elastic problem on the unit cell, eq. (3). The flexural rigidities obtained are shown to be in good agreement with the results of refined FE analyses of any RVE varying the ratio of the mortar elastic modulus to the brick elastic modulus over a wide range [11]. Also, the present approach fits the analytical results obtained by Lourenço [24], provided that the units are not excessively stiff.

2.3 Macroscopic strength domain

Assuming mortar and units to behave as rigid-perfectly plastic materials, a lower bound to the macroscopic strength domain of masonry can be obtained solving a problem of limit analysis over the RVE. The problem amounts at maximizing the load multiplier \( \lambda \) respect to the 72 moment components in the elements. In addition to the equilibrium and anti-periodicity constraints recalled in Sec. 2.1, a set of inequalities representing the yield conditions at the interfaces and within the elements must be prescribed.

Any point of the surface of the homogenized out-of-plane strength domain in the space of the macroscopic moments can be defined solving the following non-linear programming problem: Find
Optimal fiber reinforcement of masonry walls loaded out-of-plane

\[
\max \lambda \quad \text{subject to} \quad \begin{align*}
\lambda \alpha &= \frac{\sum M_{xx}^{(i)} A_i}{2ab}, \quad \lambda \beta = \frac{\sum M_{yy}^{(i)} A_i}{2ab}, \quad \lambda \gamma = \frac{\sum M_{xy}^{(i)} A_i}{2ab} \\
A_{eq}^{(i)} X &= b_{eq}^{(i)}, \quad A_{eq}^{ap} X = b_{eq}^{ap} \\
f_{E}^{i}(M_{xx}^{(i)}, M_{yy}^{(i)}, M_{xy}^{(i)}) &\leq 0, \quad i = 1\ldots24 \\
f_{I}^{i}(M_{\mu}^{(i)}, M_{\nu}^{(i)}) &\leq 0, \quad i = 1\ldots32
\end{align*}
\]

where \((\alpha, \beta, \gamma)\) are the director cosines of any vector in the space of the macroscopic moments, \(A_i\) is the area of the \(i\)-th element \((ab/8 \text{ or } ab/16)\), \(X\) is the array gathering the optimization unknowns \(\text{(elements moments and collapse multiplier)}\), \(A_{eq}^{(i)} X = b_{eq}^{(i)}\) and \(A_{eq}^{ap} X = b_{eq}^{ap}\) are sets of linear equations defining the equilibrium conditions at the interfaces and the anti-periodicity conditions for the moment field, respectively. \(f_{E}^{i}\) \(\leq 0\), \(f_{I}^{i}\) \(\leq 0\) define the strength domains of the elements and the interfaces in the spaces of the relevant moment components. These domains are evaluated solving a series of linear programming problems similar to that used to determine the out-of-plane strength domain of masonry utilized in \([25]\). Any element is subdivided along the thickness \(t\) into several layers, and a suitable strength criterion is assumed for each layer. In the applications shown in Section 4, a Mohr-Coulomb criterion in plane stress is used for both units and interfaces. Additionally, a tension cut-off and a linearized cap in compression are used at the interfaces between units. The presence of a possible vertical pre-compression (or any membrane force) can be accommodated in the evaluation of \(f_{E}^{i}\) and \(f_{I}^{i}\), modifying the out-of-plane strength domains of mortar and bricks accordingly.

3 FORMULATION OF THE TOPOLOGY OPTIMIZATION PROBLEM

Within a two-dimensional domain \(\Omega\), consider a linear elastic orthotropic body subjected to out-of-plane loads. A reinforcing material is applied on both sides of the body, with the aim of improving stiffness and strength of the underlying masonry layer. Perfect bonding between the different layers is assumed. The reinforcing layers are supposed to undergo membrane forces only, which add bending stiffness and strength to the structure owing to their lever arm. Accordingly, the stiffness of any finite element \((i)\) into which the reinforced structure is subdivided can be written as:

\[
K_{fi}(x_{i,1}, x_{i,2}) = K_{Mi} + x_{i,1}^p K_{Ri,1} + x_{i,2}^p K_{Ri,2},
\]

where \(K_{fi}\) is the stiffness matrix of the \(i\)-th reinforced finite element, \(K_{Mi}\) is the stiffness contribution of the underlying orthotropic masonry structure, whereas \(K_{Ri,1}, K_{Ri,2}\) account for the reinforcing layers placed on both sides of the masonry element. The latter contributions depend on the density of the reinforcement on each side of the \(i\)-th element, \(x_{i,1}, x_{i,2}\) \((0 \leq x_{i,1}, x_{i,2} \leq 1)\), according to the so-called SIMP law \([26]\); \(p = 3\) in the application presented hereafter \([27]\).

Aiming at minimizing the amount of reinforcement required to keep the stress in any element of the underlying masonry layer within the strength domain defined in Sec. 2.3, the discrete version of the topology optimization problem to be solved can be written as: Find
where $A_i$ is the area of the $i$-th finite element, $t_F$ the prescribed reinforcement thickness on each side of the panel, and $n$ the number of finite elements. The first constraint in eq. (6) enforces equilibrium of the reinforced structural element in weak form. The second requirement consists of a set of local constraints that enforce the macroscopic strength criterion defined in the previous section, involving the components of the moment tensor in the $k$-th element of the masonry layer. The smooth macroscopic failure surface is approximated by a set of $m$ planes: accordingly, the admissibility of the stress within the masonry layer is prescribed through a set of $m$ linear inequalities. Whereas all the inequalities prescribed by the adopted strength criterion are evaluated at each finite element, only a few are actually implemented as effective enforcements [28]. This approach allows for a significant decrease in the number of active constraints, as a very limited set of local enforcements can be selected and included in the optimization algorithm.

4 NUMERICAL APPLICATIONS

A rectangular panel, 5 m wide, 2.5 m high and 0.125 m thick, is considered in the applications. The panel is in running bond brickwork. It has simply supported vertical edges (in-plane displacements are allowed), the top edge is free and the bottom edge is fixed. The topology optimization algorithm presented in Section 3 is applied to define a possible layout of fiber-reinforcement under different values of a uniform lateral pressure.

The equivalent elastic properties of masonry assumed in the applications are $E_x = 14534$ MPa, $E_y = 12420$ MPa, $G_{xy} = 4914$ MPa and $\nu_{xy} = 0.1588$, $x$ being an axis parallel to the bed joints. The horizontal (vertical) flexural stiffness is obviously evaluated as $E_i t_i^3/[12(1-\nu_{xy}\nu_{yx})]$ ($E_i t_i^3/[12(1-\nu_{xy}\nu_{yx})]$), where $t$ is the thickness. The strength parameters of the mortar joints are listed in Table 1. Bricks are supposed to be much stronger than the mortar joints. The topology optimization algorithm aims at distributing two layers of retrofitting material 0.5 mm thick, with Young modulus $E_f = 230$ GPa and Poisson’s ratio $\nu_f = 0.2$, over both sides of the panel.

Table 1: Strength properties of the mortar joints.

<table>
<thead>
<tr>
<th>strength parameter</th>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>cohesion [MPa]</td>
<td>$c$</td>
<td>1.5$f_f$</td>
</tr>
<tr>
<td>tensile strength [MPa]</td>
<td>$f_t$</td>
<td>0.32</td>
</tr>
<tr>
<td>friction angle [deg]</td>
<td>$\Phi$</td>
<td>36</td>
</tr>
<tr>
<td>Cap in compress [MPa]</td>
<td>$f_c$</td>
<td>4</td>
</tr>
</tbody>
</table>

Under a lateral pressure $p = 0.165$ kN/m$^2$, the minimum weight solution compatible with the prescribed strength criterion is presented in Figure 2(a). Black regions stand for reinforced zones, whereas white regions for unreinforced ones. Reinforcement is mainly located along the clamped edge of the panel, in order to increase strength where maximum bending is ex-
Optimal fiber reinforcement of masonry walls loaded out-of-plane

Expected, see also the relevant principal stress directions in the fibers of Figure 2(b). Figure 3 shows contour plots of the moments $M_{xx}$, $M_{yy}$ and $M_{xy}$, computed in the masonry layer of the reinforced panel and their admissibility according to the strength criterion. The feasibility ratio is lower or, at most, equal to 1, meaning that the adopted stress-constrained formulation is effective in providing a robust control of the stress field all over the domain. The layout shown in Figure 2 has a final volume fraction $V_f$ equal to 0.25, meaning that 12.5% of the surface of each vertical side should be fiber-reinforced; the maximum out-of-plane deflection is $w = 1.59$ mm.

![Figure 2](image)

Figure 2: Stress-based design for $p = 0.165$ kN/m$^2$. (a) Optimal distribution of the fiber-reinforcement and (b) relevant principal stress directions. $V_f = 0.25$, $w = 1.59$ mm.

![Figure 3](image)

Figure 3: Stress-based design for $p = 0.165$ kN/m$^2$. (a-c) Moments (in kNm/m) in the reinforced masonry layer. (d) Feasibility ratio according to the macroscopic strength criterion.
Alternatively, the optimal reinforcement was sought using an energy-based formulation, in which the same amount of reinforcement is bonded to the panel in order to maximize its stiffness. The optimal layout is shown in Figure 4, and is basically the same one given by the previous approach (Figure 2). The energy-based procedure gathers most of the material at the center of lower edge of the panel, whereas the stress-based layout is nearly uniform along the edge. As expected, the energy-based solution is stiffer, as the maximum out-of-plane deflection is 1.35 mm (15% less than the stress-based solution). However, the moments arising in the masonry layer are not feasible with respect to the strength criterion, as the feasibility ratio is larger than 1 along most of the lower edge of the reinforced panel, see Figure 5.

![Figure 4: Energy-based design for $p = 0.165$ kN/m$^2$ and $V_f = 0.25$. (a) Optimal distribution of the fiber-reinforcement and (b) relevant principal stress directions. $w = 1.35$ mm.](image)

![Figure 5: Energy-based design for $p = 0.165$ kN/m$^2$. (a-c) Moments (in kNm/m) in the reinforced masonry layer. (d) Feasibility ratio according to the macroscopic strength criterion.](image)
Major differences between stress-based and energy-based solutions at higher values of the lateral pressure. Figure 6 shows the stress-based design achieved for \( p = 0.185 \text{kN/m}^2 \). An increased amount of reinforcement, corresponding to \( V_f = 0.7 \), is required to strengthen the panel so as to fulfill the strength criterion all over the masonry layer, see Figure 7. For the same volume fraction, the energy-based procedure gives the optimal layout shown in Figure 8. Apart from the lower strip, the achieved design is completely different from the previous one. Again, the energy-based solution is stiffer than the stress-based (1.27 vs 1.32 mm of out-of-plane deflection), but the moments arising in the masonry layer are not feasible with respect to the stress criterion, see in particular Figure 9d.

Figure 6: Stress-based design for \( p = 0.185 \text{kN/m}^2 \). (a) Optimal distribution of the fiber-reinforcement and (b) relevant principal stress directions. \( V_f = 0.70, \ w = 1.32 \text{ mm} \).

Figure 7: Stress-based design for \( p = 0.185 \text{kN/m}^2 \). (a-c) Moments (in kNm/m) in the reinforced masonry layer. (d) Feasibility ratio according to the macroscopic strength criterion.
Figure 8: Energy-based design for $p = 0.185$ kN/m$^2$ and $V_f = 0.70$. (a) Optimal distribution of the fiber-reinforcement and (b) relevant principal stress directions. $w = 1.27$ mm.

Figure 9: Energy-based design for $p = 0.185$ kN/m$^2$. (a-c) Moments (in kNm/m) in the reinforced masonry layer. (d) Feasibility ratio according to the macroscopic strength criterion.

5 CONCLUSIONS

A numerical approach for the automatic generation of optimal layouts of fiber-reinforcement for masonry walls subjected to out-of-plane loads was presented. The proposed algorithm defines the arrangement of the minimum amount of material required to keep the stress in the wall within a macroscopic strength domain. The computational burden inherent to stress-based topology optimization formulations is alleviated using a selection strategy for the stress constraints. The robustness of the proposed procedure is assessed through numerical
Optimal fiber reinforcement of masonry walls loaded out-of-plane

applications. The achieved optimal reinforcing layouts were compared with conventional solutions based on the minimization of the overall strain energy. The two formulations basically provide the same layouts when a low amount of fiber-reinforcement is required ($V_f < 0.5$). If the out-of-plane loads call for higher percentage of reinforcement, an energy-based layout is quite different from a stress-based distribution of the same amount of material. A minimization of the strain energy might provide unsafe solutions, with moments in the masonry layer exceeding 20% of the allowable values.

In the continuation of the research, compressive stresses in the reinforcing layers and debonding at the interface between reinforcement and masonry will be prevented by suitable constraints. Also, the anisotropy of the FRP layers will be taken into account.

REFERENCES


