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MASONRY WALLS BETWEEN ART AND SCIENCE: HISTORICAL BUILDING TECHNIQUES AND STRUCTURAL ANALYSIS ACCORDING TO HEYMAN'S ASSUMPTIONS

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Abstract. *In this paper a brief analysis of the most common masonry construction techniques in historical buildings and their evolution over the centuries is conducted. Through the critical interpretation of the technical literature, it is possible to identify the main orientations derived from historical treatises that allow to recognize well-executed vertical structures. For these masonry walls a theory aimed at evaluating the structural behavior is formulated. This theory is rigorous and at the same time flexible enough to be applied to different types of constructions. Based on the limit analysis, it uses* Heyman's *assumptions (masonry has no tensile strength, infinite strength to compression, infinite friction coefficient), and it allows to determine the equilibrium configurations of masonry walls through a relatively simple computational procedure. A multi-bodies model, which consists in a set of rigid blocks connected by bilateral and unilateral constraints, is presented for the analysis of walls. Given the values of settlements, geometry and external loads, it is possible to obtain the unknown generalized displacements of each block by the use of the principle of minimum energy. A simple application to a case study is also presented.*

1 THE ANCIENT ART OF BUILDING MASONRY WALLS

With regard to historical masonry constructions, walls are the first elements to be built (for ensuring the support of the horizontal elements) and the last ones to be extinguished (when the buildings they compose come to ruin). It's a fact that, even many centuries after their construction, they are able to tell their functions and highlight their layers, allow to identify the skills of their builders, the economic possibilities of their costumers and the typical construction techniques of the site where they stand. They are precious documents of Material History.

Composition and size are basic requirements for defining well-executed masonry walls. The presence or absence of cross-connections, the geometry, shape and type of components, the texture, the horizontal dispositions, the correct stagger of joints, the quantity, quality and consistency of mortar, the presence of chips, the characteristics of possible inner cores, the homogeneity of materials are not negligible details.

Over the years, different building cultures and materials gave rise to a wide variety of masonry types. At first, walls were only made of dry hewn stones. Irregular and large blocks were also employed (as for the Cyclopean defensive structures). Instead adobe constructions arose where it was not possible to extract building stones: a mixture of clay-like soil and water with straw was prepared in special forms and left to dry in the sun; the result of the operation was a number of friable and slightly resistant elements. The later discovered baking of the dough allowed to obtain more durable bricks.

Fig.1: *Opus quadratum* (Velia, Italy)

Fig.2: *Opus incertum* (*Pompeii*, Italy)

Fig.3: *Opus reticulatum* (*Aeclanum*, Italy)

Fig.4: *Opus mixtum* (Brest, France)

Fig.5: *Opus testaceum* (Ostia Antica, Italy)

Fig.6: *Opus craticium* (*Herculaneum*, Italy)

The Romans developed sophisticated techniques with mortar according to different textures and execution methods. The *opus quadratum* (consisting in the use of stones which could be cut in square blocks in order to obtain regular courses of masonry) referred to both a hewn stone wall and a squared stone coating of an irregular rough core. *Opus quadratum* dry walls were often strengthened by iron connecting elements which were fixed by pouring molten lead into receptacles that manufacturers had made in the stones. Another method involved the *opus caementicium*, i.e. a mix of hydraulic mortar and natural or artificial stone fragments of varying size, in order to obtain a compact material. Vitruvius, the most famous engineer and architect of the Roman period, indicated several types of coating for it: the *opus incertum* was made up of irregular rough stones; the *opus reticulatum* instead consisted of the set of truncated pyramid-shaped stones (whose smaller bases were drowned into the inner core and larger bases sides were inclined at 45 degrees to the horizontal) giving the wall a typical neat texture which ensured the vertical stagger but tended to crack easily because of the instability of the bed joints; if brick edges were also employed, the technique was called *opus mixtum*; if the wall facing was only made up of bricks, it was called *opus testaceum*. Vitruvius also defined the *opus craticium*, consisting of a wooden frame structure with lightweight stone or brick or clay filler panels.

The need to build more durable walls brought to pay more attention to the stagger of vertical joints according to the isodomic arrangement deriving from the Greek models, with rectangular blocks positioned alternately in both the longitudinal and transverse directions. The blocks with the longer side in sight were called orthostates, those with the shorter side in sight were called diatones.

The rules of art remained quite unchanged over the centuries, at least until the post-Baroque time. They dictated the minimum requirements that masonry walls must ensure in terms of durability and efficiency.

In relation to the only elements used, the most of historical treaties divided masonry walls with mortar into:

- brick walls: they are made up of brick elements which are arranged in continuous and staggered courses and bound together with mortar joints; they can be single or multiple wythe walls and respect the stagger of vertical joints thanks to the use of sub-bricks;
- rough stone walls: they are made up of irregular stones arranged as much as possible in horizontal rows; cavities are filled with mortar and in case of large joints with stone or brick fragments too;
- hewn stone walls: they are obtained with regular parallelepiped-shaped stones interposing a thin layer of mortar;
- mixed walls (with stones and bricks): in a first case, bricks are arranged in horizontal rows interposed at 80-160cm, in a second case, they are used as single course elements for the local leveling of the texture.

Obviously the conservation status of the components and the degradation of mortar greatly affect the stability of the wall, but the effects can differ depending on the building techniques adopted.

The rediscovery of historical treaties dedicated to masonry, from Vitruvius's work until the nineteenth-century manuals, would help not only to understand the evolution of the ancient construction concepts, but also to regain possession of a reading code which seems to have been lost by now. The in-depth knowledge of the characteristics of masonry walls is necessary in order to better interpret the structural behavior of historical buildings, and it allows to recognize the composition and the critical points of masonry walls through the only visual inspection of their external facing.

2 THE BOUNDARY VALUE PROBLEM AND COMPATIBILITY CONDITIONS FOR RIGID NO-TENSION MATERIALS

For well-executed walls it is possible to make some assumptions which lead masonry to a rigid no-tension material.

2.1 Constitutive restrictions and equilibrium problem

It is assumed that the body $\Omega \in \Re^n$ (here n = 2), loaded by the given tractions $\underline{\mathbf{s}}$ on the part $\partial\Omega_N$ of the boundary, and subject to given displacements $\underline{\mathbf{u}}$ on the complementary, constrained part of the boundary $\partial\Omega_D$, is in equilibrium under the action of such given surface displacements and tractions, besides body loads $\underline{\mathbf{b}}$ and distortions $\underline{\mathbf{E}}$ (the set of data being denoted: ($\underline{\mathbf{u}}$, $\underline{\mathbf{E}}$; $\underline{\mathbf{s}}$, $\underline{\mathbf{b}}$)), and undergoes small displacements $\mathbf{u}$ and strains $\mathbf{E}(\mathbf{u})$.

We point out that the masonry structure is identified with the set: $\Omega \cup \partial\Omega_D$, i.e. it is considered closed on ∂ΩD and open on the rest of the boundary.

We consider that the body **Ω** is composed of Rigid No-Tension material, that is the stress **T** is negative semidefinite

$$\mathbf{T} \in Sym^{-}, \tag{1}$$

the effective strain $\mathbf{E^*} = \mathbf{E}(\mathbf{u}) - \underline{\mathbf{E}}$ is positive semidefinite

$$\mathbf{E^*} \in Sym^{+}, \tag{2}$$

and the stress **T** does no work for the corresponding effective strain **E***

$$\mathbf{T} \cdot \mathbf{E^*} = 0\ . \tag{3}$$

In order to avoid trivial incompatible loads ($\underline{\mathbf{s}}$,$\underline{\mathbf{b}}$), we assume that the traction $\underline{\mathbf{s}}$ satisfy the condition

$$\underline{\mathbf{s}} \cdot \mathbf{n} < 0,\ or\ \underline{\mathbf{s}} = \mathbf{0},\ \forall \mathbf{x} \in \partial\Omega_N\ , \tag{4}$$

Notice that in the plane case (n=2) conditions (1), (2), can be rewritten as

$$tr\ \mathbf{T} \leq 0\ ,\ det\ \mathbf{T} \geq 0\ , \tag{5}$$

$$tr\ \mathbf{E^*} \geq 0\ ,\ det\ \mathbf{E^*} \geq 0\ . \tag{6}$$

2.2 Statically admissible stress fields

An equilibrated stress field **T** (that is a stress field **T** balanced with the prescribed body forces $\underline{\mathbf{b}}$ and the tractions $\underline{\mathbf{s}}$ given on $\partial\Omega_N$) satisfying the unilateral condition (1) (that is conditions (5)), is said *statically admissible* for a RNT body. The set of statically admissible stress fields is denoted *H* and is defined as follows

$$H = \{\mathbf{T} \in S(\Omega)\ s.t.\ div\ \mathbf{T} + \underline{\mathbf{b}} = \mathbf{0}\ ,\ \mathbf{T}\,\mathbf{n} = \underline{\mathbf{s}}\ on\ \partial\Omega_N\ ,\ \mathbf{T} \in Sym^{-}\}, \tag{7}$$

$S(\Omega)$ being a function space of convenient regularity. Since for RNT materials, discontinuous and even singular stress fields will be considered, one can assume $S(\Omega) = M(\Omega)$, that is the set of bounded measures.

For Elastic No-Tension materials a sensible choice is $S(\Omega) = L^2(\Omega)$, that is the function space of square summable functions. Actually the space $M(\Omega)$ is much larger than $L^2(\Omega)$, that is the set of functions which compete for equilibrium is richer for RNT than for ENT materials; this fact makes easier for RNT materials the search of statically admissible stress fields.

2.3 Kinematically admissible displacement fields

A compatible displacement field $\mathbf{u}$, that is a displacement $\mathbf{u}$ matching the given displacements $\underline{\mathbf{u}}$ on $\partial\Omega_D$ for which $(\mathbf{E}(\mathbf{u}) - \underline{\mathbf{E}}) \in Sym^+$, i.e. such that the *effective strain* satisfies the unilateral conditions (6), is said to be *kinematically admissible* for a RNT body.

The set of kinematically admissible displacement fields is denoted K and is defined as follows:

$$K = \{\mathbf{u} \in T(\Omega^*)\ s.t.\ \mathbf{u} = \underline{\mathbf{u}}\ on\ \partial\Omega_D\ ,\ (\mathbf{E}(\mathbf{u}) - \underline{\mathbf{E}}) \in Sym^+\}, \tag{8}$$

where $\Omega^* = \Omega \cup \partial\Omega_D$ and $T(\Omega^*)$ is a function space of convenient regularity.

Since for RNT materials discontinuous displacements can be considered, one can assume $T(\Omega^*) = BV(\Omega^*)$, that is the set of functions of bounded variation (the functions whose gradient belongs to $M(\Omega^*)$, i.e. functions $\mathbf{u}$ admitting finite discontinuities). We restrict to the subset of $BV(\Omega^*)$, consisting of displacement fields $\mathbf{u}$ having finite jumps on a finite number of regular arcs. Actually, as we shall see, we will need only to consider discontinuous functions $\mathbf{u}$ whose jump set is the union of a finite number of segments.

2.4 Compatibility of loads and distortions

The data of a general boundary value problem for a RNT body can be split into two parts

$$\ell \leftrightarrow (\underline{\mathbf{s}}, \underline{\mathbf{b}}) \approx loads\ , \tag{9}$$

$$\ell^* \leftrightarrow (\underline{\mathbf{u}}, \underline{\mathbf{E}}) \approx distortions\ . \tag{10}$$

The *equilibrium problem* and the *kinematical problem* for RNT materials, namely the search of admissible stress or displacement fields for given data, are essentially independent, in the sense that they are uncoupled but for condition (3).

It has to be pointed out that, for RNT bodies, there are non-trivial compatibility conditions, both on the loads and on the distortions; that is the existence of statically admissible stress fields for given loads, and the existence of kinematically admissible displacement fields for given distortions, is submitted to special conditions on the data.

The definition of compatible loads and distortions is rather straightforward:

$$\{\ell\ is\ compatible\} \Leftrightarrow \{H \neq void\}\ , \tag{11}$$

$$\{\ell^*\ is\ compatible\} \Leftrightarrow \{K \neq void\}. \tag{12}$$

Therefore the more direct way to prove compatibility, both for loads and distortions, is to construct a statically admissible stress field or a kinematically admissible displacement field.

To prove the existence of a solution to the boundary value problem for a No-Tension body, the compatibility of ℓ and ℓ^* is necessary but not sufficient, since the further condition

$$\mathbf{T} \cdot \mathbf{E}^*(\mathbf{u}) = 0\ , \tag{13}$$

must be satisfied (this is the material restriction (3)). Then one can say that a possible solution to the boundary value problem is given, if there exist a statically admissible stress field and a kinematically admissible displacement field, which are reconcilable in the sense of condition (3).

3 MASONRY STRUCTURES AS MULTI-BODIES

In the present section a method for the analysis of the effects of settlements and distortions on masonry-like structures (i.e. structures composed of no-tension material in the sense of Heyman) is presented. As described in Section 2, the three basic assumptions of Heyman's model for masonry are:

- tensile stresses (forces) are forbidden, and therefore the material can separate freely (with zero energy expended).
- the material can withstand stresses of infinite intensity (infinite strength to compression, i.e. no possibility of crushing)
- friction coefficient is infinite: no sliding on separation lines.

The restriction to small strains and displacements is introduced. Based on these assumptions, the infinitesimal strain can be a bounded measure.

In the present paper we consider special measures whose Cantor part is void, that is to measures represented by line Dirac deltas with support on a finite number of regular arcs of finite perimeter. As a consequence of this last assumption, the structure is composed of non-intersecting, rigid polygonal pieces connected by unilateral constraints along the common interfaces. The shape of these pieces, that is of the skeleton of the interfaces (a segmentation of the domain), should be actually unknown, and part of the sought solution.

As a first simple case we assume that the pieces are fixed (in the form of a finite number of specified bounded polygons of finite perimeter) and that the loads are given as conservative (potential) forces and moments applied at the centroid of each polygonal element. Under this simplifying assumption the structure becomes a set S of simple rigid bodies connected by unilateral and bilateral constraints. The unilateral constraint translate the incompenetrability condition along the interfaces. The bilateral constraint enforce the non-sliding assumption along the interfaces.

For such a structure we can consider two kinds of data:

- kinematical data: ($\underline{\mathbf{u}}$, $\underline{\mathbf{E}}$), that is given displacements at the constrained boundary, given eigenstrains;
- statical data: ($\underline{\mathbf{s}}$, $\underline{\mathbf{b}}$), that is given tractions at the loaded boundary, given body forces.

The first set ($\underline{\mathbf{u}}$, $\underline{\mathbf{E}}$) represents the datum for the kinematical problem of the given structure, the kinematical problem being the search of a rigid body displacement $\mathbf{u}$ of the polygonal pieces composing S, compatible with the unilateral and bilateral constraints inter-connecting the pieces and connecting the pieces to the constrained boundary. The set of all the possible rigid body displacements satisfying the constraints is denoted K and is called the set of kinematically admissible displacements. In particular cases, K can be void, that is the structure (for this shape and fixing conditions) cannot accommodate the given kinematical datum with a rigid body displacement. It must deform.

The second set ($\underline{\mathbf{s}}$, $\underline{\mathbf{b}}$) represents the datum for the statical (equilibrium) problem of the given structure, the statical problem being the search of the constraint reactions $\mathbf{R}$ arising at the unilateral and bilateral constraints inter-connecting the pieces and connecting the pieces to the constrained boundary (restricted by the "compression" assumption at the unilateral constraints), in equilibrium with the given loads. The set of all the possible reactions satisfying the "no-tension" constraint and in equilibrium with the loads is denoted H and is called the set of statically admissible reactions. In particular cases, even H can be void, that is the reactions (for this structure and fixing conditions) cannot balance the given loads under the no-tension assumption, and the pieces composing S must change either their linear or their angular momentum.

The kinematical and statical problems for such a structure are coupled, in the sense that, given the assumption of zero dissipation on any interface, the work of the reactions for the displacements both at the internal and at the boundary interfaces must be zero.

In general there will be infinite elements **u** of *K* and infinite elements **R** of *H*, and the no-work assumption gives a criterion to select (may be not uniquely), among them, a couple ($\mathbf{u}^0$, $\mathbf{R}^0$) that is called: solution of the kinematical and statical problem. Notice that, restricting to rigid blocks having fixed interfaces the sets *H* and *K* become finite dimensional.

There is a way to select variationally such a couple. The idea is to introduce the potential energy of the structure (that is minus the potential energy of the loads, i.e. the scalar product of the loads and couples applied at the centroids of the pieces for the displacement parameters of each piece), call it $E(\mathbf{u})$, and minimize $E(\mathbf{u})$ over the set *K*.

Can write

$$E(\mathbf{u}) = \langle \mathbf{p},\mathbf{u} \rangle \quad , \tag{14}$$

a linear function of the displacement parameters of the elements, where **p** represents loads, and

$$E(\mathbf{u^0}) = min\ E(\mathbf{u}) \ , \ \ for\ \mathbf{u} \in K. \tag{15}$$

This is a linearly constrained minimization problem for a linear function, which is solved with the simplex method.

4 A CASE STUDY: *PALAZZO MARINELLI* IN NAPLES (ITALY)

Fig.7: Main facade of *Palazzo Marinelli* (Naples, Italy)

Palazzo Marinelli is a palace in the Neapolitan district of *Montecalvario*. Set in the sixteenth-century Spanish Quarters, it consists of a ground floor, a mezzanine floor and four upper floors. The vertical structures are made up of tufa blocks and mortar. All the vertically and horizontally aligned openings are rectangular, except for the grand entrance portal, covering the height of the two lower floors, which is supported by a round arch, and the shops gates supported by segmental arches.

The masonry building has a foundation settlement which affects all the piers of the main facade, except for the right end one.

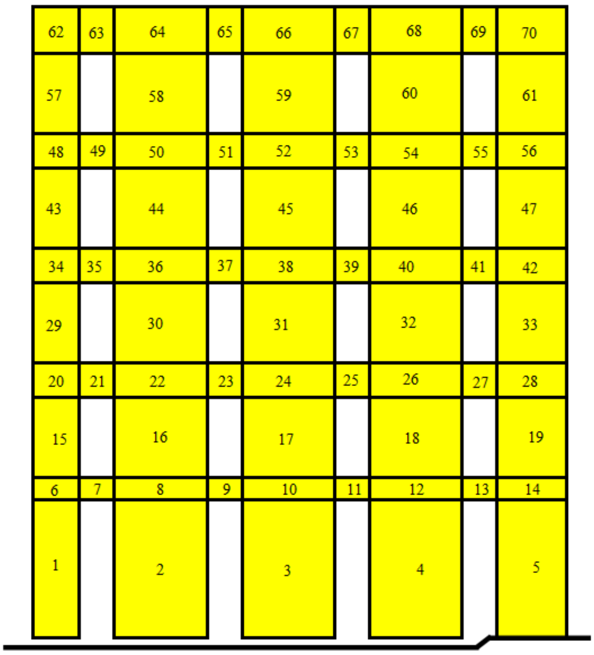

Fig.8: Simplified initial geometry of the wall with settlements

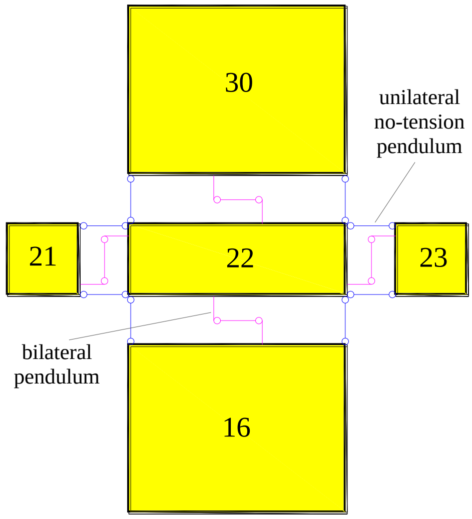

Fig.9: Interface constraints

In Figure 8 the geometry of the wall and the numeration of panels is reported. The number of elements is n = 70.

The generalized displacement is

$$\hat{\mathbf{u}} = \{u(1), v(1), \varphi(1), \ldots, u(n), v(n), \varphi(n)\}\ , \qquad (16)$$

where u(i) is the horizontal displacement, v(i) is the vertical displacement, φ(i) is the rotation referred to the centroid of panel(i).

The corresponding generalized dual force is

$$\mathbf{F} = \{H(1), V(1), M(1), \ldots, H(n), V(n), M(n)\}\ , \qquad (17)$$

where H(i) is the horizontal force, V(i) is the vertical force, M(i) is the moment acting in the centroid of panel(i). In particular the self-load only is considered, that is

$$H(i) = 0,\ V(i) = P(i),\ M(i) = 0,\ \textit{for any}\ i = 1, 2, \ldots, n, \qquad (18)$$

where P(i) is the weight of the panel(i).

The bilateral and unilateral constraints depicted in Figure 9 are considered on any interface (which in this case are considered as fixed). They produce the following system of equality and inequality constraints on $\hat{\mathbf{u}}$:

$$\boldsymbol{\Gamma}'\, \hat{\mathbf{u}} = 0\ , \qquad (19)$$

$$\mathbf{\Gamma''}\,\hat{\mathbf{u}} \leq \delta\,, \tag{20}$$

δ being the vector of generalized distorsions, corresponding in the present case to a given pure vertical displacement of the footing (except for the right end pier), of value -0.1m.

The set of kinematically admissible generalized displacement is

$$K = \{\ \hat{\mathbf{u}}\ s.t.\ \mathbf{\Gamma'}\,\hat{\mathbf{u}} = 0,\ \mathbf{\Gamma''}\,\hat{\mathbf{u}} \leq \delta\}. \tag{21}$$

The minimizer $\mathbf{u}^0$ is found through the minimization of the energy

$$E(\hat{\mathbf{u}}) = -\ \mathbf{F} \cdot \hat{\mathbf{u}}\,, \tag{22}$$

over the set K, through the simplex method.

The displacement corresponding to the solution $\mathbf{u}^0$ is depicted in Figure 10.

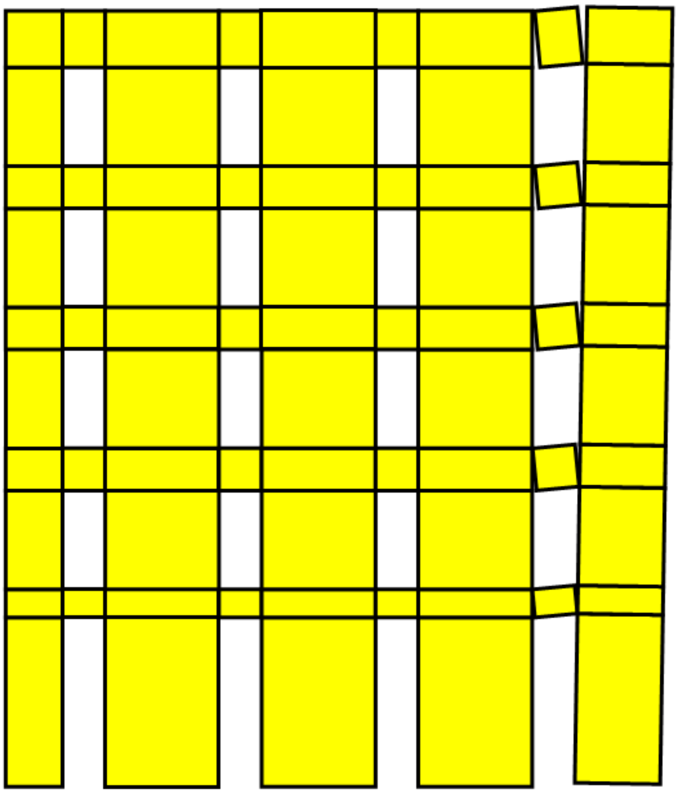

Fig.10: Wall configuration after displacements

5 CONCLUSIONS

While in the most strictly historical context the recently consolidated attention to the logic of conservation of masonry buildings has developed an increasing sensitivity to construction techniques, in structural analysis a certain inertia undeniably remains in accepting the diversity of masonry as several existing standards, which apparently are still prisoners of the strait-jacket imposed by the models for framed structures, demonstrate. The simplified model due to Heyman captures the main aspects of the behaviour of masonry. Adopting the hypothesis of no tensile strength and no slipping and avoiding the difficulties inherent to the mechanical description of brittleness and friction, Heyman's model allows to apply the static and kinematic theorems of limit analysis, thus leading to the study of masonry buildings in a consolidated theoretical context. In a climate where the principles of conservation must be respected along with those of structural safety, more accurate analysis methods should be performed to ensure low-impact and high-efficiency restoration techniques. In this paper an original approach to masonry structural analysis was presented: once recognized a well-executed masonry building, it was possible to study its walls as sets of rigid blocks and search its equilibrium configuration by the use of the principle of minimum energy.

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