STRUCTURAL ANALYSIS OF TIMBER VAULTED STRUCTURES WITH MASONRY WALLS

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Abstract. A historic building type composed of churches with vaulted timber structures and masonry walls emerged in Peru during the Hispanic Viceroyalty (16th – 19th century). Past earthquakes have shown that these structures are vulnerable, especially when their integrity has been jeopardized by centuries of weathering. The 2007 Pisco earthquake caused severe damage and even the collapse of several churches in Peru. This earthquake evidenced the urgency of analysing the seismic response of this type of structures in order to investigate whether these buildings require a retrofitting solution or merely a good plan of maintenance. A numerical model of the whole structure of the Cathedral of Ica was developed by the authors for this purpose. In this model, the timber joints governing the seismic response of the timber framing are simulated by means of translational and rotational springs with experimental values of stiffness. The nonlinear response of the masonry walls is simulated by means of the Drucker-Prager material model. The model of the cathedral shows important deformations at the top of the lunettes, in the mortice and tenon joints that connect the planked arches with beams running in the longitudinal direction of the church. These joints failed during the 2007 Pisco earthquake. The numerical response of the walls also well agree with the damage observed after the earthquake. The research reported in the present paper is part of the ‘Earthen Architecture Initiative – Seismic retrofitting Project in Peru’, which is a collaboration project of The Getty Conservation Institute, the University College London, the Pontificia Universidad Católica del Perú and the Ministerio de Cultura del Perú.
1 INTRODUCTION

A historic building type composed of churches with vaulted timber structures and masonry walls emerged in Peru during the Hispanic Viceroyalty (16th-19th century) [1]. Comparable structures were also built in other Latin-American countries and in Europe, such as in Spain, Italy and France from the 16th century onwards [2]. Although the construction characteristics and structural roles of several components may differ among the systems, similar modeling strategies are applicable to all cases. Historic timber structures have highly nonlinear behaviour, with large permanent deformation due to the presence of carpentry or nailed joints. To the authors’ knowledge, the structural behaviour of Peruvian historic churches has been researched in the past mainly from an historic and architectural viewpoint [1] while structural analysis aimed at determining the global structural response have been carried out using simplifying assumptions. Proaño et al. [3], for instance, assumed the joints as either rigid or pinned in the modelling of the Cathedral of Lima. This assumption can substantially overestimate or underestimate the real response of the structure and lead to unnecessary retrofitting measures. The need of appropriately simulate the semi-rigid behaviour of timber joints when modelling of historic timber structures has been clearly highlighted in [4, 5]. Tsai and D’Ayala [4], for instance, assessed the seismic performance of Taiwanese Dieh-Dou temples by means of a finite element model in which the material was assumed to behave as linear-elastic and joints were simulated by means of translational and rotational springs with adequate stiffness obtained from experimental testing. A similar approach was also applied by Parisi and Piazza [5] in the analysis of traditional carpentry joints of roof structures in Italy.

This paper aims to present a numerical modelling approach that can be applied to the study of the seismic vulnerability of historic masonry constructions with timber vaulted structures. This methodology is applied to the analysis of the Cathedral of Ica and the numerical results are compared with the damage observed in this church after the 2007 Pisco earthquake. This research is part of the ‘Earthen Architecture Initiative – Seismic retrofiting Project in Peru’, which is a collaboration project of The Getty Conservation Institute, the University College London, the Pontificia Universidad Católica del Perú and the Ministerio de Cultura del Perú.

2 THE CATHEDRAL OF ICA: CONSTRUCTION SYSTEM AND DAMAGE AFTER THE 2007 PISCO EARTHQUAKE

The Cathedral of Ica (IC), shown in Fig.1, is representative of churches built in the coastal regions of Peru during the Hispanic Viceroyalty. It was severely affected by an earthquake of 7.9 moment magnitude that occurred in the 15th of August 2007. The building plan is composed of a timber choir loft, central nave with aisles covered by timber vaults and domes, a transept bay and altar covered by timber arches and a crossing with a central timber dome.

Figure 1: The Cathedral of Ica after the 2007 Pisco Earthquake – a) front façade (East); b) longitudinal wall (North); and c) nave with partially collapsed timber vault.
The front brickwork façade is flanked by two bell towers, which are made of a timber framing structure placed on top of a 0.7 m high brickwork base (see Fig. 1a). The front façade has a low slenderness ratio of 3.4 [6]. However, its slenderness dramatically increases in the pediment, where a horizontal crack formed during the Pisco earthquake. The lateral walls are made of adobe laid on mud mortar with a slenderness ratio of 3.4-6.8 (see Fig. 1b). The walls have a rubble stone masonry foundation and a brickwork base course. Both types of masonry are laid in sand and lime mortar. The timber framing of IC is shown in Fig. 2. The vault of the nave is composed of principal and secondary timber arches and lunette’s arches and ribs.

Timber-framed pillars are supporting the vaults, domes and beams. These pillars are composed of several timber posts, which are connected together by means of horizontal and diagonal timber elements. Three different pillar’s layouts are present in the church, as shown in Fig. 3. The pillars between the nave and the lateral aisles have also a central trunk made of guarango, a hardwood specie indigenous of Peru. This timber specie was also used to make the bracing of the pillars. The arches and lunette’s ribs, which are composed of several arc-shaped planks, are made of cedar. The other timber members, such as the beams and pillars’ posts are made of sapele. All pillars have a brickwork base course which is 0.7m high.

The interior and exterior surface of the timber structure is covered with cane and plaster. The canes are attached to the timber framing by means of nails and leather straps. The top of the roof has an additional layer of sand, lime and cement mortar, which was introduced after the construction. The timber framing and the masonry walls of IC are connected mainly at two locations: i) connection of the front façade with the 1st bay of the timber framing by the sitting of the choir loft floor’s joists on the façade; and ii) connection of the transversal beams
of the crossing bay with the longitudinal adobe walls. The interaction of the timber structure with the longitudinal walls caused the formation of vertical cracks in the North wall during the Pisco Earthquake. Further details of the construction system of IC can be found in [7].

![Figure 3: Representative timber pillars of IC – a) pillars of the crossing’s bay, supporting the central dome; b) central pillars of the nave, which separate the nave from the lateral aisles; and c) pillars of the lateral naves, which are adjacent to the lateral walls.](image)

### 2.1 Timber Joints

The arches forming the vault, the lunette’s ribs, the lunette’s arches and the domes’ ribs and rings are composed of two alignments of timber arc-shaped planks, which are connected together by means of plain lap joints with 4 wrought handmade nails (Fig. 4a).

![Figure 4: Representative timber joints of IC – a) plain lap joints with nails; b, c) mortice and tenon joints connecting the beam at the top of lunette with the arches and ribs; d) mortice and tenon joints connecting the pillars to the beams, notched joints and leathered connection of beams; e) mortice and tenon joint with pegs connecting the horizontal bracing to the pillars’ posts; and f) nailed connection of diagonals with pillar’s post.](image)
Arches and ribs of the vault are connected to beams at the top of lunette by means of mortice and tenon joints (Fig. 4b and Fig. 4c) of varied layout and geometry. One mortice of the beam receives the tenons of both the secondary arches and the lunette’s ribs (see Fig. 2 and Fig. 4b). This connection failed during the 2007 Pisco Earthquake due to the failure of the deteriorated beam at the top of lunette in shear (see Fig. 1c, Fig. 4b and Fig. 4c). The lunette’s ribs are nailed to the lunette’s diagonals. These joints present either 1 or 2 nails, depending on the position of the rib – ribs near the centre of the vault have 1 nail and the others have 2 nails. The pillars’ posts are connected to longitudinal and transversal beams by means of mortice and tenon joints (Fig. 4d) at the top of the columns. Notched joints are connecting the transversal beams to the longitudinal beams (Fig. 4d). These beams are reinforced near the supports with a short span beam of similar cross section that is connected to them by means of leather straps (Fig. 4d). The horizontal bracing of the pillars is connected to the posts by means of pegged mortice and tenon joints (Fig. 4e). The diagonals of the pillars are connected to the posts by means of nailed joints (Fig. 4f).

3 NUMERICAL MODELLING OF THE CATHEDRAL OF ICA

3.1 Modelling of the masonry walls and towers

The failure of the masonry parts of IC is related to the propensity of the material to undergo plastic deformation. Hence, a Drucker-Prager material model [8] is used with the mechanical properties shown in Table 1. These parameters are obtained through experimental work performed by PUCP [9] or from literature [e.g. 10]. The walls are modelled by means of volume elements with full continuity between transversal walls. The timber part of the towers is modelled assuming the hypothesis of full connection between all timber members, since the timber structure does not present important permanent deformations after the Pisco Earthquake and neither the timber joints failed nor does the interaction with the brickwork base seem to be poor. The timber part of the towers is therefore modelled by means of volume elements with a linear elastic material model with the properties of guarango (see Table 2).

Nonlinear static (pushover) analyses with a model of the masonry walls and towers isolated from the timber framing are performed. This hypothesis is reasonable since on-site observations show no evidence of full connection of the timber framing with the masonry walls in such a way that the response of the masonry parts are substantially influenced by the response of the timber framing.

Table 1: Mechanical parameters for material characterization of adobe and brickwork [based on ref. 9 and 10].

<table>
<thead>
<tr>
<th>Masonry</th>
<th>Specific weight (kN.m⁻³)</th>
<th>Elastic modulus (GPa)</th>
<th>Poisson ratio</th>
<th>Cohesion (kPa)</th>
<th>Friction angle (rad)</th>
<th>Tension cut-off (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adobe</td>
<td>1.6</td>
<td>0.1</td>
<td>0.2</td>
<td>44.5</td>
<td>0.5</td>
<td>30</td>
</tr>
<tr>
<td>Brickwork</td>
<td>1.6</td>
<td>0.21</td>
<td>0.2</td>
<td>111</td>
<td>0.6</td>
<td>100</td>
</tr>
</tbody>
</table>

3.2 Modelling of the timber framing

In the case of the timber framing, the main issue regards the ability of the model to reproduce the flexibility of the system due to the timber joints, as the deformation of the framing is governed by the stiffness of the joints. Hence, the simulation of the timber members as beam elements with isotropic and linear material models is reasonable for mezo-models whose aim is to assess the global response and hence local phenomena can be overlooked. This simplifi-
cation is required for large models at a professional environment and it is also supported by the observation on site that notwithstanding some deterioration of the materials, no stress induced failure of the timber was observed on site. Material properties of the wooden species are indicated in Table 2. The weight of the roof cover (1.7kN/m² on the top of the vault and domes and 2.5kN/m² on the top of the aisles) is simulated as mass on the top of the structure.

Table 2: Mechanical properties for material characterization of the timber framing of IC [based on ref. 9].

<table>
<thead>
<tr>
<th>Species</th>
<th>Specific weight (kN.m⁻³)</th>
<th>Elastic modulus (GPa)</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sapele</td>
<td>4.1</td>
<td>10.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Guarango</td>
<td>9.1</td>
<td>16.9</td>
<td>0.3</td>
</tr>
<tr>
<td>Cedar</td>
<td>3.3</td>
<td>9.4</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Sensitivity analysis with local models of IC were initially developed in order to identify the critical joints of the structure [11]. The following timber joints were identified as important and modelled in greater detail: i) nailed joints connecting together the planks of the arches; ii) mortice and tenon joints connecting the arches and lunette’s ribs with the beam at the top of lunette, the vault’s members with longitudinal beams and the pillars’ posts with longitudinal beams; iii) pegged mortice and tenon joints connecting the horizontal bracing with the pillars’ posts; and iv) nailed joints connecting the diagonal bracing with the pillars’ posts. The modelling approach for the in-plane and out-of-plane response of the planked arches is shown in Fig.5 and reported in [12].

Figure 5: Approach for the numerical modelling of the in-plane and out-of-plane response of planked arches.

\( K_{\text{per}} \) and \( K_{\text{par}} \) denote the shear stiffness of the nails in the direction perpendicular and parallel to grain, respectively. \( K_{\text{rot}} \) denotes the rotational stiffness of the pair of nails. Elastic springs with values of stiffness obtained by means of experimental work performed at Universidad Politécnica de Madrid [13] are used to simulate the in-plane response of the joints (Table 3). Rigid elements are used to connect the two alignments of planks (arch i and arch j).
in such a way that the discontinuous arch behaves as one continuous arch of thickness twice
the thickness of one plank as far as the out-of-plane translation, rotation and torsion are con-
cerned. This assumption is acceptable when considering the observed deformation of the
arches and the layout of the connections in which the nails are clamped at the ends.

Table 3: Experimental stiffness of the springs used to model the timber joints [based on ref. 9 and 13].

<table>
<thead>
<tr>
<th>Joint</th>
<th>Translational stiffness (kN.m(^{-1}))</th>
<th>Rotational stiffness (kN.m.rad(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mortice and tenon</td>
<td>-</td>
<td>30 (parallel to grain)</td>
</tr>
<tr>
<td>Mortice and tenon with pegs</td>
<td>10000</td>
<td>45 (perpendicular to grain)</td>
</tr>
<tr>
<td>Nailed joints of diagonals</td>
<td>-</td>
<td>20</td>
</tr>
<tr>
<td>Nailed joints of arches</td>
<td>4100 (parallel to grain)</td>
<td>5.4</td>
</tr>
<tr>
<td></td>
<td>2900 (perpendicular to grain)</td>
<td></td>
</tr>
</tbody>
</table>

Mortice and tenon joints are simulated by releasing the degrees-of-freedom (DOF) corre-
spondent to the rotation of the tenon around an axis parallel and perpendicular to the grain of
the beam containing the mortices. Two elastic springs are then introduced between the timber
members, one with the rotational stiffness of the joint parallel to grain and the other with the
rotational stiffness of the joint perpendicular to grain, using the values of Table 3. The pegged
mortice and tenon joints are simulated by releasing the translation of the elements in the direc-
tion of the centreline of the horizontal bracing and introducing a spring with the stiffness indi-
cated in Table 3. The nailed joints of the diagonals are simulated by releasing the in-plane
rotation of the diagonal corresponding to the opening/closing of the connection, and introduc-
ing a spring with the stiffness indicated in Table 3. These values of stiffness were obtained by
means of experimental work performed at PUCP [9].

The choir loft and aisles’ joists are assumed as continuously connected to the transversal
and longitudinal beams. On-site survey and photographic evidences show the presence of lon-
gitudinal timber members, which are nailed to the principal arches, connecting adjacent bays
of the nave. If it is assumed that this longitudinal bracing is effective the result will be an in-
crease in stiffness in the longitudinal direction and a coupling of the local vault sway modes in the transversal direction. It should be noted that the timber structure is cladded in canes nailed to the timber by using leather straps and mud plaster. However, the stiffening
effect of this cladding system is modest and hence this component is not directly simulated in
the models, if not in terms of added weight. The longitudinal bracing is modelled as truss ele-
ments connecting the principal arches of the bays, as shown in Fig. 2. Circumferential ribs
connecting the meridians of the domes are also included in the model. They are shaped so as
to connect two adjacent ribs and they reduce torsional effects in the domes. All the connec-
tions within the domes and between those and supporting structure are assumed as continuous.

4 STRUCTURAL ANALYSIS OF THE CATHEDRAL OF ICA

4.1 Natural frequencies and modal shapes

The natural frequencies, periods and modal shapes of IC, considering both the timber and
masonry parts are shown in Table 4 and Fig. 6. The structure is characterised by many local
modes with close periods due to the lack of a rigid diaphragm and the difference in stiffness
of the various parts of the timber framing. The vault is considerably more flexible than the frames composed of pillars and beams. The mezzanine bay is stiffer than the other bays of the
nave due to the presence of the mezzanine. The timber framing vibrates in the longitudinal direction (X-direction) with a frequency that is almost half of the frequency of vibration of the brickwork façade. The transversal vibration of the first three bays of the vault (mode 1) occurs with a natural frequency 60% lower than the natural frequency of the last two bays of the vault. Furthermore, the response of the timber framing is largely independent of the response of the masonry walls. However, some level of interaction occurs for the transversal vibration of the central dome’s bay, the longitudinal anti-symmetrical vibration of the timber framing, the transversal vibration of the North wall and the longitudinal vibration of the façade. The frequency associated to these modes is slightly greater, i.e. the structure is stiffer, than the corresponding frequencies of the timber framing when analysed as independent of the masonry walls. The interaction of the timber with the masonry walls is further shown by the increase of the participating modal masses associated to those modes. It can be seen in Fig.6 that modes 10 and 19 have the participation of the central dome’s bay and of the first bay of the nave. Similarly, mode 41 has the participation of the mezzanine’s bay and of the first bay of the nave.

Table 4: Natural frequencies and periods of the Cathedral of Ica.

<table>
<thead>
<tr>
<th>Mode no.</th>
<th>Description</th>
<th>Period (s)</th>
<th>Participating modal masses (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Transversal symmetrical vibration of the vault.</td>
<td>1.420</td>
<td>1.56</td>
</tr>
<tr>
<td>10</td>
<td>Transversal vibration of the North wall.</td>
<td>0.498</td>
<td>7.84</td>
</tr>
<tr>
<td>11</td>
<td>Longitudinal vibration of the nave (transept bay vibrates in the opposite direction of the nave).</td>
<td>0.474</td>
<td>2.65</td>
</tr>
<tr>
<td>12</td>
<td>Transversal anti-symmetrical vibration of the vault.</td>
<td>0.451</td>
<td>0.83 (vertical)</td>
</tr>
<tr>
<td>19</td>
<td>Transversal vibration of the South wall.</td>
<td>0.409</td>
<td>8.56</td>
</tr>
<tr>
<td>41</td>
<td>Longitudinal vibration of the façade.</td>
<td>0.262</td>
<td>3.59</td>
</tr>
</tbody>
</table>

Figure 6: Modal shapes of the Cathedral of Ica.
4.2 Nonlinear analysis of the masonry walls and towers

The distribution of maximum principal stresses in the masonry walls and towers of IC is shown in Fig. 7 for an equivalent acceleration of 0.3g applied in both the Y and X directions. Major cracking of the walls is due to out of plane bending rather than in plane shear. This is to be expected in long unrestrained adobe walls. Tension stresses are greater than the tension strength of the material at the following locations, which are marked in Fig. 7: 1) connection of the base of the towers with the longitudinal walls; 2) lower part of the longitudinal masonry walls, near the interface of adobe with brickwork; 3) top corners of the façade; and 4) connection of the pediment with the façade. These results well agree with the damage observed in IC after the 2007 Pisco Earthquake, which are illustrated in Fig 8.1 to 8.4.

Figure 7: Maximum principal stresses distribution in the walls and towers of IC after pushover analysis.

Figure 8: Damage in the walls of IC after the 2007 Pisco earthquake.
4.3 Modal superposition analysis of IC

Response spectrum analyses with modal superposition are performed with the model of IC. Fig. 9 shows the transversal (Y-direction) displacement of the structure when the North-South response spectrum of the 2007 Pisco earthquake recorded in Ica [14] is applied in the transversal direction. The results show that the second bay counting from the central dome is the most vulnerable bay of the nave’s vault. A maximum displacement of 0.16m occurs in the beam at the top of lunette. This deformation is 14% greater than the displacement obtained when the timber framing is analysed as independent of the masonry walls. The connections at the top of lunette present the greatest deformation, which explains the failure of this connection during the Pisco earthquake notwithstanding the fact that severe deterioration has substantially decreased the shear strength of the beam. However the condition of other joints was equally poor but they did not fail. The displacement of the timber frames that support the vault is relatively modest when compared with the deformation of the vault. The bracing of the posts of the pillars and between pillars makes these frames much stiffer than the vault.

Figure 9: Transversal displacement distribution (Y-direction) in IC after modal superposition analysis using a North-South response spectrum of the 2007 Pisco Earthquake.

5 CONCLUSIONS

The structure of the Cathedral of Ica can be investigated from the viewpoint of a system composed of two sub-structures that interact with each other: i) the timber framing; and ii) the masonry walls and towers. The presence of positive connections between the two substructures was only observed in-situ at a few locations. A perfect bilateral connection at these locations between the timber framing and the masonry walls was simulated; however, in reality this interaction depends on the friction developed at the interface between the lateral beams and the North wall and the mezzanine’s joists and the façade. The fact that the first bay of the nave, next to the central dome’s bay, did not collapse might indicate that the interaction be-
between the timber framing/masonry walls is not as strong as modelled. The response of IC might therefore lays in-between the response of the global model with both timber and masonry structures and the response of the timber structure and the masonry walls modelled as independent of each other.

Those sub-structures require different modelling approaches in order to build a global model that is feasible at a practical level and provides results with a good level of confidence. In the case of the timber framing, the main issue regards the ability of the model to reproduce the flexibility of the system due to the carpentry joints, as the deformation of the framing is governed by their stiffness and capacity. For the masonry walls, as the failure of these elements is related to the capability of the material to undergo plastic deformation, the Drucker-Prager material model is used and nonlinear static (pushover) analyses are performed.

The results of the pushover analyses performed with the model of the masonry walls and towers show that the model is able to reproduce the damage observed in-situ after the Pisco earthquake. The model shows important concentration of stresses at the following locations:

- Connection of the pediment with the East façade;
- Connection of the longitudinal adobe walls with the brickwork base of the towers;
- Near the interface between the base course and the adobe walls due to out-of-plane bending of the North and South walls.

Modal superposition analyses performed with the global models of IC considering both the timber framing connected and not connected with the masonry walls show the following:

- The use of linear analysis to study the seismic response of historic timber structures is adequate as long as the most important degrees-of-freedom at the location of the joints are governed by stiffness values that appropriately reproduce the response of the joints. Thresholds for these degrees-of-freedom were identified by experimental tests, so it is now possible to check whether the joints have failed or not. The timber members behave as continuously connected for the other degrees-of-freedom;
- Notwithstanding the few connections between the timber framing and the masonry walls, the effect of this interaction is not negligible. This influence strongly depends on the response spectrum used. The analyses performed with response spectra from the 2007 Pisco earthquake show that the timber framing is subjected to greater deformations when the connections with the masonry walls are simulated. However, this influence is also conditional to the verification that the connection do not create state of stresses greater than the ones allowed by the friction present;
- The most vulnerable bay of the vault of the nave is the second one counting from the central dome. The numerical results show that the only bay that did not collapsed during the 2007 Pisco earthquake (1st bay next to the central dome) is subjected to lower structural demands;
- The connection at the top of lunette, composed of a beam with many mortices closely spaced, where each mortice receives two or three tenons, is subjected to the greatest deformations. Even if the beam is in good condition, the great level of relative displacement might be sufficient to cause the pull-out of the tenons.

In situ surveys have evidenced that canes and mud are used as plastering rather than acting as infill of the timber frame. It represents an internal and external cladding surface that does not substantially contribute to the stiffness of the whole structure. Its contribution is especially modest when the building is analysed at the ultimate limit state, when large displacements are expected. For this reason, it can be omitted from modelling choices if not as added weight.

Future work will address the detailed assessment of the joints in terms of their performance and ultimate capacity by using the results of these analyses and applying a failure criterion for each type of joint.
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