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A NON LINEAR STATIC ANALYSIS APPROACH WITH RIGID BLOCK MODELS FOR HISTORICAL MASONRY CONSTRUCTIONS

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Abstract. *This work presents a formulation for non linear static analysis of rigid block models. The objective of this work is to develop a tool for the structural assessment of historical masonry constructions. Rigid block models have proven to be effective for the analysis of both, simplified models and macro models of unreinforced masonry elements and structures. This modelling strategy captures well the proneness of masonry to localised cracking and is numerically efficient in the sense of requiring significantly less degrees of freedom compared to finite element models. The formulation uses numerical integration at interfaces and classical non linear solution techniques. The use of this strategy in the analysis of a previously studied masonry wall enlightens a number of its strengths and weaknesses. Work is in progress and more elaborated constitutive models have to be included for both, tension and compression behaviour. Also, we need to implement more sophisticated and efficient non linear analysis techniques in order to improve the performance of the numerical tool.*

1 INTRODUCTION

Structural safety assessment of historical buildings and monuments under the action of extreme events, as earthquakes, has been a very active research issue in past decades. Unreinforced masonry buildings, mainly historical constructions, have structural behaviour significantly different from structures designed and constructed according to recent codes and practice. Among these differences are the reduced ductility of ancient masonry constructions, their lack of quality control during construction, and the fact that they have normally faced a number of modifications, reconstructions, repairing and other interventions, among others. Therefore, modern simplified methodologies focused on seismic performance assessment of framed, ductile buildings are not directly applicable to ancient masonry structures [1]. Highly sophisticated analysis tools, as non linear finite element [2, 3] or discrete element methods [4, 5], are generally unpractical because they require large amounts of time and highly specialized personnel, as well as expensive software; in general, they are suitable for research activities only. There are, as well, a number of simplified methods for the non linear analysis of unreinforced masonry buildings. For instance, Lagormasino et al. [6] developed the software TREMURI, based on an equivalent frame model, which is a tool for the non linear analysis of masonry buildings. Parisi and Augenti [7] developed a macro-element, also for the analysis of masonry wall buildings. These tools are excellent for the purpose they were developed, nevertheless, buildings and monuments with complex geometries cannot be analysed with them.

Besides, rigid block model approaches have produced good results for historical constructions. Rigid block models allow us to simplify significantly the structural model by reducing the number of degrees of freedom and retaining a good accuracy in the results. There are several limit analysis developments based on rigid block models both, two-dimensional [8-14] and three-dimensional models [15-20]. Limit analysis approaches provide good approximations to the model overall strength and failure mechanisms. Nevertheless, a more accurate and useful structural assessment requires a closer approximation to the non linear behaviour of the structure, including capacity curves.

Discrete element family of methods use rigid block models (most of them) and provide very good results for the dynamic behaviour of structural models. These methods were originally proposed for studying rock massive movements and, as mentioned, they are not practical for routine structural assessment of masonry constructions. The main reason is that discrete element approaches invert a large amount of computational effort in looking for new contacts between blocks. This is an important issue in geotechnical engineering, but normally is irrelevant in structural problems.

Casolo and Peña [21, 22] proposed the rigid elements for dynamic analysis, out-of-plane and in-plane, respectively, of two-dimensional models of unreinforced masonry walls. This proposal is suitable for practical structural assessment, nevertheless, is restrained to two-dimensional models, as mentioned.

The proposal presented in this occasion is akin to the rigid element method, extended to three-dimensional rigid block models and, by the time, restricted to static non-linear analysis. We assume that the model experiments small displacements and, therefore, the formulation does not include the search for new contacts between blocks. The interface behaviour is elastic-perfectly plastic, with limited tensile and compressive strengths. This interface behaviour is suitable for the analysis of dry jointed masonry (with zero tensile strength). In the near future we expect to include a more elaborated model with softening behaviour both, for tension and compression failure modes.

2 FORMULATION

The proposed procedure follows the traditional incremental-iterative strategy for non linear analysis problems [23]. We start from an equilibrium state, named *i*, equation 1. Here $\boldsymbol{f}_{ext}^{i}$ is the vector of external forces applied to the block centres of mass at time *i* and $\boldsymbol{f}_{int}^{i}$ is the vector of forces resulting from the generalized stresses.

$$\boldsymbol{f}_{ext}^{i} - \boldsymbol{f}_{int}^{i} = \boldsymbol{0} \tag{1}$$

The external forces at a block are three force components and three moment components. They consist on a constant part, $\boldsymbol{f}_{c}$, normally the structure self weight and all the dead loads, and a variable part $\boldsymbol{f}_{v}$, which is the load that causes the model failure, for instance a horizontal load distribution. We assume that the variable loads vary proportionally to a normalized vector $\hat{\boldsymbol{f}}_{v}$ therefore equation 2 expresses the variable load vector at time *i*.

$$\boldsymbol{f}_{v}^{i} = \lambda^{i}\hat{\boldsymbol{f}}_{v} \tag{2}$$

Equation 3 is the expression of the internal force vector in terms of the equilibrium matrix $\boldsymbol{C}^{T}$, and the generalized stress vector $\boldsymbol{Q}^{i}$. In this formulation we use an interface element with numerical integration. We regard each integration point as an individual element with three stiffness components; therefore, the generalized stress vector has two shear forces and a normal force for each integration point (s_1, s_2, n). The normal force is positive in tension, and the shear forces are perpendicular to each other.

$$\boldsymbol{f}_{int}^{i} = \boldsymbol{C}^{T}\boldsymbol{Q}^{i} \tag{3}$$

A generalized strain component corresponds to each one of the stress components, namely, two sliding displacements and a normal displacement (q_1, q_2, q_n), for each integration point. The generalized strain components are gathered into the strain vector, $\boldsymbol{q}$, which is related to the displacements vector, $\boldsymbol{u}$, by compatibility, equation 4. Where $\boldsymbol{C}$ is the compatibility matrix and is, of course, the transpose of the equilibrium matrix. Orduña [24] mentions how to assemble the compatibility matrix for this interface-rigid block model.

$$\boldsymbol{q}^{i} = \boldsymbol{C}\boldsymbol{u}^{i} \tag{4}$$

When the analyst applies an increment to the external loads in order to pass from step *i* to step *i+1*, $\boldsymbol{f}_{ext}^{i+1} = \boldsymbol{f}_{ext}^{i} + \Delta\boldsymbol{f}_{ext}^{i}$, and uses a linear approximation to the increment of the internal forces, then equation 5a holds. Here, $\Delta\boldsymbol{u}^{i}$ is an approximation to the increment on the displacements and $\boldsymbol{K}_{T}^{i}$ is the tangential stiffness matrix at time *i*, given by equation 5b. Matrix $\boldsymbol{k}_{T}^{i}$ is a diagonal matrix with the tangential stiffness values, at time *i*, for each one of the integration point components in the model.

$$\begin{gathered}(\boldsymbol{f}_{ext}^{i} + \Delta\boldsymbol{f}_{ext}^{i}) - (\boldsymbol{f}_{int}^{i} + \boldsymbol{K}_{T}^{i}\Delta\boldsymbol{u}^{i}) = \boldsymbol{0} \\ \boldsymbol{K}_{T}^{i} = \boldsymbol{C}^{T}\boldsymbol{k}_{T}^{i}\boldsymbol{C}\end{gathered} \tag{5}$$

The stiffness values of the elements are calculated accordingly to the constitutive model adopted. In this case, we used an elastic-perfectly plastic model both, for shear and normal behaviour. Figure 1 illustrates the model for normal behaviour. Here f_t and f_c are the tensile and compressive strengths, respectively; and k_n is the normal stiffness. While the element behaves within the linear regime, the tangential stiffness is k_n. Outside the linear behaviour region, the stiffness theoretically is zero, nevertheless, in order to avoid numerical instabilities, equation 6 provides the tangential stiffness assigned in these cases; where ϵ is a small positive factor. In the present work, we assumed that no residual displacement exists once the element enters the plastic region. The arguments supporting this assumption are: (1) for dry masonry, the tensile strength is zero and, effectively, if separation exists and unloading occurs, the joint closes without normal stress and; (2) compressive strength is very high and crushing is improbable. More sophisticated and realistic models will be implemented in the near future.

$$k_{nT} = \epsilon k_n \tag{6}$$

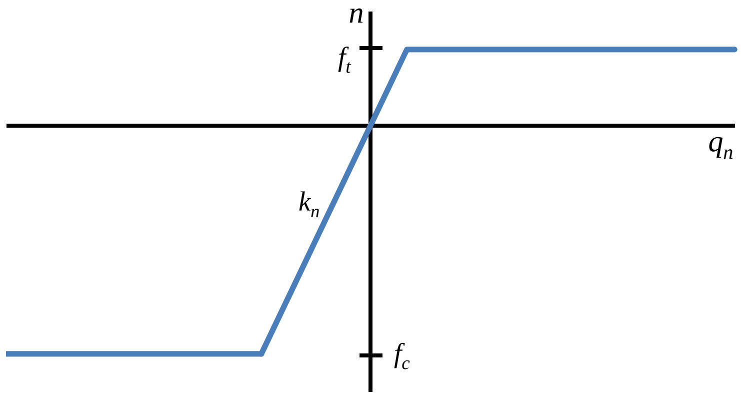

Figure 1: Normal direction constitutive model

Figure 2 shows the shear model adopted. We used the Coulomb model, so equation 7a provides the value of the shear strength, s_y, where c is the strength provided by cohesion and μ is the friction coefficient. Figure 2a illustrates that residual displacements, q_{sr}, are considered, as they represent sliding between blocks. While the shear forces lie inside the yield surface, Figure 2b, the tangential stiffness is k_s. When the shear forces reach the yield surface, the tangential stiffness is taken as in equation 7b.

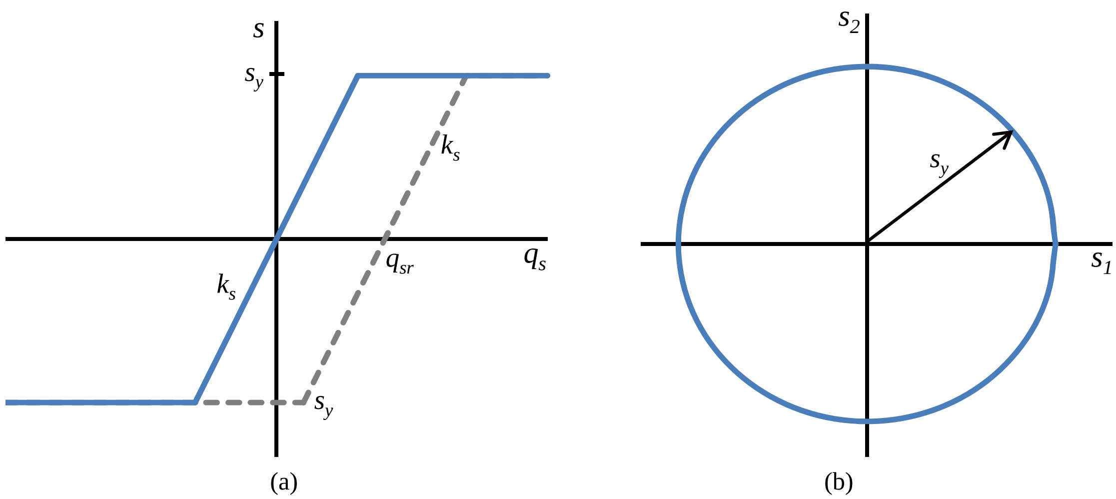

Figure 2: Shear constitutive model; (a) shear force vs. Sliding displacement curve and; (b) yield surface

$$s_y = c + \mu(-n) \tag{7}$$

$$k_{sT} = \epsilon k_s$$

Once equation 5a is solved, we apply an iterative procedure in order to correct the displacements and equilibrate the unbalanced forces [23].

3 EXAMPLE: WALL WITH OUT-OF-PLANE LOADING

This section presents the results of the analysis of a dry jointed masonry wall, figure 3a. This model was used by Orduña and Lourenço [18] in order to validate their limit analysis solution procedure; they also performed a non linear finite element method (FEM) analysis. Portioli et al. [19] used the model for comparison purposes as well. The wall is subject to its own weight, as constant load, and a horizontal out-of-plane variable loading, proportional to the weight. The wall rests on a horizontal surface and one of its vertical edges has a horizontal restriction. The wall dimensions are: 1.053 m high, 0.630 m width and 0.071 m thick. While the block dimensions are 0.081×0.210×0.071 m (height×length×thickness). Table 1 presents the mechanical properties of the model. It is also important to mention that the analysis performed for this example used four integration points per interface, located at their corners. This corresponds to a linear Lobatto quadrature.

Figure 3b shows the failure mechanism obtained by the non-linear procedure proposed in this work. Figures 3c-d present the failure mechanisms that result from a non-linear Finite element Method (FEM) analysis [Orduña 2005b] and by the limit analysis procedure proposed by Orduña [2005a-b], respectively. The three failure mechanisms are quite similar and are also comparable to that obtained by Portioli [2013].

Table 1: Model wall mechanical properties

Volumetric weight [kN/m^3]	E_m [N/mm^2]	μ [mm]	c [N]	f_t [N/mm^2]	f_c [N/mm^2]
20	1,000	0.7	0.0	0.0	∞

Table 2 presents the failure load factors obtained by different procedures. Regarding the FEM result as the more reliable one, the present procedure overestimates the failure load by 16%. This difference can be attributed to the small number of integration points used in the non-linear rigid block analysis, compared to the significantly more populated interface model used in the FEM analysis. Also the localization of the integration points at the interface corners contributes significantly to the overestimation of the model overall strength, since this distribution of “contact points” produces the highest torsional strength at an interface. With regard to the limit analyses results, we can say that the Orduña and Lourenço [18] failure load factor is the closest result to the FEM analysis with an error of 3%, while the result of Portioli et al. [19] underestimates this parameter by 13%.

Table 2: Failure load factors

Non-linear rigid block analysis	FEM [18]	Limit analysis [18]	Limit analysis [19]
0.243	0.210	0.216	0.182

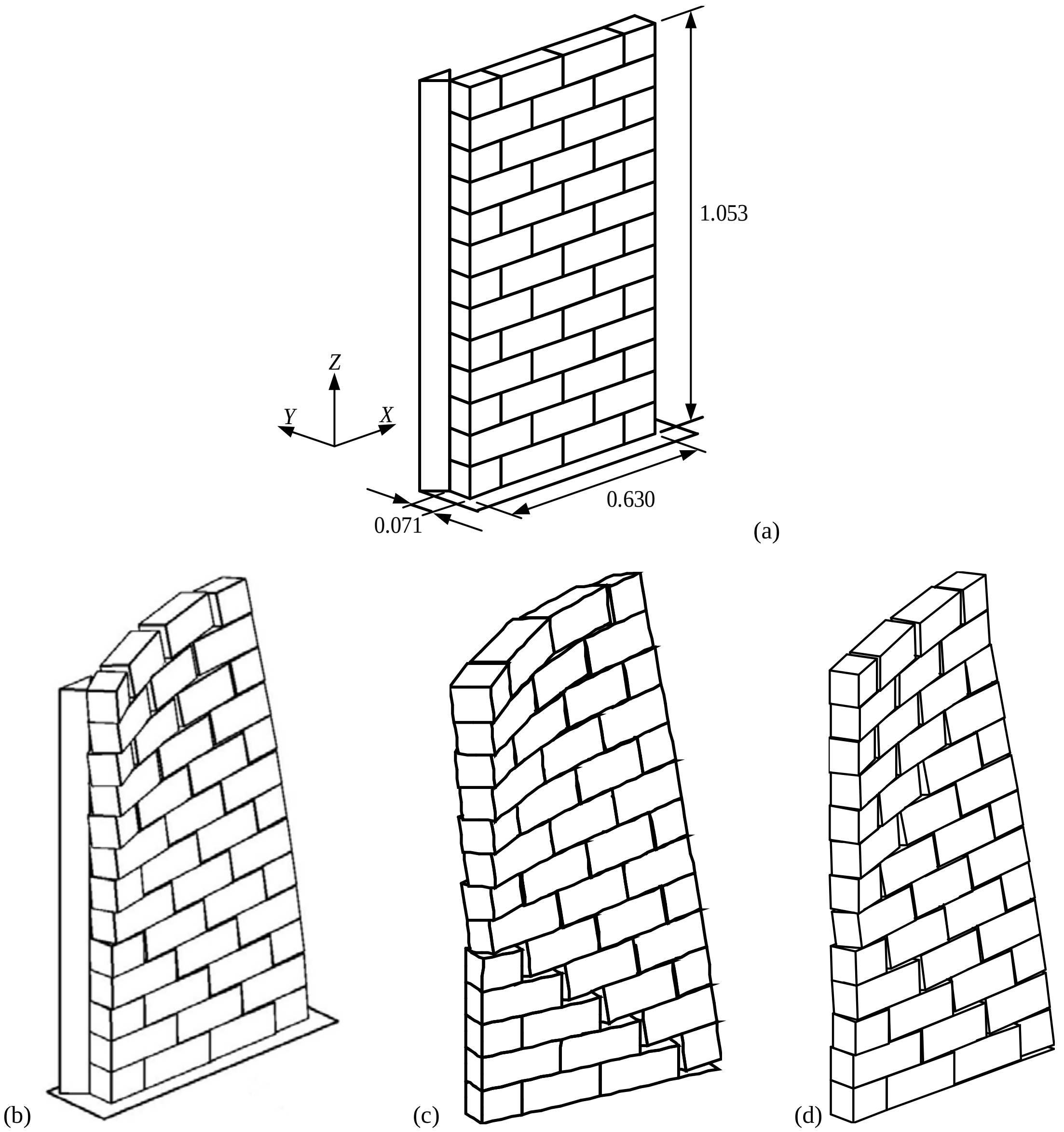

Figure 3: Dry-jointed masonry wall; (a) model; failure mechanisms by: (b) rigid block non-linear analysis; (c) FEM; (d) limit analysis

The parameter ϵ that appears in equations 6 and 7b deserves the following comments. This parameter was introduced in order to avoid zero stiffness values that could produce a singular or near singular tangent stiffness matrix, and, therefore, numerical instabilities. The reader can easily identify that when $\epsilon = 0$, we would be using a Newton-Raphson iteration procedure; while using $\epsilon = 1.0$, would conduct to a linear elastic iteration scheme. In practice we used values between 0.01 and 0.10. We observed that different values of this parameter produce faster or slower convergence, or even lack of convergence, of the iteration procedure at different stages of the non-linear analysis.

4 CONCLUSIONS

- This document presents a non linear analysis formulation based on rigid block models, focused to the structural assessment of ancient masonry structures by means of performance based procedures.
- By this time, the implementation of the formulation uses a linear Lobatto quadrature and elastic-perfectly plastic constitutive models for both, normal and shear behaviour.
- This implementation predicts a failure mechanism quite similar to those predicted by other analysis procedures as FEM and limit analysis.
- The present implementation significantly overestimates the failure load factor compared to the FEM result.
- The previous undesirable result is attributed mainly to the numerical integration scheme used; therefore, we expect that the implementation of higher order numerical integration schemes will produce better results.
- Other planned improvements to the numerical implementation include: (1) more elaborated and realistic normal constitutive models; (2) other non linear analysis strategies focused both, to accelerate the numerical convergence and to capture overall softening behaviour in the model.

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