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NUMERICAL SEISMIC RESPONSE ANALYSIS OF MULTIDRUM ANCIENT COLUMNS

L. Scandolo, S. Podestà

University of Genoa
Via Montallegro 1, 16145 Genoa, Italy
{lorenzo.scandolo, stefano.podesta}@unige.it

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Abstract. *This paper presents a numerical investigation of the seismic response of multidrum columns, type of system that shown high non-linear response and large sensitivity to input parameters. For these reasons only numerical approach can be used, in particular in this case, a Discrete Element Method (DEM) is utilized, adopting the bi-dimensional commercial available software UDEC.*

The monolithic column of the Romulus Temple (Rome, Italy) is used as reference column and gradually it was divided to investigate the influence of the number of drums in the seismic response.

Incremental Dynamic Analysis (IDA) in the time domain are performed with the model assumptions of rigid bodies and frictional joint. The models was excited at the base with 7 seismic records that are chosen in the European Strong Motion Database with a spectrum-compatible propriety associated to the hazard of the Roman site. Since damping plays a key role in the numerical analysis, 4 different values of mechanical damping are used.

The results show the high variance of the PGA that leads to the first collapse of the columns to different earthquake excitation and the different behavior related to the number of drums. Effectively the monolithic configuration is not always the most vulnerable one as a high number of blocks does not imply more strength. Moreover seismic records with high spectrum displacement at long period present very low acceleration scale factor at the collapse of the column.

It is interesting to notice that spectral displacements for all of the used records show a trend to join a very close range of displacement at long periods.

1 INTRODUCTION

Italy has a wide archaeological cultural heritage that is not negligible with respect to architectonics assets and deserves a suitable protection against earthquakes. Today there are a lot of procedures in Codes to analyze normal structures but for archaeological finds there is a lack of a knowledge path and analysis strategies to protect them in the course of time.

The large number of finds present in archaeological areas, often with recurrent typological characteristics, need the development of strategies to define the most vulnerable typologies and the evidences with high seismic risk within them.

The age of the archaeological evidence cannot be considered as a test of seismic safety since excavations and transformations occurred in recent years, had reset de facto the seismic history.

An up-to-date challenge is the definition of performance levels that are essential to obtain the risk of an asset. If for ordinary buildings a prearranged performance level can be reached with measures of reinforcement (improvement), for archaeological finds this cannot be taken. Indeed needed restorative measures can modify the knowledge from an archaeological point of view, like archaeological stratigraphy, leading to a loss of the evidence.

For this reason the choice of the protection level cannot be settled a priori but only after having obtained the capacity of the system and having related it to the admissible reinforcement programs. In substance it is not possible defining performance level which is too far from the capacity of the structures, un less the exposure is reduced or to modify the structural behavior (obviously without altering archaeological stratifications).

Masonry structures can be studied with different kind of methods, that are here divided following two criteria: scale of analysis and type of description of masonry continuum. A not exhaustive example is synthetized in Table 1.

Models developed at material scale are oriented to describe in an accurate way the complex behavior of masonry solids. At this scale, a fundamental role is played by the composite nature of the material, which may be considered whether like heterogeneous or homogenous. If it is considered as heterogeneous, discrete interface models are usually adopted. This means that each single constituent of the material (blocks and joints) is modelled separately and then assembled with the others by mean of interface elements. This is a very accurate modelling approach, but may require large computational efforts. If the material is considered as homogeneous, a more synthetic description of the material is provided, by mean of continuum constitutive laws. The laws can be defined adopting two different approach, the phenomenological and the micromechanical one.

The driving idea of models developed at element scale is to identify, within the masonry continuum, portions of structure subjected to recurrent damage modes. In the discrete model the behavior of a set of masonry bodies, connected through interfaces is considered. Commonly the bodies are rigid and the non-linear behavior is concentrated in interfaces, but can also have an elastic or elasto-plastic law). In the continuous model the system is divided in macroscopic structural elements that are defined from the geometrical and kinematic point of view through finite elements. The macro-elements represent damage, cracking, sliding and rotations in predefined zones which are characterized based on mechanical assumptions and implementation of, more or less sophisticated, non-linear constitutive laws.

Table 1: Models for masonry structures.

	Continuous models	**Discrete models**
Material scale	Continuum constitutive law	Interfaces models
Element scale	Structural elements	Macro-blocks models

Probably today the most used method for archaeological finds are the discrete one. This can be explained noting that evidences are a part of a complex structures that had suffered the consequences of natural and anthropic actions. Moreover same structures were built as an assemblage of stone block that, by them nature, are already divided in bodies and the crack patterns are prearranged. An examples of these cases can be found in the rest of old masonry walls, that have not more a three-dimensional structures, and isolated standing columns as a portion of ancient temples.

For these reasons interface model seems reasonable for a wide class of historical masonry. In this work this approach is adopted, in which the non-linear response is driven by the behavior of joints. In this model masonry is viewed as a discrete system where the blocks are considered as rigid and infinitely resistant bodies interacting with non-linear frictional joints.

Also rocking block model (macro-blocks model) can be very useful to analyze single block systems since a lot of archaeological evidences can be considered to have a monolithic behavior, hence they can be studied with the Housner model.

2 DYNAMIC OF ROCKING SYSTEMS

The behavior of single block under seismic load was analyzed the first time by Housner in 1963 [1]. Let us consider a rocking block with dimensions referred to Figure 1. The block is symmetric and rests on a rigid base with a high friction coefficient that prevents sliding.

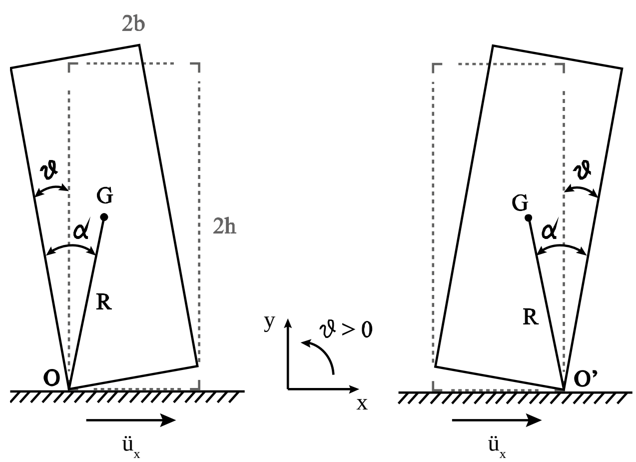


Figure 1: Reference system for rocking block.

The equation of motion can be written in a compact form that describes positive and negative rotations through the use of sign function.

$$I_O\ddot{\theta} = -\mathrm{sgn}(\theta)WR\sin\left(\alpha - \mathrm{sgn}(\theta)\cdot\theta\right) + W\frac{\ddot{u}_x}{g}R\cos\left(\alpha - \mathrm{sgn}(\theta)\cdot\theta\right) \tag{1}$$

where I_O is the mass inertia moment with respect to O, R is the distance between the barycenter G and the point of rotation O, W is the weight of the block, ϑ is the rotation, α is the "static" critical angle, g is the gravity acceleration and $\ddot{u}_x$ is the seismic input.

The transition from positive rotation to the negative one is governed by an impact that reduce the velocity of the block. The velocity after the impact is lower than before the impact and their ratio is called coefficient of restitution c. The value of the coefficient of restitution can be obtained (2) if the conservation of angular momentum about the new center of rotation is written, just before and after the impact.

$$c = 1 - \frac{2MR^2\alpha^2}{I_O} \tag{2}$$

Experimental tests on free standing masonry wall show how the value obtained with (2) is overestimated by 5% [3] due to plastic deformations in the contact surface. It is credible that this reduction of the coefficient of restitution should not be applied for systems constituted by stone block with dry joint like multidrum columns.

If the external excitation has a simple form the equation (1) can be solved analytically however seismic acceleration requires a numerical approach to solve the rocking motion. Nevertheless near source earthquakes can be well described with a sinusoidal waves so closed solutions for evaluating synthetic parameter of the response is even now studied [2,3].

Archaeological finds do not have necessarily a rectangular or a symmetric shape, since the response is highly non-linear, it is important to develop a model, starting from the Housner one, to take into account the different geometric property when it is pivoting to O or to O' [4].

The complex of a system, that seems easy, forces to switch from an analytical approach to a numerical one to study structures with a number of interacting blocks greater than one. Some author have studied an assemblies of two rocking block showing a difficulty to extend the model to system with more bodies [5,6].

For this reason, to describe systems with high number of block, but also to describe sliding phenomena, it is required a different approach. In many case is useful thinking to discrete element model that are able to take into account all the intrinsic peculiarity of multi-block systems under seismic excitation. In this work, a DEM approach is used to analyzed the behavior of columns described below.

3 THE TEMPLE OF ROMULUS

Figure 2: Temple of Romulus.

Around the year 309, a circular vestibule measuring 16 m in diameter was built in front of the Bibliotheca Pacis, commissioned by Massenzio (Figure 2). It was introduced by a curved portico (esedra) with four facing columns, monolithic with Corinzian capitals. There were niches and statues between them. Two of the columns can still be seen in-situ while the other two are be found in the second chapel on the left in S. Peter's Basilica.

The rotunda, covered by a cupola with a later lantern (1638), was flanked by two rectangular halls with apses.

This building came to be called the Temple of Romulus, because it was erroneously identified with the mausoleum of Romoletto, the son of Massenzio, who died when he was five.

The rotunda was not directly line up with the Flavian Hall. It was so placed for symmetrical reason, that is, to align the façade of the vestibule with the Via Sacra.

In the years 526-530, Pope Felix IV transformed the Bibliotheca Pacis into a Christian Basilica dedicated to Sts. Cosmas and Damian. The Pope found that the rotunda made an excellent atrium to the Flavian Hall "converted" in a church. For this reason an access between the two buildings was created.

3.1 The columns

The two remain columns (Figure 3), have a similar plinth and shaft while only one has the capital and a part of trabeation. The right column has a height of 1.09 m for the plinth, 0.45 m for the base and 7.09 m for the shaft. Instead left columns has a height of 1.06 m for the plinth, 0.45 m for the base and 7.09 m for the shaft; moreover on the top presents a 0.96 m of capital and a trabeation with a height of 1.62 m. The columns are tapered (base diameter equal to 0.86 m while at the top equal to 0.78 m) and have a very light entasis.

The columns are built in Cipollino marble, that is characterized by flat or wavy strips with variable thickness, on the order of few centimeters. This material can be schematized by a dense succession of layers of calcite and a mineral with planar structure.

a) b)

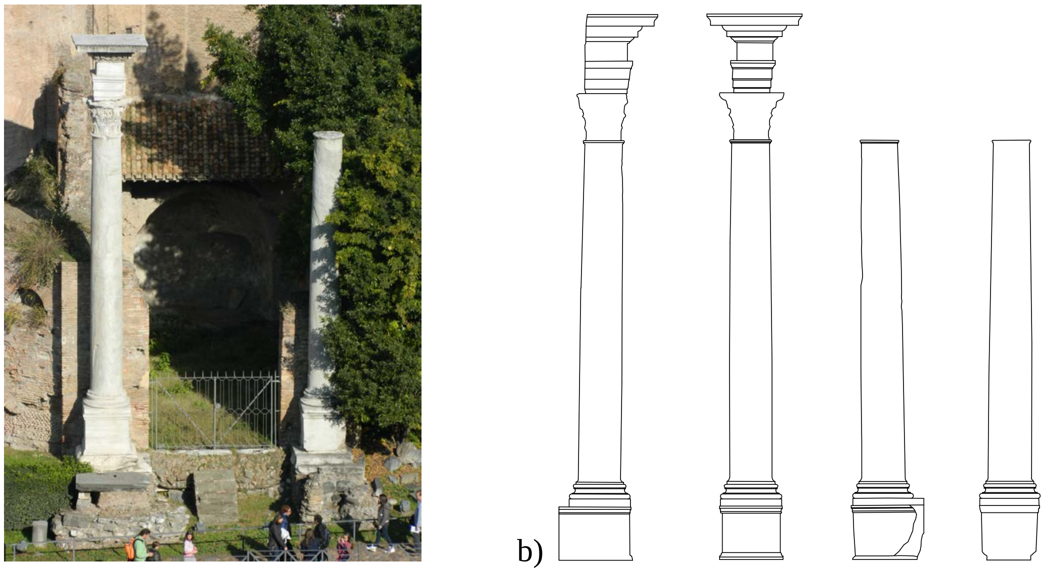


Figure 3: a) Overall view; b) west and south façade for column 2 (left), east and south for column 1 (right).

The two columns in the year 2000 were subjected to a diagnostic campaign to evaluate frequencies and damping through vibrational tests. The use of an instrumented impact hammer to excite the system leads to an elastic response of the structures and it is not able to activate rocking mechanism. This aspect will be used in the numerical DE model when the mechanical damping will be defined.

Table 2 shows the results of dynamical investigation on first mode (direction x is in the plane of west or east façade while direction y is in the plane of south façade):

Table 2: First modes of vibration for the two columns and relative damping coefficients.

	Column 1	**Column 2**
Dir x (Hz)	4.39	1.86
Damping (%)	1.82	2.57
Dir y (Hz)	4.40	2.44
Damping (%)	1.97	2.11

It is worth noting that column 1 have the first two natural frequencies equal with respect to the two orthogonal directions. This means that the shape of the column can be assumed to have an axial symmetry. In the column 2, the presence of capital and the architrave, with an eccentric position, determines a pronounced difference in natural frequencies in the two directions.

4 NUMERICAL ANALYSIS OF MULTIDRUM COLUMNS

Focusing only on column 1 numerical analyses are performed with an interface discrete model. The analyses are carried out using a bi-dimensional distinct element program, UDEC 5.0. A distinct element method is a numerical model capable of describing the mechanical behavior of assembled bodies. It is a model that can be enclosed in the family of discrete element methods since it allows finite displacements and rotations of discrete bodies, their complete detachment and the contacts are recognized automatically.

In a distinct element method the contact are deformable while bodies can be rigid or deformable. The solution algorithm uses an explicit scheme to solve the equations of motion directly.

4.1 Reference models

Column 1 was analyzed in reference to different configurations: the monolithic one (Figure 4a) and dividing the column in several drums, with the same height (Figure 4b). This choice has been introduced to understand the vulnerability increment of the system increasing the number of drums, in which it is divided. This information can be very useful since from a restoration reinforcement point of view, metallic element were used as the column core.

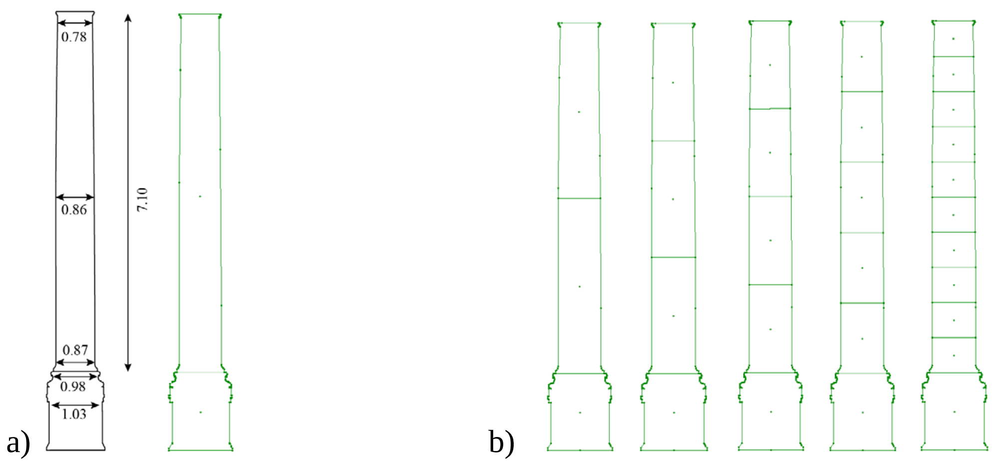


Figure 4: a) Simplified geometry of column 1 and DEM; b) DEM of multidrum columns with 2, 3, 4, 5 and 10 drums respectively.

The drums are considered as rigid so only one propriety is necessary to be defined, that is the material density, assumed equal to 2704 kg/m^3. For the joints a Mohr-Coulomb criterion is chosen and the below parameter are assigned:

- Normal stiffness $k_n=5\cdot10^{10}$ Pa/m;
- Shear stiffness $k_s=5\cdot10^{10}$ Pa/m;
- Frictional angle $\varphi=35°$;
- no cohesion;
- no dilation.

A specific problem with contact schemes is the unrealistic response that can result when block interaction occurs close to, or at, two opposing block corners. Numerically, blocks may become locked or hung-up. This is a result of the modelling assumption that block corners are sharp or have infinite strength. In reality, crushing of the corners would occur as a result of a stress concentration. Explicit modelling of this effect is impractical. However, a realistic representation can be achieved by rounding the corners so that blocks can smoothly slide past one another when two opposing corners interact. Corner rounding is used in UDEC by specifying a circular arc for each block corner. The analyses are performed using a value of 1 mm for the corner rounding. The solution between Housner model and UDEC are performed to define a correct corner rounding.

4.2 Numerical analysis

Numerical analyses are made applying 7 natural time-histories of earthquakes to the models of the column, according to an incremental dynamic scheme (IDA). The records (Figure 5) are chosen in the European Strong Motion Database with a spectrum-compatible propriety associated to the hazard of the Roman site. This operation was done with the software Rexel [7]. The time history are assigned to the plinth of the column as velocity, defining a zero last value in order to prevent a continue translation of the system at the end of the excitation.

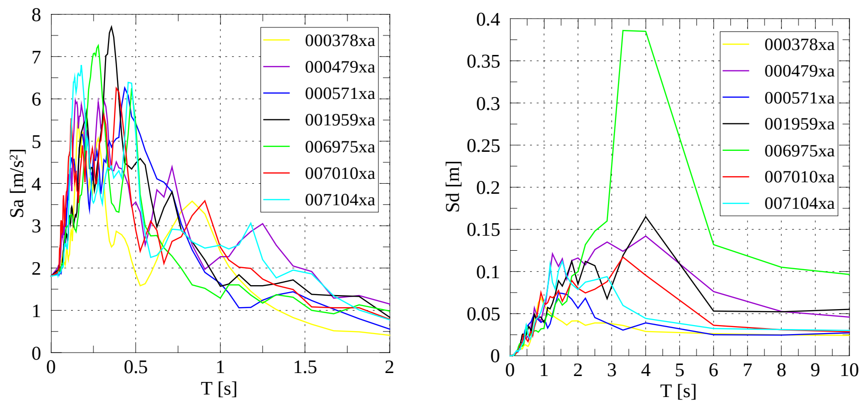


Figure 5: Response spectrum (ξ=5%) for the 7 seismic signals.

The choice of damping is a critical point in distinct element analysis. In literature different configurations were used, sometimes linked to computational time consuming [8,9,10,11,12]. Rayleigh damping in UDEC is considered for dynamic analysis, so the damping matrix components are obtained as a linear combination of mass matrix, by an α coefficient and by a β coefficient for the stiffness matrix. The mass proportional term works like a damper linked to the ground and any blocks while the stiffness proportional one like a damper that is connected among the blocks.

Four different configurations are analyzed, modifying the α and β coefficients (Table 3).

Table 3: Damping parameters.

	α	**β**
Case 0	0	0
Case 1	6.28E-5	4.92E-4
Case 2	0.08	4.92E-4
Case 3	0.5	6.66E-4

Case 0 is the configuration with no damping. Experimental results highlight a good response during the strong motion but an unrealistic behavior after the maximum excitation that leads to very high rotations at the ending part of the signal.

Case 1 is a configuration with stiffness proportional damping only (mass damping is negligible). The value of β is defined comparing the solution of the column with Housner model and the result obtained with the distinct element model considering a sinusoidal excitation.

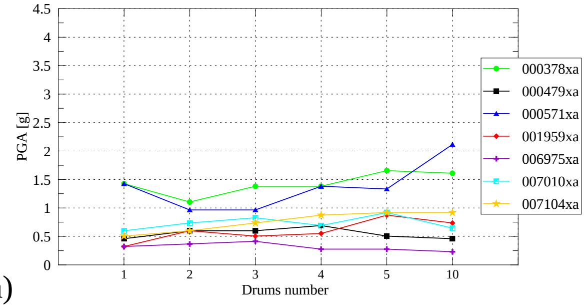
These two cases are representative of the behavior of a structure that exhibits a rigid block response. In the rigid block theory developed by Housner the impact is the only dissipative mechanism allowed and it is defined by the coefficient of restitution (2). UDEC works with the hypothesis of conservation of the moment of momentum so the energy dissipation associated to the impact is implicitly considered in the numerical solution. The diagnostic campaign performed on the columns shows a dissipative mechanism linked to the elasticity of the material. For this reason two more cases are investigated.

Case 3 is a configuration with mass and stiffness proportional damping. The stiffness damping coefficient is equal with case 2 while mass proportional coefficient was defined starting from the result of vibrational campaign. Analyzing the result of the response of the column with the Housner model it has been studied the reduction of restitution coefficient necessary to have a increment of a “viscous” damping equal a 1.82% (result of in-situ tests for column 1) obtained with logarithmic decrement. After that a value of mass proportional coefficient is defined comparing the Housner solution with the numerical distinct element method one.

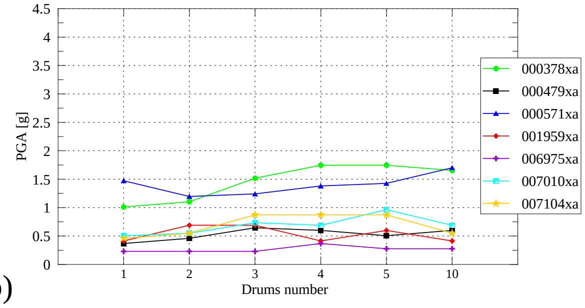
Case 4 is a configuration obtained assigning a damping of 1.82% at a frequency of 4.39 Hz directly.

4.3 Results

The results of IDA are represented in term of PGA max supportable for the 6 systems. As shown in Figure 6 there is a high dispersion of acceleration that leads to collapse if a drums number is set. Considering a single record there is not an increment of PGA increasing the drums number generally and the value of collapse acceleration is quite similar to the other drums configuration, except a few rare cases. The behavior of the curves is associated to the records and it is not possible to find an unambiguous trend.



a)



b)

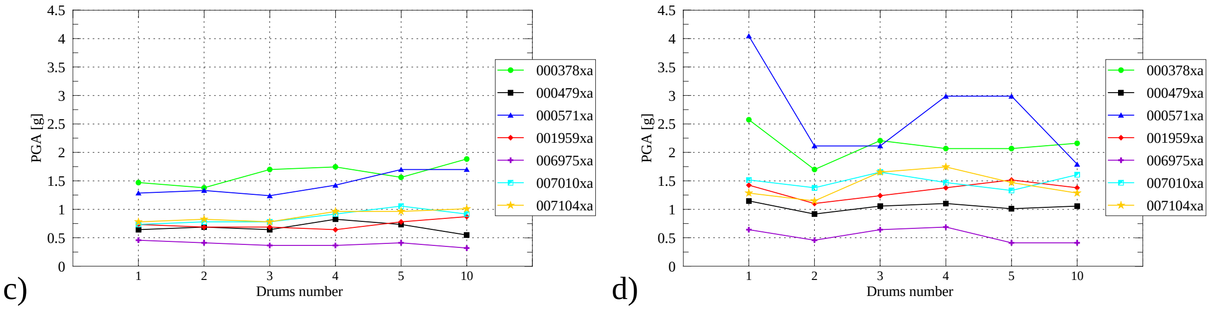


Figure 6: PGA before first collapse of the systems for: a) Case 0; b) Case 1; c) Case 2 and d) Case 3.

For each case of damping the minimum values of PGA is extracted and represented depending of the drums number (Figure 7). Always record 006975 defines the minimum collapse acceleration for all the cases. If you do not consider Case 0, the Case 2 is limited by the other two cases. From Case 1 to Case 3 the mass proportional coefficient increases so they can be considered as ordered by increasing damping. Indeed the stiffness term damps only high frequency associated to impact between blocks.

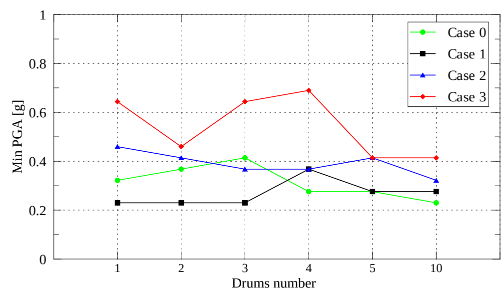


Figure 7: Min PGA for the 7 records depending of drums number.

On the other hand Case 0 do not have damping and this lead sometimes to a numerical instability or a not real response of the structure. In many cases there is a high value of sliding (Figure 8) that can lead to an increment of energy dissipation, that involves sometimes an increment of max PGA, or to a reduction of capacity of rotation of the drums that brings forward the collapse of the system. Moreover in some cases the maximum rotational response is occurred in the ending part of the input signal, when the part with major energy is already expired.

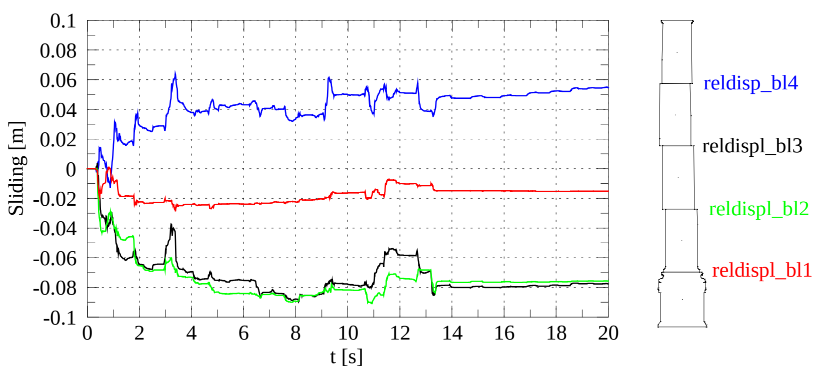


Figure 8: Relative drums displacement for record 000571 with a PGA equal to 1.1 g.

As mentioned before PGA is not a good intensity measure. If the maximum time history admissible by the systems is plotted in spectral displacement component, is possible to observe a trend to converge to a small range of value at long periods (Figure 9). Record 006975 is always the most severe signal, from a PGA point of view, and effectively has a very high displacement at long period (Figure 5). In a spectral displacement chart there is not a large difference at the collapse among the other records. The spectra are found out using the damping associated to Rayleigh relation, but this do not affect the long period displacement.

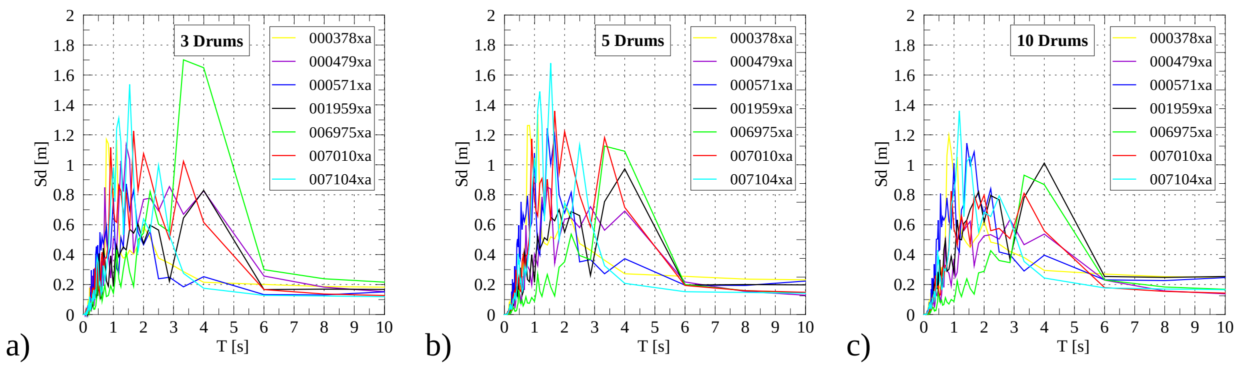


Figure 9: Displacement response spectrum before the first collapse for: a) Case 0; b) Case 1 and c) Case 2.

5 CONCLUSIONS

This study has analyzed the behavior of a multidrum columns subjected to earthquake excitation. First of all the analyses have highlighted that acceleration is not a good intensity measure because there is a large dissimilarity in the PGA of collapse changing the record. Subsequently the results do not show a link between PGA and number of drums so system with a high number of block unnecessarily have a better response. Moreover different configuration of damping was also investigated changing the Rayleigh's parameters. Numerical solution obtained without damping suffers from too much sliding and high response associated to the input after strong motion part. Considering three different cases of α-parameter the systems show much strength with the increase of the value of mass proportional damping.

Analyzing the time-histories before the first collapse in a spectral displacement vs period chart it is possible to notice a convergence of the displacement at high periods. In fact records with high spectral displacements at high period, with respect to the other signals, shown a lower acceleration scale factor.

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