

SEISMIC BEHAVIOUR OF FOUR ITALIAN MASONRY BUILDINGS: COMPARISON BETWEEN DIFFERENT MODELLING

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Abstract. *The paper deals with the problem of evaluating the in-plane seismic capacity of unreinforced masonry structures. These evaluations have been obtained by means of two different approaches: limit analysis and non-linear static analysis (Pushover), applied to four case studies of typical Italian multi-storey masonry buildings*

The Pushover analyses were performed by using three different structural softwares: Tremuri and 3DMacro, the most common commercially used software, and Abaqus, more generally used for research purposes. With reference to the main façade of the building case studies, the comparison is provided in terms of horizontal strength, horizontal top displacement and collapse mechanism.

The aim of this study is both to verify advantages and disadvantages of the different modelling examined and to point to limitations in the structural commercial software on the market at present.

1 INTRODUCTION

Nowadays, the evaluation of the seismic safety of historical buildings and retrofitting design still deserve attention, due the fact that they are so widespread. In fact, historical buildings were constructed according to ancient well-established rules, without having to perform any explicit structural analysis. Furthermore, modelling and analysing of masonry structures are complex tasks, due to the anisotropic and non-homogeneous material properties, as well as frequent modifications of the static scheme, occurring over the centuries as a consequence of additional structural elevation, openings in the bearing walls, etc. In this regard, the Italian technical code [1] explicitly requires the evaluation of structural safety. This must be included in the structural report with the safety level achieved through retrofitting and/or the possible limitations to be impose of on the use of the building. For these assessments, the Italian technical codes [1] and [2] recommend using linear or non linear kinematic analysis, and/or pushover analysis. In this paper both kinematic and pushover analyses are used for evaluating the horizontal capacity of unreinforced masonry structures. The Pushover analyses were performed by using three different structural software: Tremuri and 3DMacro, the most common commercially used software, and Abaqus, more generally used for research purposes. With reference to the principal façade of the building case studies, the comparison is provided in terms of horizontal strength, horizontal top displacement and collapse mechanism. The comparison of the numerical results has pointed to some limitations in structural computer codes.

2 THE BUILDING CASE STUDIES

In order to compare the results carried out from the different modelling examined in this paper, four historical buildings have been analyzed; they have typical geometrical, typological and mechanical characteristics of Italian buildings constructed during the XVII-XVIII centuries. In detail, the examined buildings are:

- A) Palazzo Scarpa (in the following Building A), it was built in Naples in 1906;
 - B) Ex Pretura, it was built in Naples in XVII cent. (in the following Building B);
 - C) Palazzo Centi, it was built in the historical centre of L'Aquila in XVIII cent. (in the following Building C);
 - D) De Amicis School in L'Aquila, XIII cent. (in the following Building D).
- More information on these buildings can be found in [3-5].

3 NON-LINEAR STATIC ANALYSIS AND MODELLING WITH DIFFERENT COMMERCIAL CODES

The non-linear static analysis (pushover) is nowadays a very common professional design tool for the evaluation of the seismic performance of masonry structures. It is based on displacement-based seismic assessment, where the expected deformation demands are computed by means of a response spectrum analysis of an equivalent single degree-of-freedom system, which are then compared with the deformation capacities at given performance levels. The result of the non-linear static analysis is provided in terms of “capacity curve”, which is the curve base shear-to-horizontal displacement of a control point that usually corresponds to the mass centroid of the roof [6].

Different approaches are used by the computer codes for the non-linear modelling of masonry structures. In several widespread computer codes in Italy, according to the norms of the Italian technical code [1], the non-linear modelling of the masonry building is made through the “Equivalent Frame”: The walls are divided in vertical panels (piers), horizontal panels (spandrel beams) and intersection panels between the piers and the spandrels. The piers and spandrels are modelled respectively as columns and beams of 2D frame, while the intersection

panels are modelled as rigid links. This structural modelling allows the use of the lumped plasticity model with plastic hinges for bending and shear in pre-defined points of the structure (at the bases and tops of the columns, and at the ends of the beams). This structural modelling allows a non-linear incremental collapse analysis of the masonry walls with a reduced computational effort than bi or three-dimensional finite elements modelling.

In this paper three different structural software have been used: Tremuri; 3DMacro and Abaqus. The main information regarding the modelling strategy is provided in the following sub-sections for each computer code.

3.1 Tremuri

The Tremuri program, provided by S.T.A. Data S.r.l, is one of the most common software in Italy for the structural analysis of masonry buildings. The modelling implemented in Tremuri is based on the non-linear macro-element model, each representing a whole masonry panel. This structural modelling was first proposed by Gambarotta and Lagomarsino [7] and later developed with Penna [8], Galasco et al. [9], and Lagomarsino et al. [10]. It permits, with a limited number of degrees of freedom, the representation of the two main in-plane masonry failure modes - bending-rocking and shear-sliding (with friction) mechanisms - on the basis of mechanical assumptions. This model considers, by means of internal variables, the shear-sliding damage evolution, which controls the strength deterioration (softening) and the stiffness degradation. Figure 1 shows the three sub-structures in which a macro-element is divided: two layers, lower (1) and upper (3), in which the bending and axial effects are concentrated; the central part (2) suffers shear-deformations and presents no evidence of axial or bending deformations. A complete 2D kinematic model should take into account the three degrees of freedom for each node “i” and “j” on the extremities (axial displacement w , horizontal displacement u and rotation ϕ), and two degrees of freedom for the central zone (axial displacement δ and rotation φ). It is assumed that the extremities have an infinitesimal thickness.

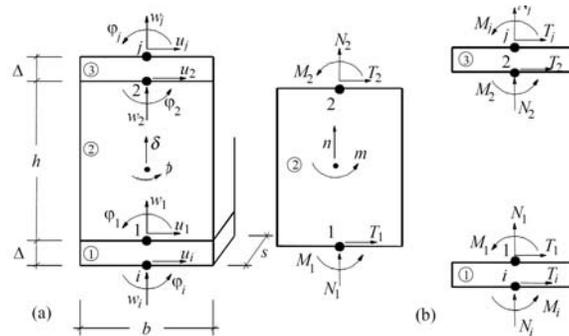


Figure 1: Kinematic model for the macro-element [8].

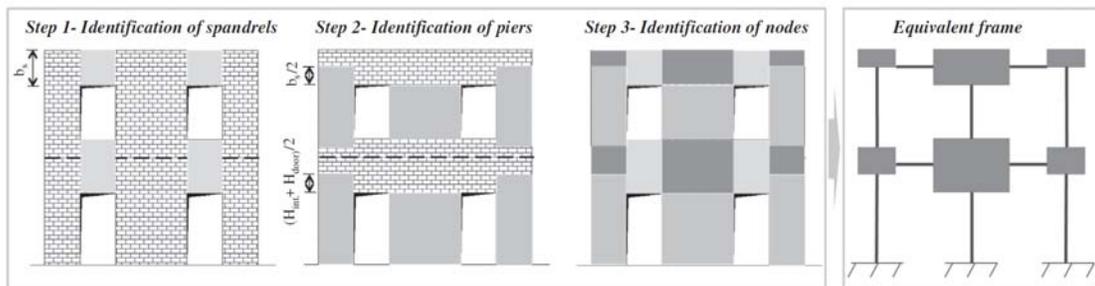


Figure 2: Example of equivalent frame idealization in case of regularly distributed openings [10].

Tremuri divides each wall in macroelements and simulates the wall in an equivalent frame as described above. Figure 2 summarizes the main steps of the frame simulation procedure in a regularly perforated masonry wall. From the identification of spandrels and piers (steps 1 and 2), defined on the basis of the vertical alignment and overlap of openings, to that of nodes (step 3). The geometry of the rigid nodes comes out directly from the previously defined elements that are connected to them.

Once having idealized the masonry wall into an assemblage of structural elements, Tremuri considers these as non-linear beam elements with lumped inelasticity simulation (bilinear elastic perfectly plastic behaviour). The deformation and non-linear response are concentrated in spandrels and piers, whereas the nodes are considered as portion rigid, which connect the deformable ones (step4). In fact, earthquake damage observation shows that only rarely and with very irregular shape or very small openings, do cracks appear in these areas of the wall.

3.2 3DMacro

The 3DMacro program, like the Tremuri program, is also based on macromodelling of the masonry walls, developed by Gruppo Sismica S.r.l. and supported by numerous research papers since 2005 [11-13].

The basic element of the proposed simplified approach has a simple and easy-comprehensive mechanical scheme, Figure 3. It is represented by an articulated quadrilateral consisting of four rigid edges connected by four hinges and two diagonal non-linear springs. Each side of the panel can interact with other panels or elements or supports by means of a discrete distribution of nonlinear springs, denoted as interface. Each interface consists of N nonlinear springs, orthogonal to the panel side, and an additional longitudinal spring which controls the relative motion in the direction of the panel edge. Given a simple masonry wall, it is possible to define a minimum number of panels which form it. 3DMacro couples each panel with the defined macroelement but the model can also be refined by using more panels in order to better describe the kinematics of the masonry walls.

In spite of its great simplicity, such a basic mechanical scheme is able to simulate the main in-plane failures of a portion of masonry wall subjected to horizontal and vertical loads: the flexural failure, the diagonal shear failure and sliding shear failure. The flexural failure mode, associated to the progressive rupture of the panel in the tensile zone and/or to the crushing of the panel in the compressive zone, can be reproduced by the transversal springs of the interfaces that simulate the axial and flexural deformability of the masonry panel. All the axial and flexural properties of the portion of masonry are lumped in the transversal springs that are calibrated by assuming a non-symmetric elasto-plastic behaviour with limited deformability.

The diagonal-shear failure mode and the consequent formation of diagonal cracks along the directions of the principal compression stresses, can be governed by means of the two diagonal nonlinear springs which have the role of simulating and predicting the nonlinear shear response of the modelled masonry portion. To simulate the shearing behaviour, an elasto-plastic constitutive law with a Turnsek & Cacovic yielding surface is considered.

The sliding-shear failure mode is associated to the sliding of the masonry panel in its own plane and it can be controlled by the longitudinal nonlinear springs of the interfaces. This longitudinal springs are characterised by a rigid plastic constitutive law with a Mohr-Coulomb yielding surface.

Each panel exhibits three degrees of freedom, associated to the in-plane rigid body motion, plus a further degree of freedom needed for description of the panel deformability. Hence the total degrees of freedom of a N panels structural scheme are $4N$. As Lagrangian parameters, the four displacements of the rigid edges along their own directions have been chosen.

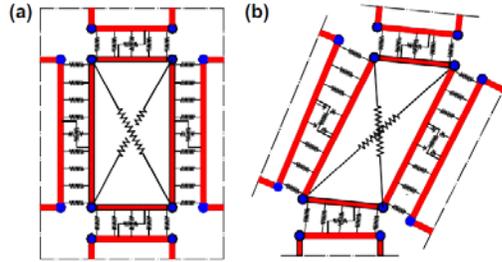


Figure 3: The basic macroelement: a) undeformed configuration; b) deformed configuration [11].

3.3 Abaqus

The Abaqus computer code allows modelling masonry structures by using the “concrete smeared model” that is based on a simple yield surface with isotropic hardening and associated flow when the state of stress is predominantly compressive. It uses damaged elasticity to account for the cracking, the occurrence of which is defined by a so-called “crack detection surface”. This failure surface is assumed to be a simple Coulomb line written in terms of the first and second stress invariant. Neglecting the reduction of stiffness caused by inelastic straining, which is very important for masonry, the model is not capable of predicting cyclic response. Moreover, the use of associated flow usually leads to over-prediction of volume strain [14, 15]. The concrete model basically requires the stress–strain curve in compression to be defined in tabular form as a function of plastic strain, the shape of the failure surface via the “failure ratios” option and the post-cracking tensile behaviour defined by the “tension stiffening” option. Indeed this last feature makes no sense for masonry, however a small amount of tensile resistance should be provided in any case to avoid numerical instability problems.

In detail, the “concrete smeared model” uses the classical concepts of plasticity theory: strain rate, decomposition into elastic and inelastic strain rates, elasticity, flow and hardening. In the definition of the compression yield, the value of the magnitude of each nonzero principal stress in biaxial compression and the stress magnitude in uniaxial compression (σ_{bc}/σ_c) is given on the 1st “failure ratio” data line. In the same way, the ratio of the uniaxial tensile stress at failure to the uniaxial compressive stress at failure (σ_t/σ_c) is given on the 2nd “failure ratio” data line. In the definition of the flow, the value given on the 3rd “failure ratio” option is representative of the ratio of ε_{pl} in a monotonically loaded biaxial compression test to ε_{pl} in a monotonically loaded uniaxial compression test. In tension, cracking dominates the material behaviour. The model uses a “crack detection” plasticity surface in stress space to determine when cracking takes place and the orientation of cracking. Damaged elasticity is then used to describe the post failure behaviour of the material with open cracks. Regarding the crack orientation, although some models have been proposed (fixed model with orthogonal cracks, rotating model, fixed model with multidirectional cracks), the used model by Abaqus is the first one. The perpendicular to the first crack that occurs in a point is parallel to the maximum principal tension stress. The model remembers this direction so that the following cracks could form only in a direction perpendicular to the first one. The value of the tensile failure stress σ_I in a state of biaxial stress when the other nonzero principal stress σ_{II} , is at the uniaxial compression ultimate stress state, is defined by the 4th “failure ratio”.

4 NON-LINEAR STATIC ANALYSES

4.1 Tremuri

The principal façades of the four case studies are herein analysed, at first, with the commercial code Tremuri 5.5.208.

The needed input data required by the software are the geometric parameters and mechanic parameters of the masonry wall. Providing also the seismic parameters, the software automatically define the loads acting on the structure: self weight and seismic force.

The Tremuri software use a simplified macromodelling, as explained in section 2.1. It is not necessary to define a mesh, in fact the software automatically divides every wall in spandrels, piers and nodes simulating these as elements of an equivalent frame.

The non-linear analysis is conducted increasing the loads in monotonic mode, and then deriving the horizontal displacement of the structure. Once the conventional displacement is exceeded, which is calculated automatically, the structure is considered to have collapsed. Then the software provides the horizontal force-horizontal displacement curve, which represents the capacity curve.

The Figure 4 shows the capacity curves (in terms of collapse multiplier and displacement divided by global height) for the principal façade of the case studies.

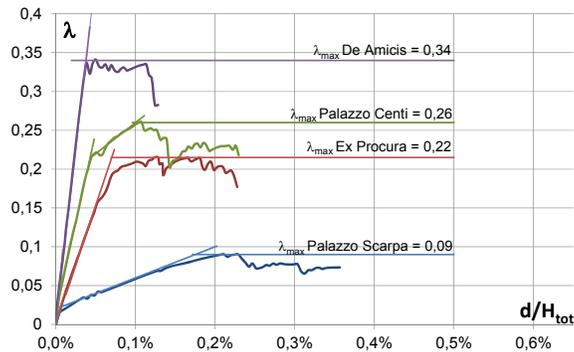


Figure 4: Pushover curves obtained with Tremuri computer code.

It is noted that the maximum base shear forces and the corresponding values of the collapse multiplier are lower in Neapolitan buildings (Palazzo Scarpa and Ex Procura) because they have a higher value of the global pier slenderness (B/H_{tot}), than the buildings located in L'Aquila. The value of the collapse multiplier is equal to $\lambda=9\%$ for Palazzo Scarpa and $\lambda=34\%$ for the De Amicis primary school. The table 1 summarizes the numerical results of the analyses and provides for each case studies: 1) the collapse multiplier; 2) the corresponding displacements (d_{max}); 3) the displacements divided by global height.

Table 1: Tremuri numerical results.

	$\lambda=F_{max}/W_{tot}$	d_{max} [cm]	d_{max}/H_{tot}
a) Palazzo Scarpa	9%	9.75	0.36%
b) Ex Procura	22%	5.9	0.23%
c) Palazzo Centi	26%	3.8	0.23%
d) De Amicis	34%	1.7	0.11%

These results show that the Tremuri code is able to research the structural inelastic behavior, in fact this software achieves notable values of maximum displacements.

Figure 4 also shows the linear approximation of the capacity curves. For the capacity curve of De Amicis School, the linear approximation provides a bilinear diagram; for the other case studies the capacity curves can be simplified by tri-linear diagram. The reduction of the slope shows that the structural stiffness decreases due to the progressive spandrels damage. Consequently the collapse largely occurs due for the failure of the spandrels except for the De Amicis School that collapses for the shear failure in the wall piers.

4.2 3DMacro

The 3DMacro code uses a similar procedure to the ones described for the analysis with Tremuri. So also the main data given to 3DMacro, are the geometry and the material of the walls. The material parameters are the same values provided to Tremuri so the results carried out are comparable.

The loads on the buildings are given by the weight of the masonry and by static forces proportional to the masses.

3DMacro is able to model the walls through a more refined mesh, as explained in the section 2.2. The use of a more refined mesh is not mandatory, however in some cases it can provide more accurate results and a better description of the collapse mechanism, but in the case studies described it is not necessary.

The results obtained with 3DMacro are summarized in the capacity curves plotted in the Figure 5 and in numerical values provided in the Table 2.

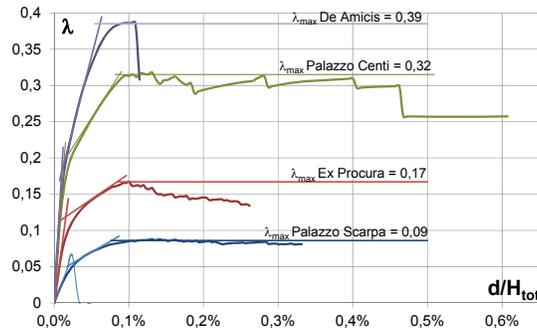


Figure 5: Pushover curves obtained with 3DMacro computer code.

Table 2: 3DMacro numerical results.

	$\lambda = F_{\max}/W_{\text{tot}}$	d_{\max} [cm]	d_{\max}/H_{tot}
a) Palazzo Scarpa	9%	9	0.3%
b) Ex Procura	17%	6.8	0.26%
c) Palazzo Centi	32%	10	0.6%
d) De Amicis	39%	1.9	0.13%

The 3DMacro results are similar to the Tremuri results, as it is noted comparing Table 1 and Table 2, but 3Dmacro provides higher values of the maximum displacement (divided by global height). Furthermore the linear approximation for 3Dmacro capacity curves gives tri-linear diagrams representing the case studies. It denotes three levels of the walls stiffness.

With 3DMacro the damage and the causes of the collapse are still connected to spandrels shear crisis except of the shear crisis of the piers in the De Amicis school.

4.3 Abaqus

The principal façades of the four case studies were finally analysed with the computer code Abaqus 6.10, that has provided the finite element models of the 4 masonry walls.

The needed data to conduct analysis with this software are: 1) the geometrical features of the wall; 2) the mesh, 3) the mechanical properties of the masonry material; 4) the loads acting on the wall; 5) the constraint conditions.

The masonry walls have been modelled using the S4R thick shell linear element. The average size of the mesh is 0.25m. Sensitive analyses have been conducted in order to establish the optimised size and shape of the mesh.

The walls are constrained on the base with a fixed support and loaded at first step with the self weight and in the second step with a horizontal body forces applied incrementally.

The non-linear analysis provides the values of the maximum base shear force and the maximum displacement of a control point located in top of the wall. In the Figure 6 the capacity curves obtained using these results are shown.

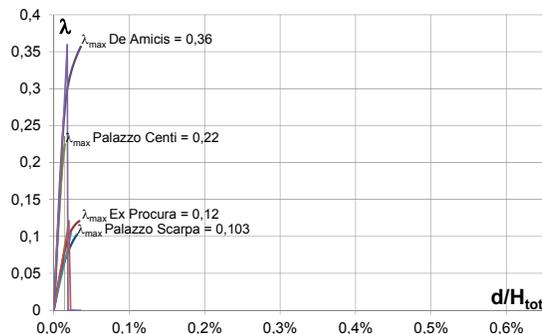


Figure 6: Pushover curves obtained with Abaqus computer code.

It is noted that Abaqus code amounts to values of the maximum displacement lower than those obtained with Tremuri and 3Dmacro. In fact, over a certain level of damage, the numerical modelling performed to Abaqus has many difficulty of convergence, so the software aborts the analysis. Therefore, Abaqus seems to be not able to evaluate the realistic maximum displacement of the wall. But this software can be used to define a realistic value of the maximum base shear force (Table 3).

Even though results are not similar, the causes of collapse are the same the two other software. The development of the plastic deformation in the model indicates that, with the exception of De Amicis school, the collapse of the façades is still given by the shear failure of the spandrels.

Table 3: Abaqus numerical results.

	$\lambda = F_{\max}/W_{\text{tot}}$	d_{\max} [cm]	d_{\max}/H_{tot}
a) Palazzo Scarpa	10%	0.85	0.03%
b) Ex Procura	12%	0.9	0.03%
c) Palazzo Centi	22%	0.25	0.015%
d) De Amicis	36%	0.53	0.04%

5 LIMIT ANALYSIS

A simplified assessment of the horizontal strength of masonry structures can be made by using the limit analysis approach. It consists in: (i) defining the possible collapse mechanisms; (ii) evaluating the seismic capacity, i.e. the value of horizontal force corresponding to the ac-

tivation of the mechanism; (iii) comparing the seismic capacity with the seismic demand. According to the Italian Code CM'09 [2], the application of the linear kinematic analysis is based on the following assumptions: null tensile strength; infinite compression strength; sliding of a stone or of a part of the structure cannot occur.

The global collapse mechanism of the generic masonry wall of Figure 7a, can be considered the in-plane failure mode. It is characterized by the formation of hinges at the ends of the girders and at the base of the piers (Figure 7b). The corresponding multiplier of the horizontal actions (λ) can be evaluated by means of the following closed form expression:

$$\lambda = \frac{B}{H} \cdot \frac{\sum_{i=1}^{n_b+1} W_{p,i} + \sum_{j=1}^{n_b \cdot n_s} W_{s,j}}{\left[n_s \cdot \sum_{i=1}^{n_b+1} W_{p,i} + \left(n_s + 1 - \frac{t}{H} \right) \cdot \sum_{j=1}^{n_b \cdot n_s} W_{s,j} \right]} = \frac{B}{H} \cdot \frac{1}{n_s + \left(1 - \frac{t}{H} \right) \cdot \frac{\sum_{j=1}^{n_b \cdot n_s} W_{s,j}}{W_{tot}}} \quad (1)$$

Where B , H and t are the parameters that define the wall geometry (Figure 7a), n_b is the number of bay spans, n_s is the number of storeys, $W_{p,i}$ is the weight of the wall pier i^{th} , $W_{s,j}$ is the weight of the spandrel j^{th} , W_{tot} is the total weight.

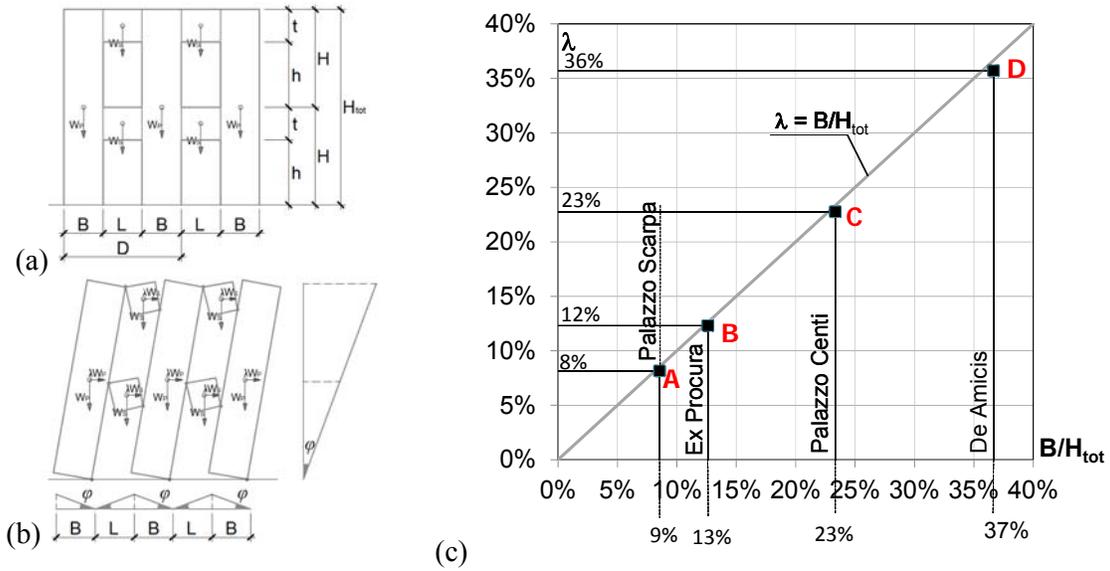


Figure 7: Limit analysis: (a) geometry of masonry wall; (b) frame collapse mechanism; (c) horizontal collapse multipliers for the principal façades of the building case studies.

The application of Eq. (1) to the principal façades of the building case studies allows obtaining the values of the horizontal collapse multiplier λ plotted in Figure 7c as a function of the B/H_{tot} ratio. It can be noted that the bullet points are allocated very close to the bisector line ($\lambda=B/H_{tot}$). This is because the contribution of the spandrels to the horizontal capacity is negligible than the contribution of the piers. In fact the façades are characterized by low values of the $A_{spandrel}/A_{tot}$ ratio, variable in the range 10%-16%. In other words, when the total weight of the spandrels is negligible, i.e. $\sum_{j=1}^{n_b \cdot n_s} W_{s,j} / W_{tot} \cong 0$, the Eq. (1) provides the following approximated expression of the collapse multiplier: $\lambda=B/(n_s \cdot H_{tot})=B/H_{tot}$, corresponding to the bisector line equation.

6 PUSHOVER VS. LIMIT ANALYSIS: COMPARISON AND DISCUSSION

Figure 8 shows the comparison between the capacity curves carried out by using Tremuri, 3DMacro and Abaqus and the horizontal lines corresponding to collapse multiplier evaluated with the limit analysis (Eq. (1)). The comparison shows that the seismic capacities assessed using the non-linear analysis are not very different from those measured with the closed-form equations proposed in this paper.

In details the scatter between the results is lesser than 15% using 3DMacro, and is equal to 6-7% using Abaqus and Tremuri, respectively. It follows that the limit analysis can be used as simple tool for checking the results of more complex analysis, often less manageable (pushover analysis), closely linked to modelling used by commercial software for structural analysis.

In fact, as can be also observed by Figure 9, the comparison between the different results shows that each software provides different values of the maximum base shear force and of the maximum displacement of the control point. In details it has been notes that Abaqus it is not able to determine the correct value of the maximum displacement, whereas Tremuri and 3DMacro provide higher capacity in terms of the displacement and the base shear force.

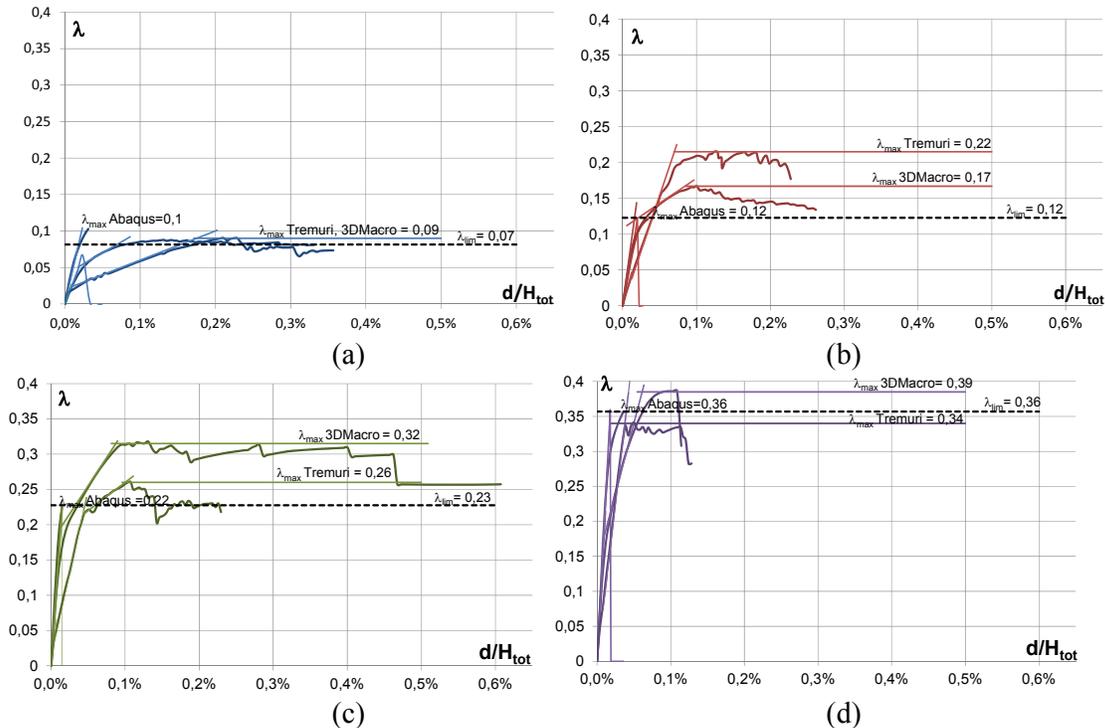


Figure 8: Comparison between Pushover and Limit Analyses: (a) Palazzo Scarpa; (b) Ex Procura; (c) Palazzo Centi; (d) De Amicis School.

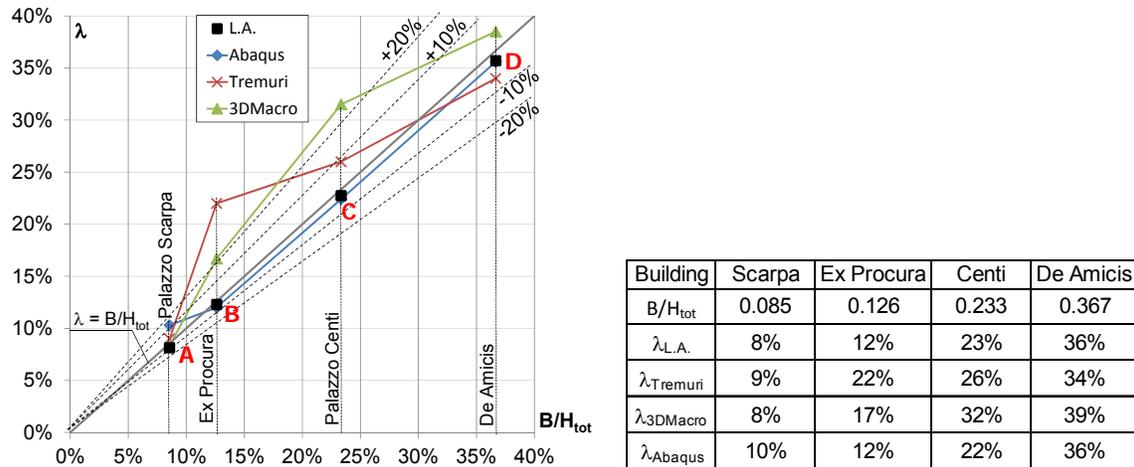


Figure 9: Non-linear analysis VS. Limit Analysis (L.A.): comparison horizontal collapse multipliers for the principal façades of the building case studies.

7 CONCLUSIONS

In this paper in-plane seismic capacity of unreinforced masonry structures is studied by adopting both non-linear static analysis and the limit analysis approach.

In particular, four case studies of typical Italian multi-storey masonry buildings have been illustrated and then analysed by using three different structural software: Tremuri and 3DMacro, and then Abaqus. With reference to the main façade of the building case studies, the comparison has been provided in terms of horizontal strength, horizontal top displacement and collapse mechanism.

Comparison of the results of pushover analyses shows that the used computer codes provide different response parameters, with maximum scatters of 10% in terms of multiplier collapse and 0.59% in terms of horizontal displacement.

These scatters are due to the different modelling adopted by the used computer codes and indicate that further experimental and numerical studies are needed to better understand the problem of evaluating the seismic capacity of masonry structures. Finally, it is pointed out that the Limit Analysis approach can be useful for providing benchmark values to be used for verifying non-linear structural analysis of masonry structures.

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