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INFLUENCE OF THE DIVINE PROPORTION IN THE SEISMIC BEHAVIOR OF RELIGIOUS HISTORICAL BUILDINGS LOCATED IN MORELIA CITY, MEXICO

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Abstract. *In this paper the structural behavior of the naves of thirteen religious structures built between the sixteenth and eighteenth centuries on firm soil of the historical downtown of the city of Morelia, which were modified geometrically following the Golden Ratio is studied. The naves were characterized by macroelements in the transversal and longitudinal sections of the building which were analyzed using eleven seismic records and nonlinear analysis. In all cases the seismic records were scaled for the maximum ground acceleration expected on site for return a period of 475 years. Finally damage degrees and damage index for each case were established and the influence of the Golden ratio or Divine Proportion over the original structures was analyzed.*

INTRODUCTION

The Mexican cultural heritage is formed almost by 85000 buildings, of which about 6000 are located in the State of Michoacan and approximately 1100 of them are located in the historic downtown of the city of Morelia. So this built heritage conform the basis of the great monumental richness of the city, which has earned it´s recognition from UNESCO as a world heritage city since the year of 1991. However in Mexico the information relative to the structural behavior of this type of construction in order to perform a correct diagnosis for intervention purposes is scarce.

Therefore within the multidisciplinary context that involves the study, conservation, restoration and rehabilitation of these buildings, the part corresponding to the seismic and structural engineering represent a very particular challenge. The city of Morelia is located in an area of high seismic activity because the Michoacán coast forms part of the Pacific ring of fire. This city is located approximately 340 km from the subduction zone in the Pacific Ocean. Additionally to subduction earthquakes, Morelia city suffers local and normal type earthquakes; particularly these last, although they tend to have intermediate hypocentral distances have shown a highly destructive character, an example clear is the earthquake on June 19, 1858, which collapsed several buildings and caused major damages to religious structures such as the Cathedral of Morelia [1] a situation that has not been presented in recent earthquakes.

This article is part of a series of efforts to better understand the structural behavior expected for heritage constructions in high seismicity zones of Mexico.

DEFINITION OF SEISMIC DEMAND

Ten seismic records were used and they were scaled to the maximum acceleration expected of the soil at the site, as well as an artificial register compatible with a uniform seismic hazard spectrum of the site; these parameters were obtained from a study of probabilistic seismic hazard performed for the site, which casts a likely maximum value for peak ground acceleration of 1.6 m/s^2 for a return period of 475 years.

Most of the seismic records were obtained from the Mexican Strong Motion Database and only earthquakes with a maximum spectral response linked to high frequencies were used, since it has been observed that these earthquakes are more aggressive for old masonry structures of small to middle size. Likewise it has been observed after several earthquakes around the globe that this type of construction usually collapses by zones and not in a extensive manner, because the macroelements with short periods tend to suffer greater energy demands. Table 1 shows the dynamic characteristics of the considered earthquakes. It should be noted that two of the records involved were obtained by local seismic stations, one of them in the station located on firm ground of University City, University Michoacana de San Nicolás de Hidalgo, and the other on the University Vasco de Quiroga Morelia's firm ground, which are zones with great similarity with the ground of the historic centre of the city and are located a few hundred meters of the latter.

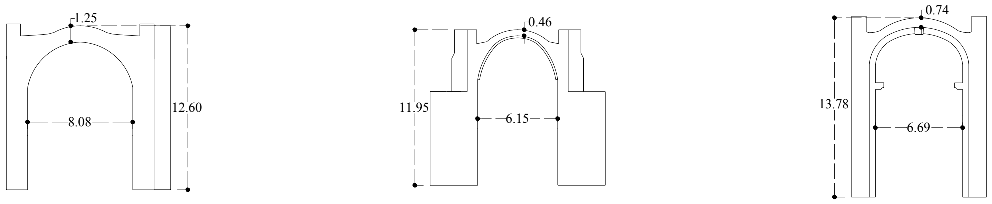
Table 1: Seismic demand

Name	Date	Direction	Dt (seg)	PGA (m/s^2)	Magnitude	Station	Type
10D94	10-12-94	N-S	0.005	0.17339	6.3	Chilpancingo	Real
14995	14-09-95	E-O	0.005	0.46609	7.3	Chilpancingo	Real
15796	15-07-96	N-S	0.005	0.21080	6.5	Chilpancingo	Real
21103	21-01-03	N-S	0.005	0.182	7.6	Chilpancingo	Real
25498	25-04-98	N-S	0.005	0.10546	5.2	Gutiérrez	Real
AC020996	02-09-96	N-W	0.01	0.0701	-	Acapulco	Real
AC310393	31-03-93	S-O	0.01	0.046	5.0	Acapulco	Real
CU110197	11-01-97	E-O	0.01	1.186	6.9	Cu UMSNH	Real
PU150699	15-06-99	E-O	0.01	1.991	6.5	Puebla	Real
UV200498	20-04-98	E-O	0.005	0.075	5.9	UVAQ	Real
CATE475	-	-	0.01	1.6	-	-	Artificial

It is important to mention that for practical purposes analysis, the duration of the seismic records was reduced to only consider the significative duration. As it can be seen in the table 1, the earthquake of Puebla of June 15 of 1999 had a maximum acceleration of the ground of 1.991 m/s2 which is greater than the expected probabilistic acceleration for 475 years of recurrence, so we decided to use this seismic record without any modification.

DEFINITION OF MACROELEMENTS

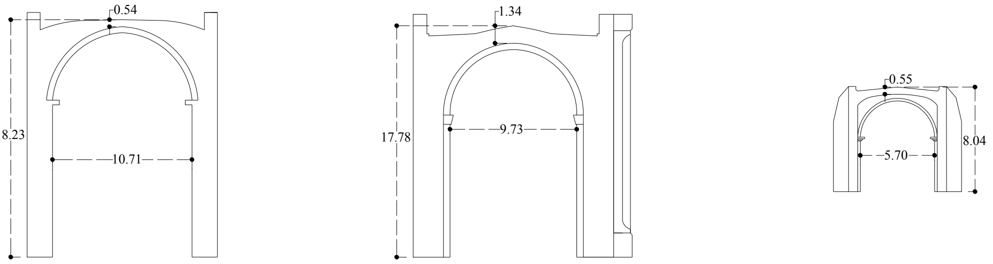
A two-dimensional approach was chosen for the construction of macroelements both transverse and longitudinal. Figure 1 shows the original transversal sections of the macroelements and Figure 2 the modified geometries with the golden ratio [6]. In the same manner, Figures 3 and 4 shows the original and modified geometries for the longitudinal macroelements of the building stock. Additionally, for the definition of the macroelements the most vulnerable zones of the buildings were modeled, based in the fact that the stiffer zones will suffer the largest stresses and deformations during earthquakes.



(a) Capuchinas Church (1734)

(b) La Cruz Church (1680)

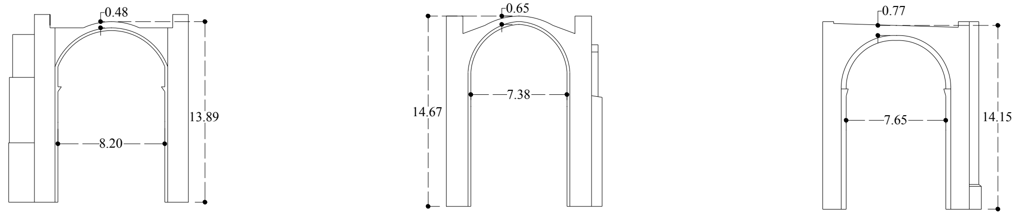
(c) La Merced Church (1736)



(d) Franciscano de St. Buenaventura Church (1585)

(e) St. José Church (1760)

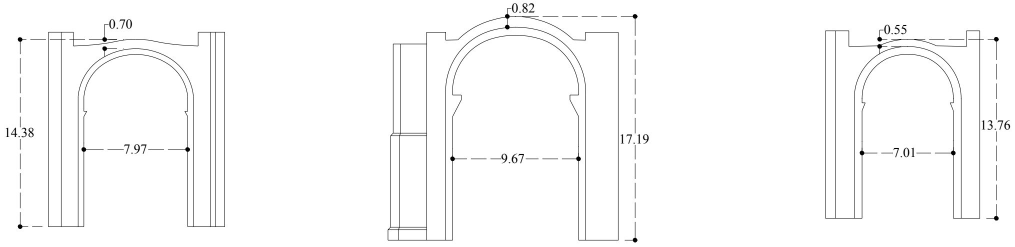
(f) St. Juan Bautista Church (1696)



(g) St. Francisco Javier Church (1660)

(h) El Carmen Church (1596)

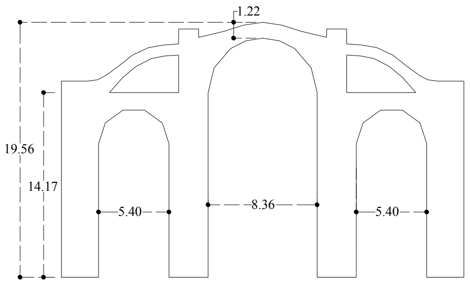
(i) Las Monjas Church (1732)



(j) Saint Rosa de Lima Church (1757)

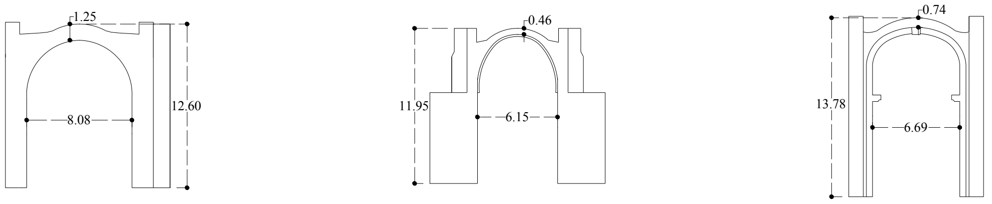
(k) St. Agustin Church (1550)

(l) Guadalupe Church (1708)



(m) Transfiguration Cathedral (1660)

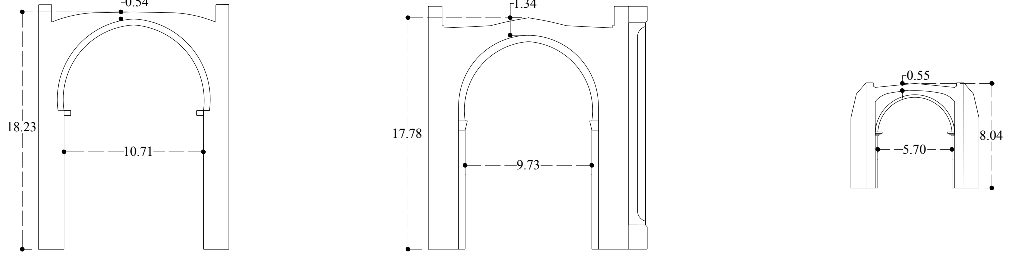
Figure 1: Transversal macroelements with actual geometry.



(a) Capuchinas Church
(1734)

(b) La Cruz Church
(1680)

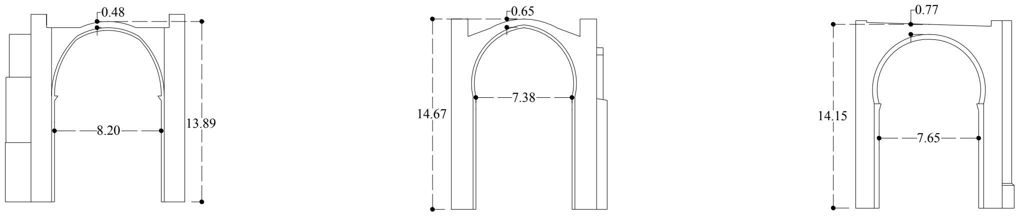
(c) La Merced Church
(1736)



(d) Franciscano de St.
Buenaventura Church (1585)

(e) St. José Church
(1760)

(f) St. Juan Bautista Church
(1696)



(g) St. Francisco Javier Church
(1660)

(h) El Carmen Church
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(i) Las Monjas Church
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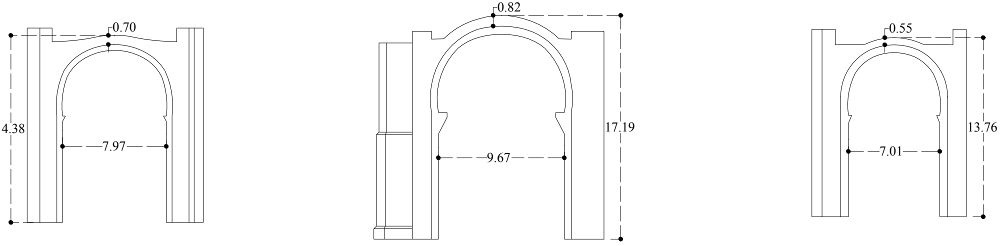
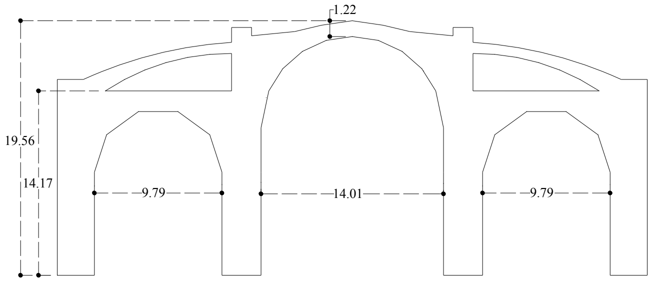


(j) Saint Rosa de Lima Church
(1757)

(k) St. Agustín Church
(1550)

(l) Guadalupe Church
(1708)

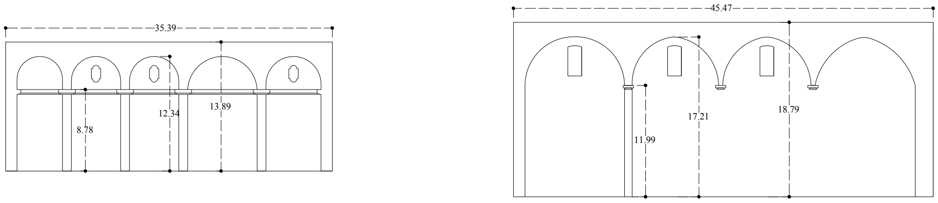


(m) Transfiguration Cathedral
(1660)

Figure 2: Transversal macroelements modified with the Golden proportion.

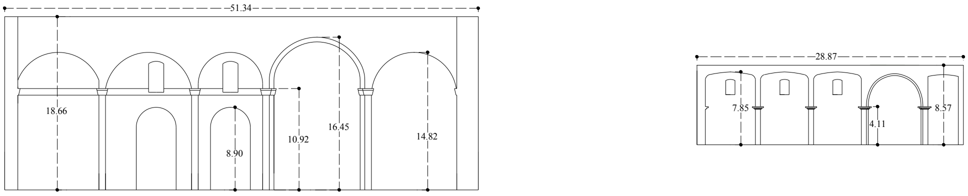
(a) Capuchinas Church
(1734)

(b) La Cruz Church
(1680)



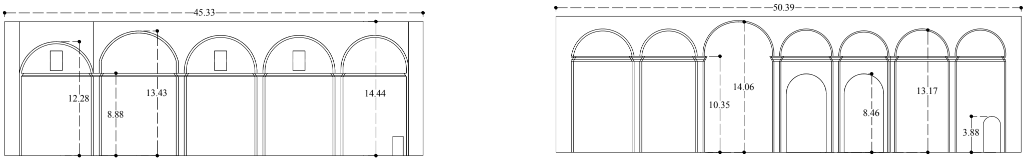
(c) La Merced Church
(1736)

(d) Franciscano de St. Buenaventura Church
(1585)



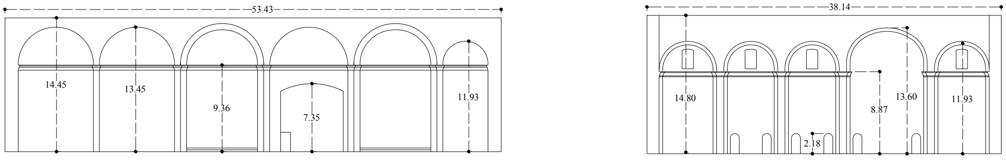
(e) St. José Church
(1760)

(f) St. Juan Bautista Church
(1696)



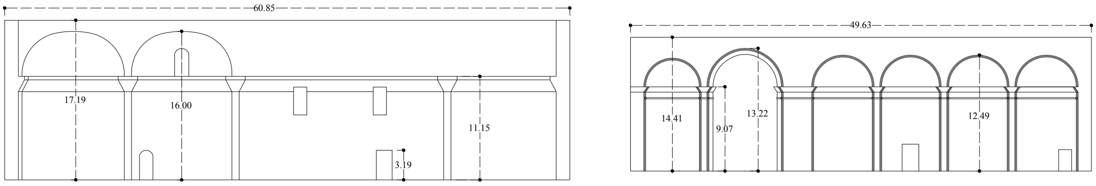
(g) St. Francisco Javier Church
(1660)

(h) El Carmen Church
(1596)



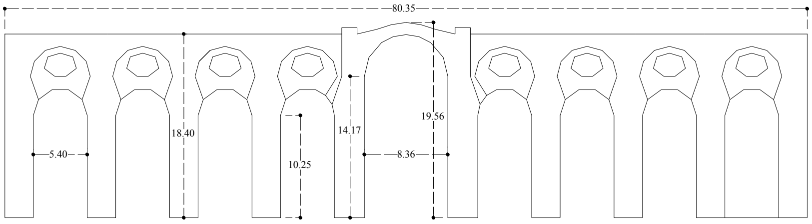
(i) Las Monjas Church
(1732)

(j) Saint Rosa de Lima Church
(1757)



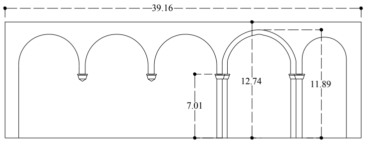
(k) St. Agustin Church
(1550)

(l) Guadalupe Church
(1708)

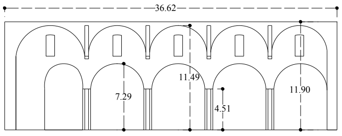


(m) Transfiguration Cathedral
(1660)

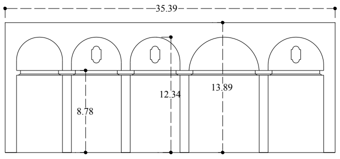
Figure 3: Longitudinal macroelements with actual geometry.



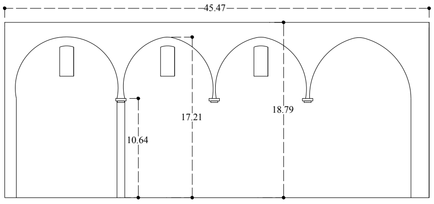
(a) Capuchinas Church
(1734)



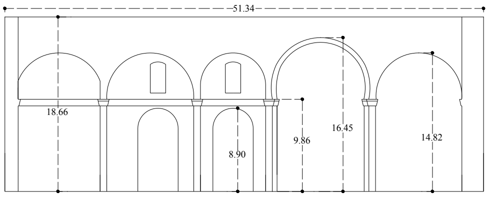
(b) La Cruz Church
(1680)



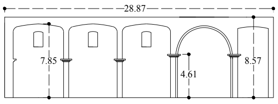
(c) La Merced Church
(1736)



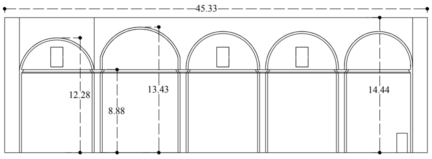
(d) Franciscano de St. Buenaventura Church
(1585)



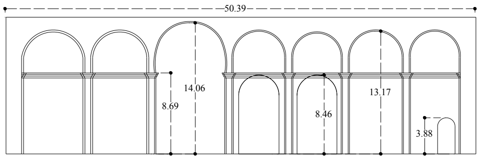
(e) St. José Church
(1760)



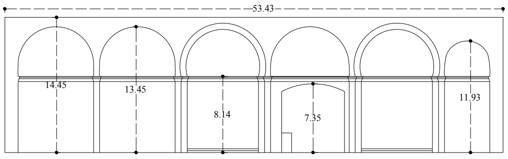
(f) St. Juan Bautista Church
(1696)



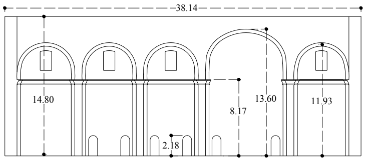
(g) St. Francisco Javier Church
(1660)



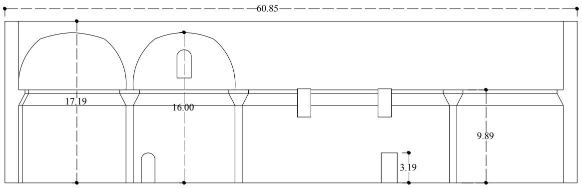
(h) El Carmen Church
(1596)



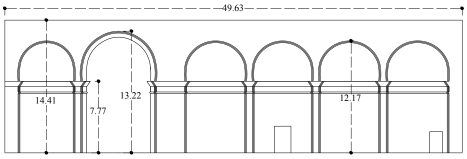
(i) Las Monjas Church
(1732)



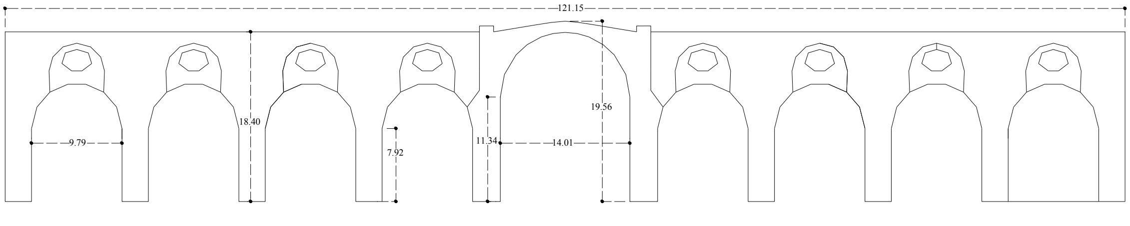
(j) Saint Rosa de Lima Church
(1757)



(k) St. Agustín Church
(1550)



(l) Guadalupe Church
(1708)



(m) Transfiguration Cathedral
(1660)

Figure 4: Longitudinal macroelements modified with the Golden proportion.

Most of the actual cross sections are provided with buttresses, for this reason we choose these macroelements as the workbench for the transversal directions of the buildings; however, there are some cross sections where the buttresses are not presented as is the case of Merced church, the St. Francisco of St. Buenaventura church and the Cathedral. Most of these structures are ex-convent buildings where the cloister is attached to the temples; this structural characteristic was not considered at this stage of the investigation but will be considered in future research. Despite this, the two-dimensional modeling based on the definition of macroelement provides a fair idea of the general behavior of the structure.

For the generation of the sections with the golden ratio the actual height of each building was left as a fixed parameter, in order to keep the mass of the buildings quite similar to the mass of the actual sections, so this characteristic changed the rise/span relationship of the arches as well as the height of walls and buttresses as it shown in Figure 5, it is important to mention that for the Cathedral the spans of the macroelements were also modified with the golden ratio. Let us remember that the golden ratio is also denoted with the letter φ (phi) and a simple way to define this proportion is using Equation 1, where the letter "a" represents the height from the ground to the intrados of the key-stone, the letter "b" corresponds to the height from the ground to the springing of the arch and the letter "c" the rise of the arch (Figure 5).

$$\varphi = \frac{d}{a} = \frac{c}{d} = 0.6180339887 \ldots \tag{1}$$

Actual transversal section

Golden transversal Section

Figure 5: Generating golden sections in the d/a relationship.

STRUCTURAL ANALYSIS OF MACROELEMENTS

The structural analysis for each macroelement was performed using the rigid element formulation which is a simplification for linear and non-linear analysis of unreinforced masonry structures, considering that the elements have the kinematics of rigid body with two linear displacements and one rotation. These connections are two axial springs and one shear device connected to the common side between two rigid elements [3, 4].

The meshing criteria for each macroelement were based on the geometry of each studied section conforming a mesh as regular as possible. The thickness for arches, walls and buttresses were obtained directly from the actual structures and they stayed without any change both for actual geometries as the modified ones with the golden proportion. It is noteworthy that the stiffening effect provided by the vaults, apse and narthex were not included in the macroelements. The mechanical properties used in the analyses are shown in Table 2.

Table 2: Mechanical properties

	Young's Modulus (MPa)	Poisson's ratio	Density (Kg/m^3)
Stone	1000	0.2	1800
Fill material	500	0.2	1600

DYNAMIC RESPONSE

Nonlinear time history analysis

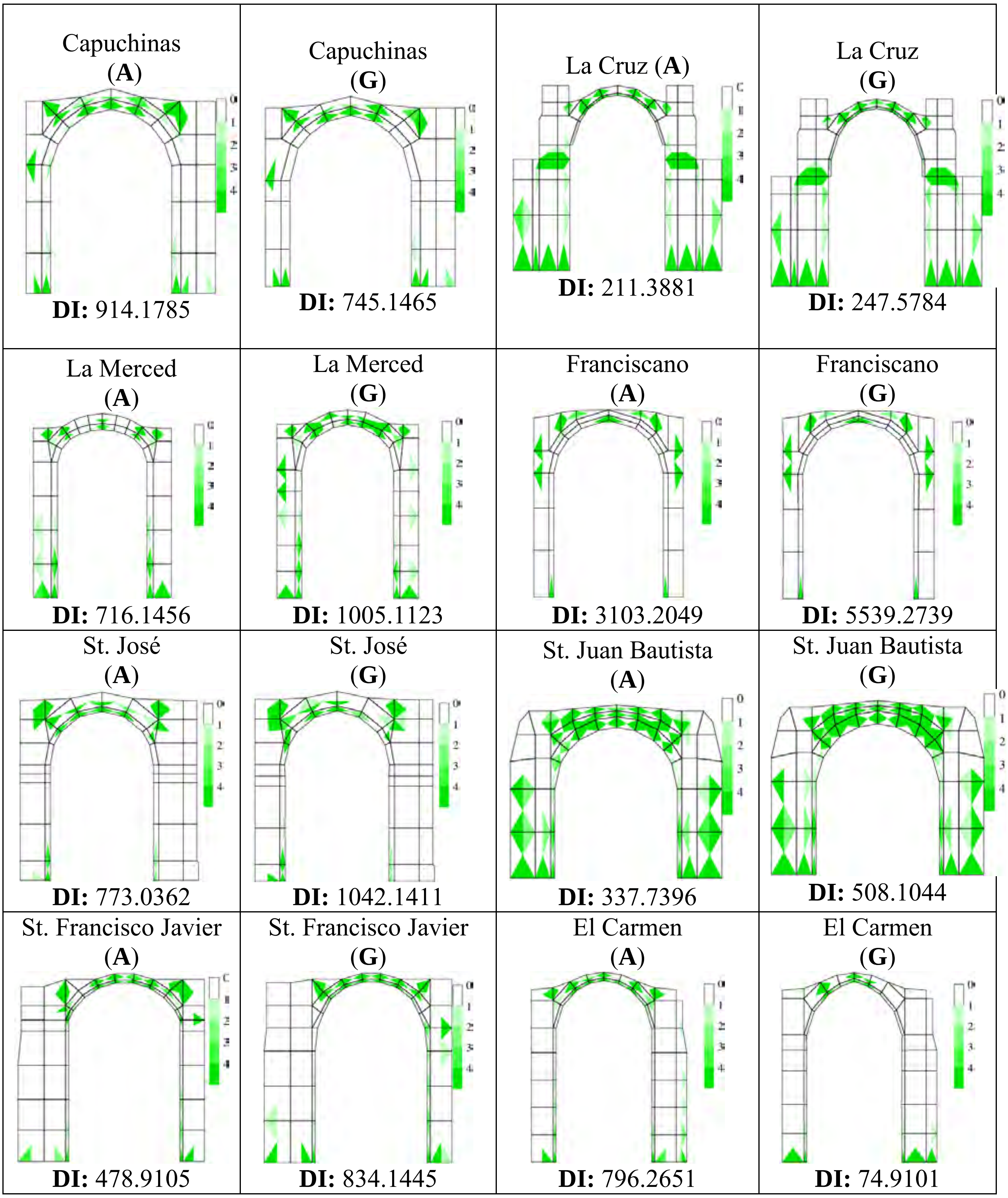
A non-linear time-history analysis was performed for the macroelements using the RIGID v0.4.1 [4]. Eleven seismic records were scaled to the probabilistic peak ground acceleration expected for the site and only the horizontal acceleration component was considered; however it is important to mention that some of the studied macreoelements failed by compression at the supports, so for this reason the maximum ground acceleration was decreased in these cases in order to know the status of pre-compression failure since the program does not generate results once it has occurred.

Lateral displacement, dissipated energy, load capacity and damage degrees were obtained for the macroelements. With the above parameters were estimated damage indexes of according to the criterion proposed by Ang and Wen [5], which mentions that the failure of the masonry is mainly fragile, which means that a damage indicator may be the maximum lateral drift; however the authors mention that after application of several loading cycles the walls suffer a total failure, which means that another indicator of damage can be energy dissipation; based on the above mentioned the authors proposed equation 2 to calculate the damage index (**DI**) for the unreinforced masonry taking as an indicator of damage the energy dissipation.

$$DI = \frac{U_m}{U_f} + \frac{\varepsilon}{q_u * U_f} \int dE \tag{2}$$

Where Um is the maximum lateral drift, ∫dE is the total energy dissipated, Uf is the failure displacement under monotonic load, qu is the ultimate load capacity and ε is an experimental parameter with value of 0.075. It should be noted that a static non linear pushover analysis is necessary in order to estimate the Uf value.

Figures 6 and 7 shows the damage degrees and damage indexes obtained for the actual (**A**) and modified macroelements following the golden ratio (**G**). The presented results correspond to the maximum earthquake values for each structure. The damage indexes are presented in kJ and relative to the damage degrees cero means no damage and number four means extensive damage.



Capuchinas (**A**) **DI:** 914.1785

Capuchinas (**G**) **DI:** 745.1465

La Cruz (**A**) **DI:** 211.3881

La Cruz (**G**) **DI:** 247.5784

La Merced (**A**) **DI:** 716.1456

La Merced (**G**) **DI:** 1005.1123

Franciscano (**A**) **DI:** 3103.2049

Franciscano (**G**) **DI:** 5539.2739

St. José (**A**) **DI:** 773.0362

St. José (**G**) **DI:** 1042.1411

St. Juan Bautista (**A**) **DI:** 337.7396

St. Juan Bautista (**G**) **DI:** 508.1044

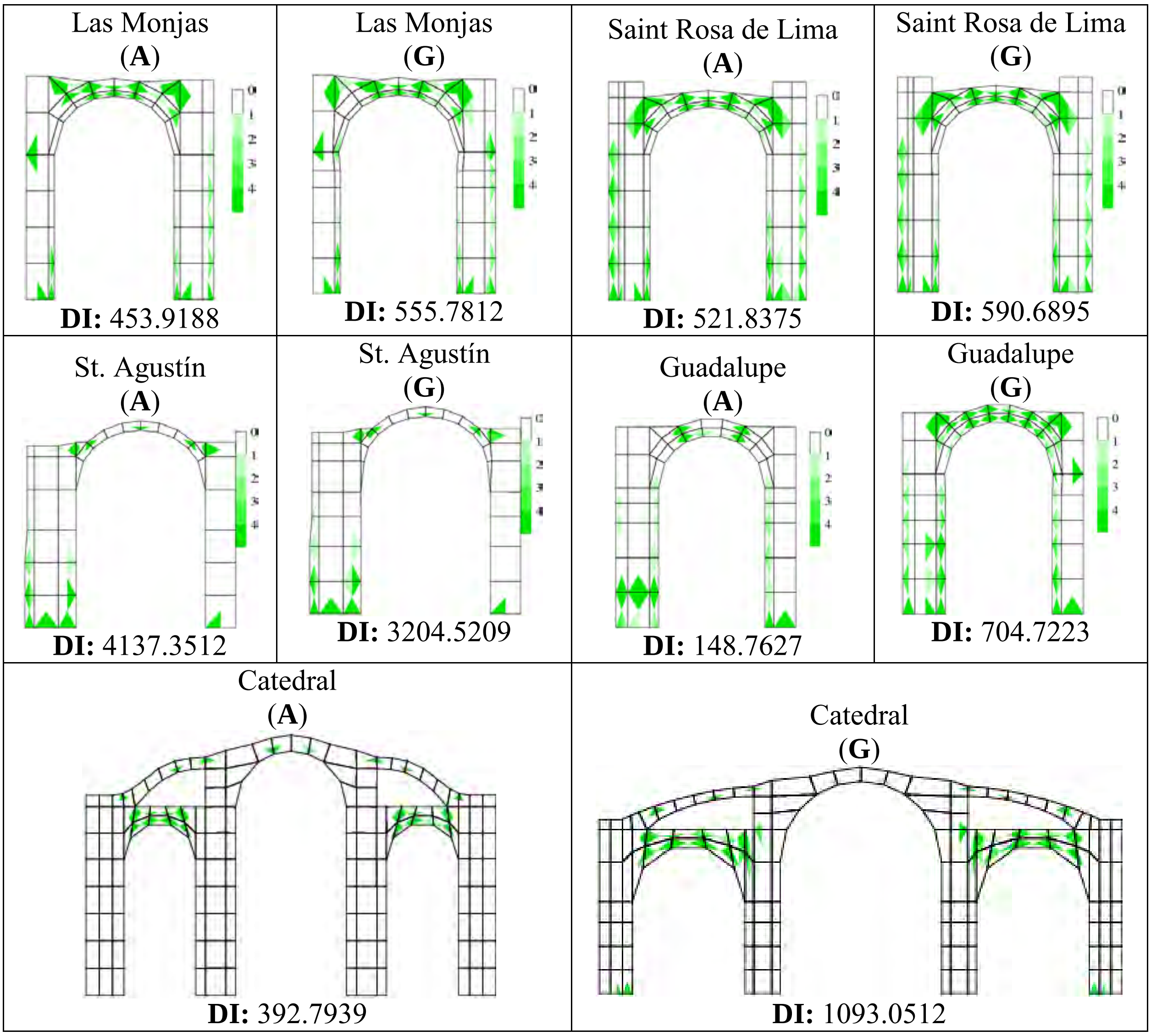
St. Francisco Javier (**A**) **DI:** 478.9105

St. Francisco Javier (**G**) **DI:** 834.1445

El Carmen (**A**) **DI:** 796.2651

El Carmen (**G**) **DI:** 74.9101

Figure 6: Comparison of damage degrees in transversal macroelements.


Las Monjas (**A**) **DI:** 453.9188

Las Monjas (**G**) **DI:** 555.7812

Saint Rosa de Lima (**A**) **DI:** 521.8375

Saint Rosa de Lima (**G**) **DI:** 590.6895

St. Agustín (**A**) **DI:** 4137.3512

St. Agustín (**G**) **DI:** 3204.5209

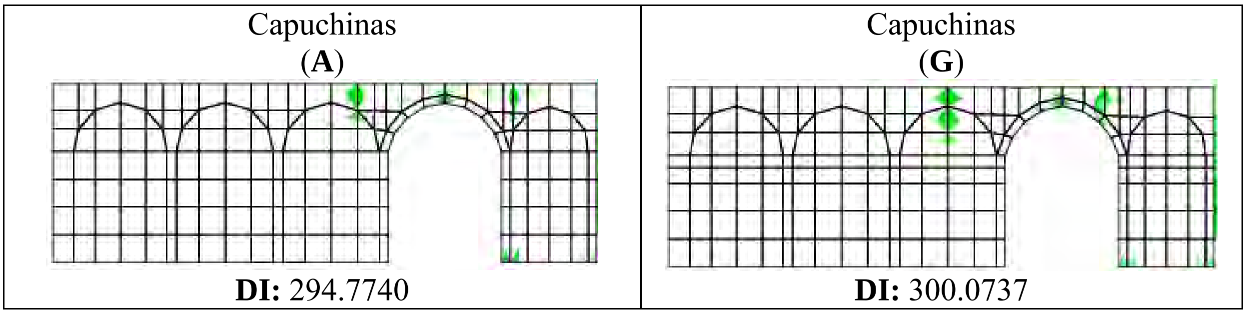
Guadalupe (**A**) **DI:** 148.7627

Guadalupe (**G**) **DI:** 704.7223

Catedral (**A**) **DI:** 392.7939

Catedral (**G**) **DI:** 1093.0512

Figure 6: Comparison of damage degrees in transversal macroelements (Continuation).


Capuchinas (**A**) **DI:** 294.7740

Capuchinas (**G**) **DI:** 300.0737

Figure 7: Comparison of damage degrees in longitudinal macroelements.

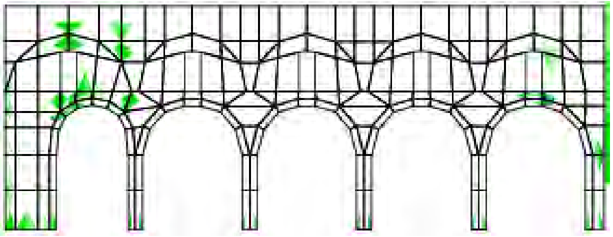
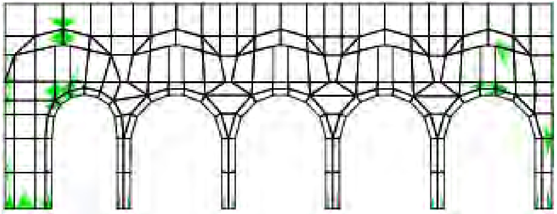
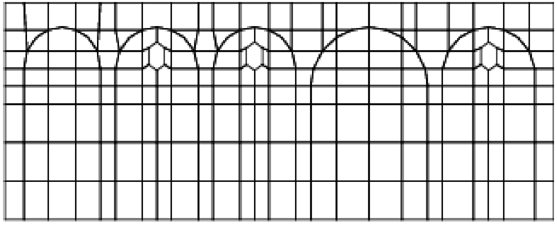
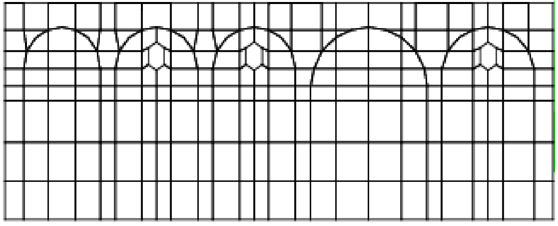
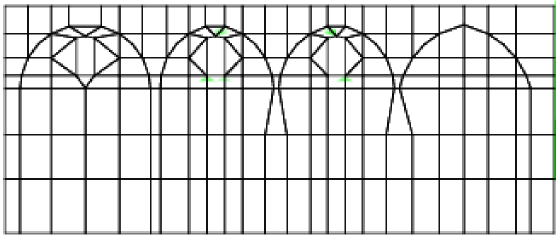
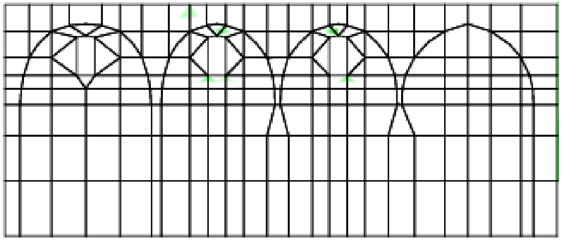
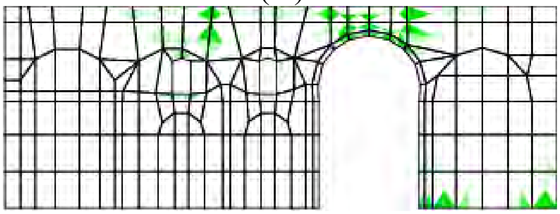
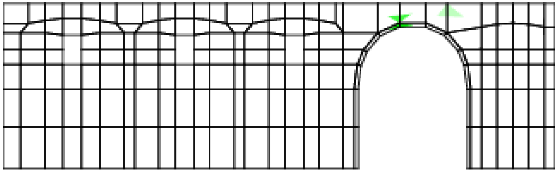
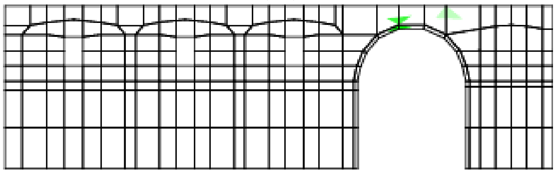
La Cruz (**A**)  **DI:** 3375.9968	La Cruz (**G**)  **DI:** 2307.5566
La Merced (**A**)  **DI:** 178.3522	La Merced (**G**)  **DI:** 160.5693
Franciscano (**A**)  **DI:** 315.9501	Franciscano (**G**)  **DI:** 393.1298
St. José (**A**)  **DI:** 318.3572	St. José (**G**)  **DI:** 291.7139
St. Juan Bautista (**A**)  **DI:** 213.5821	St. Juan Bautista (**G**)  **DI:** 53.9364

Figure 7: Comparison of damage degrees in longitudinal macroelements (Continuation).

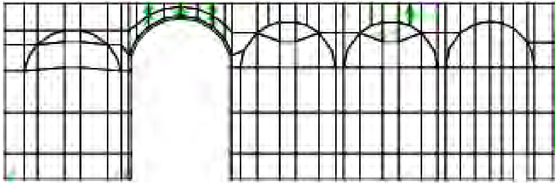
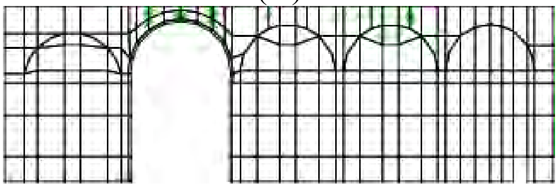
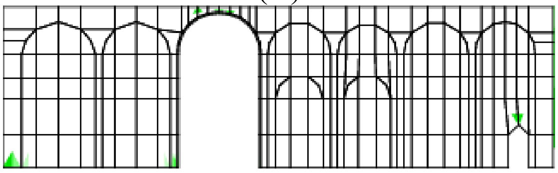
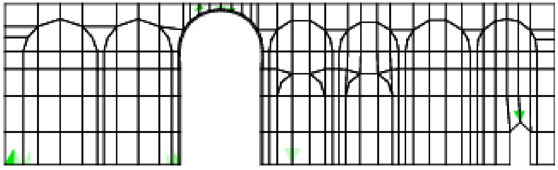
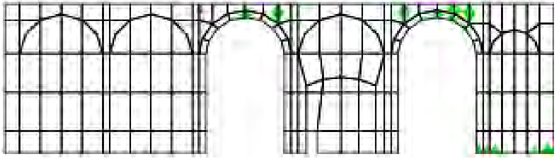
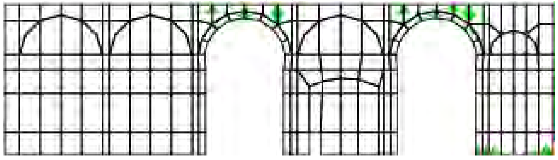
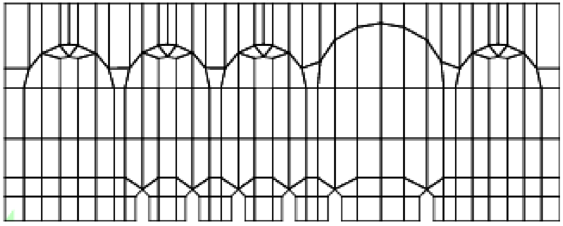
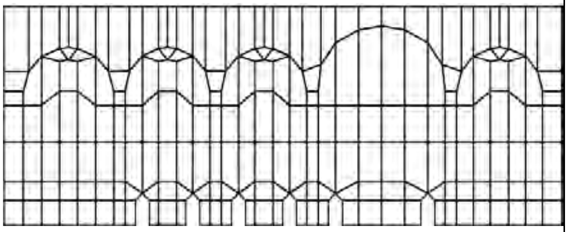
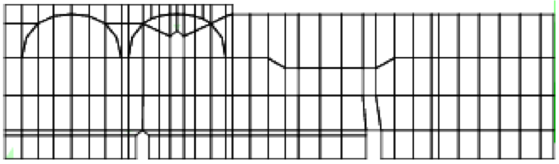
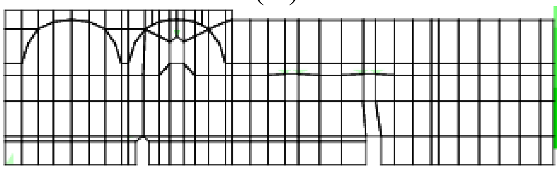
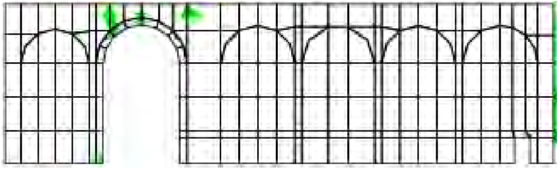
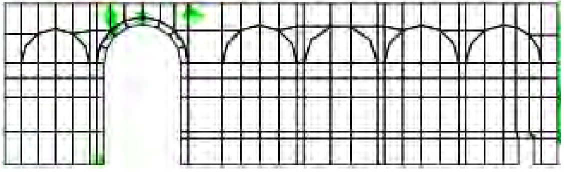
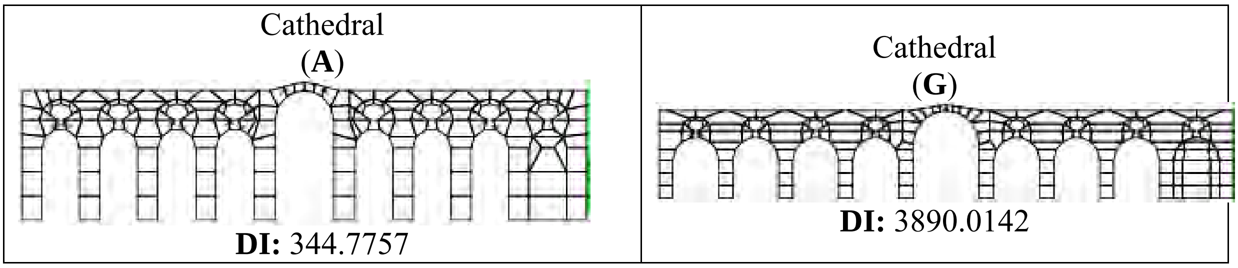
St. Francisco Javier (**A**)  **DI:** 390.5160	St. Francisco Javier (**G**)  **DI:** 605.6152
El Carmen (**A**)  **DI:** 250.1877	El Carmen (**G**)  **DI:** 237.9239
Las Monjas (**A**)  **DI:** 305.725	Las Monjas (**G**)  **DI:** 286.227
Saint Rosa de Lima (**A**)  **DI:** 712.6925	Saint Rosa de Lima (**G**)  **DI:** 191.4083
St. Agustín (**A**)  **DI:** 450.5416	St. Agustín (**G**)  **DI:** 439.4412
Guadalupe (**A**)  **DI:** 326.5443	Guadalupe (**G**)  **DI:** 327.5276

Figure 7: Comparison of damage degrees in longitudinal macroelements (Continuation).



Cathedral (**A**) **DI:** 344.7757

Cathedral (**G**) **DI:** 3890.0142

Figure 7: Comparison of damage degrees in longitudinal macroelements (Continuation).

Figures 6 and 7 shows the degree of damage caused by tension stresses and it can be seen that the majority of macroelements in study suffer damage mainly in the regions of the arches and vaults, as well as the supports of the walls and buttresses. It is important to mention that seismic records that infringed major damages to these macroelements were the July 15th 1996 and the June 15th 1999, with epicenters located in Chilpancingo, Guerrero and Puebla City respectively.

CONCLUSIONS

- The above results shows that the divine proportion is not satisfactory for the major part of the studied transverse macroelements, since that the 76.9% of the modified sections have a damage index greater than that obtained for the sections with actual geometry.
- When comparing the longitudinal macro elements can be seen that the modified sections with the golden ratio has better seismic performance, since only 38.5% of the golden cases have a greater index damage than the actual geometrical sections. It can also be noted in these sections that the damage indices and the damage degrees are quite similar between each other.
- Comparing the macro elements of actual cross sections against the modified ones with the golden ratio it can be seen little differences between damage degrees, except for the churches of St. Agustin and the Franciscan St. Bonaventure where the index rise dramatically, which is mainly attributable to the reduced thickness of the vault.
- For the macroelements of the longitudinal sections, it was observed that the most vulnerable structures are formed by arcades as in the case of La Cruz church. Likewise similar patterns of behavior between actual and golden macroelements was observed, with the exception of Transfiguration Cathedral, which was the only building were the golden proportion was implemented in the longitudinal direction.
- Finally, in the present study only the effect of the golden ratio on the rise/total height (d/a) was analyzed, so in further research the effect of the golden ration over the span/wall thickness will be discussed. Additionally it was observed the necessity of increase the number of buildings at the current research, considering structures of three or more naves in order to improve the obtained results.

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